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Measurement of β_{eff} in the Fast Critical Assembly BFS and Validation of a β_{eff} Computation Code, BETA-K

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Abstract

We have performed two experiments in the fast critical assembly BFS to measure the effective delayed neutron fraction β_{eff} values and compared the results to validate the β_{eff} computation code, BETA-K. Measurements of β_{eff} were carried out in a metallic plutonium core and a metallic uranium core with Cf^{252} source pseudo-reactivity method. Fission integrals and correction factors, which were used to obtain the experimental β_{eff} values, were calculated by using the LMR core design computation code system of KAERI. BETA-K has been developed consistently with the hexagonal Nodal Expansion Method (NEM) and it used delayed neutron data of ENDF/B-VI. By comparing the computed β_{eff} values with the measured ones, we found that the results from BETA-K agreed with the experimental values within the experimental error bound.

Key Words : effective delayed neutron fraction, fast critical assembly BFS, Cf^{252} source pseudo reactivity method, BETA-K, LMR

1. Introduction

The kinetics parameters, like the effective delayed neutron fraction β_{eff}, neutron lifetime, inverse neutron velocity, etc., play an important role in the transient calculations of Liquid Metal Reactors (LMR). In particular, β_{eff} is a key parameter of kinetics and becomes the conversion factor between calculated and measured reactivity. For preparation of these kinetics parameters, an effective delayed neutron fraction computation code consistent with Hex-Z dimensional Nodal Expansion Method (NEM, Lawrence, 1983 and Yang, 1994) has been developed. The accuracy of the computation code, dependent on the neutron solution method and delayed neutron data sets etc., was validated using the measured β_{eff} in BFS facility.

There are many experimental methods (Sakurai, 1998, etc.) to determine β_{eff}, for example, the Cf^{252} source technique, the variance to mean method, the Feynman-α method, the modified Bennett method, etc. Recently, to improve the prediction accuracy of β_{eff} of a fast reactor, an international benchmark experiment (Sakurai, 1998) was performed in the Fast Critical Assembly (FCA). In this benchmark experiment, several participants measured the β_{eff} by their own experiment and interpretation methods, but the

Table 1. Core Characteristics

Characteristics	BFS-55-1	BFS-73-1
Fuel, enrichment	Metallic Plutonium, 10%	Metallic Uranium, 18.5%
Radial blanket	Depleted UO_2	Depleted UO_2
Axial blanket	Depleted U^{238} or UO_2	Depleted UO_2
Dimension (cm)		
Core		
Number of fuel rods	362	425
Equivalence radius	50.95	55.20
Height	102.30	98.30
Thickness of radial blanket	39.50	34.93
Thickness of axial blanket	41.10	49.73

evaluated β_{eff} values were slightly different from each other. Okajima (Okajima, 1998) also found that there were some disagreements among the various delayed neutron data sets in spite of good agreement in the β_{eff} of the FCA experiment. As mentioned above, the accuracy of the evaluated β_{eff} values in the fast critical experiment depends on the experimental method, the calculation of the fission integral and the delayed neutron data set. Therefore, in order to validate the β_{eff} computation code with the results of critical experiments, first of all, we must check the capability of the determination of experimental β_{eff} value.

In this paper, at first, we will introduce the BFS critical facility and Cf^{252} source pseudo-reactivity method for measuring β_{eff} values. And we will describe how to calculate the fission integral and correction factors, etc., which were used to obtain the experimental β_{eff} values. Then, the model of β_{eff} computation code will be presented and we will discuss the computed results by comparing them with the experiment values, including the results of IPPE.

2. Measurement of β_{eff} with the Cf^{252} Pseudo-reactivity Technique

2.1. Brief Description of BFS Critical Facility

Measurements of β_{eff} with the Cf^{252} pseudo-reactivity technique were carried out in BFS-55-1 and BFS-73-1 critical assemblies mounted at the BFS-1 facility of Institute of Physics and Power Engineering (IPPE) of Russia. As part of a cooperation program between Korea Atomic Energy Research Institute (KAERI) and IPPE, the experiment in the second critical assembly was performed jointly, but the experimental data of first core was measured by IPPE alone and provided to KAERI. These assemblies have simple hex-z geometry, as shown in Figure 1. The metallic fuels are located at the center of core enclosed by axial and radial blankets. Axial blankets about 40~50cm in thickness were located at the top and bottom of the core with depleted uranium or uranium dioxide pellets. The radial blanket surrounding the core contains only

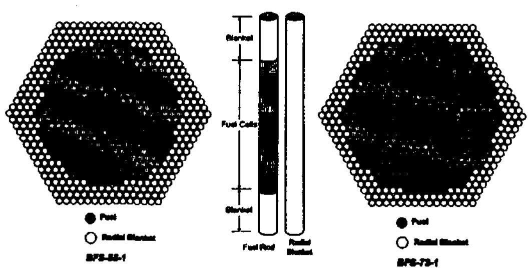


Fig. 1. Overview of the Critical Assembly BFS-55-1 and BFS-73-1

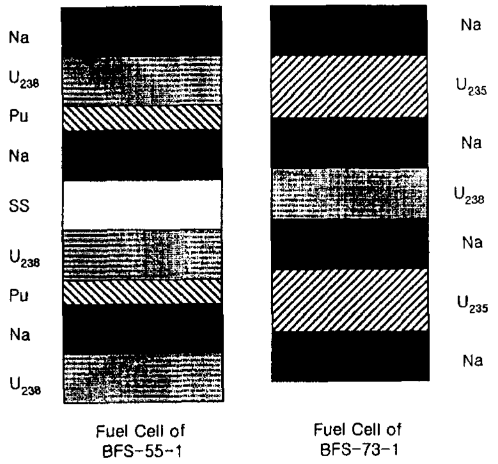


Fig. 2. Arrangement of Pellets in a Fuel Cell

depleted dioxide uranium pellets in the steel tubes. Table 1 gives a summary of the core characteristics. The fuel rod is a steel tube having 13 and 16 fuel cells 50mm in diameter. The fuel cell consists of enriched and depleted metal uranium or metal-plutonium pellets together with sodium pellets as shown in Figure 2. Therefore, BFS-55-1 and BFS-73-1 become about a 10%-enriched plutonium and 18.5%-enriched uranium core, respectively.

2.2. Cf^{252} Source Pseudo-reactivity Method

By measuring the different counting ratios between with and without external fission source in the sub-critical state, one can obtain the β_{eff} values by the following often-quoted form:

$$\beta_{eff} = \frac{S_{cf}}{\rho_\$ \cdot Q_f \cdot \bar{\nu}_{core} \cdot f} \cdot \left(\frac{F^*_{\chi,cf}}{F^*_\chi}\right), \qquad (1)$$

where

$$\bar{\nu}_{core} = \frac{\int_{core} <\nu\Sigma_f\phi> dV}{\int_{core} <\Sigma_f\phi> dV}, \qquad (2)$$

$$f = \frac{\int_{core} <\Sigma_f\phi> dV}{\int_{center} <\Sigma_f\phi> dV} = \frac{\int_{core} <\Sigma_f\phi> dV}{<\Sigma_f\phi_{center}>}, \qquad (3)$$

$$\left(\frac{F^*_{\chi,cf}}{F^*_\chi}\right) = \frac{<\chi_{cf}\phi^*_{center}> \int_{core} <\nu\Sigma_f\phi> dV}{\int_{core} <\chi\phi^*><\nu\Sigma_f\phi> dV}, \qquad (4)$$

$$Q_f = \frac{m_{cf} - m}{m} <\Sigma_f\phi_{center}>, \qquad (5)$$

and S_{cf} means the strength of a Cf^{252} source and m_{cf} and m are the counting rates of the flux level monitor with and without an external neutron source. In the above equations, ϕ^* and $\rho_\$$ denote the neutron importance and the core reactivity in dollars, respectively. Eq. (1) can be rewritten as

$$\beta_{eff} = \frac{S_{cf}}{(\rho_\$ \cdot \Delta m) \cdot \left(\frac{F^i}{m}\right) \cdot \bar{\nu}_{core} \cdot \frac{I_f}{F^i}} \cdot \left(\frac{F^*_{\chi,cf}}{F^*_\chi}\right), \qquad (7)$$

where $\Delta m = m_{cf} - m$. In the Eq. (7), I_f and F^i denote the fission integral of whole core and the absolute fission rate of a principal isotope, i.e. Pu^{239}, per mole at the center of core, respectively, and they are given by

$$I_f = \int_{core} <\Sigma_f\phi> dV, \qquad (8)$$

$$F^i = \frac{<\Sigma^i_f\phi_{center}>}{N^i}. \qquad (9)$$

In Eq. (9), Σ^i_f and N^i are the macroscopic fission cross section and the number density of a principal isotope at the center of core. To obtain the reasonable β_{eff} value, measuring quantities, for example, the sub-criticality and the absolute fission rate, must be normalized to the same flux level or they have to be replaced with other quantities that are independent of the flux level. Considering that the sub-critical reactivity is inversely proportional and the absolute fission rate is proportional to the flux level, we can find that the product of $\rho_s \cdot \Delta_m$ and F^i/m is independent of the flux level. So, Eq. (7) is more convenient than Eq. (1) to understand which quantities are to be measured in these experiments.

Table 2. Normalized Fission rate Distributions in BFS-55-1 in C/E Values

Z(cm) \ R(cm)	2.9	23.7	32.4	41.2	47.2
0.6	1.000/1.000*	0.937/0.995	0.950/0.972	0.945/0.958	0.925/0.948
22.2	1.027/1.029	1.011/1.028	0.991/1.005	0.963/0.988	0.930/0.957
32.2	1.022/ -**	1.004/ -	0.969/ -	0.956/ -	0.937/ -
40.7	0.941/0.917	0.925/0.920	0.906/0.902	0.872/0.877	0.862/0.834

*) Pu^{239}/U^{238}

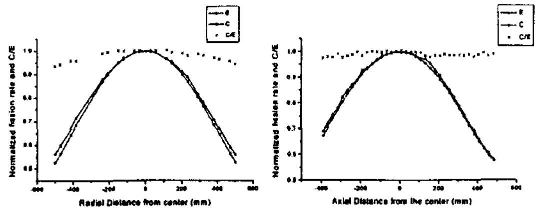
**) Not measured in these positions in the case of measuring of U^{238} fission rate

2.3. Measurements and Calculations

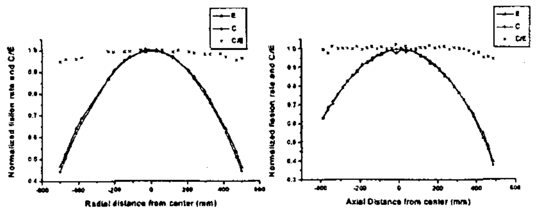
Sub-criticality and counting rates were measured by introducing the well-known Cf^{252} neutron source into the center of the core. Here counting rates can be obtained by just reading the count rate of the small fission chamber located near the center of the core and the sub-criticality $\rho_{\$}$ is also easily determined in dollars by making the sub-critical state with well-calibrated control rods. The measurement of the absolute fission rate of a principal isotope, i.e. Pu239, was detected by introducing a small size fission chamber near the center of the core. Since the small fission chamber inserted near the center of core had been calibrated using an absolute fission chamber deposited by the well-known number of Pu^{239} nuclei, the counting rate of the small fission chamber could be transferred into the absolute fission rate. The fission rate distributions of some principal isotopes, i.e. U^{235}, U^{238}, and Pu^{239} were measured by traveling small pin-type fission chambers in the core along the axial and radial directions, and all measurements were normalized to the center of the core. Since quantities such as $\bar{\nu}_{core}$, f, $(F^{*}_{x,Cf}/\bar{F}^{*}_{x})$ requires many measurements, it is more convenient to calculate them and correct with experimental correction factors. The calculations were carried out with K-CORE system (Kim, 1999), which is an integral nuclear computation system developed by the LMR development team of KAERI for LMR core design, in the homogeneous three-dimensional Hex-Z geometry. K-CORE system adopted DIF3D (Lawrence, 1983), a modern nodal expansion method, as a flux and important solution module, and it used effective 9-group microscopic cross sections generated by TRANSX (MacFarlane, 1983) code from the 80-group master library KAFAX/F22 (Kim, 1997). KAFAX/F22 was generated from JEF-2.2 nuclear data files. In the generation of the effective cross sections, there are two heterogeneity effects if we homogenize BFS fuel cells. First, the background cross section for the material of a pellet is changed because of the change in density and escape probability, which is called the heterogeneous self-shielding effect. Second, the flux will be slightly different in each pellet. Since the second effect may be negligible in the LMR having long range diffusion lengths, the cell heterogeneous effect was estimated by the first effect alone with the heterogeneous option of TRANSX code.

2.4. Experimental Results

The measurements of the fission rate distribution of U^{235}, U^{238} and Pu^{239} in the core are shown in Table 2 and Figure 3 in C/E values. There are good agreements between the



(a) Normalized fission rate distribution of U^{235}



(b) Normalized fission rate distribution of U^{238}

Fig. 3. Normalized Fission Rate Distribution in BFS-73-1

measurements and the calculations in the center of core region but the C/E values become larger as we approach the core boundary.

The measured fission distribution was used to derive an experimental correction factor for the total fission integral in the core. By the assumption that the core can be described by many homogeneous zones, we can obtain the total fission integral of the experiment as

$$I_f^{exp} = I_f^{calc} \cdot \frac{\sum_k V_k \sum_i N_k^i \cdot \bar{F}_{exp,k}^i}{\sum_k V_k \sum_i N_k^i \cdot \bar{F}_{calc,k}^i}, \tag{10}$$

where N^i_k and V_k mean the nuclide number density and the volume of zone k, and $\bar{F}^i_{exp,k}$ and $\bar{F}^i_{calc,k}$ denote experimental and calculated average fission rate of isotope i in zone k, respectively. Since these corrections are at most a few percent and the contribution of the blanket may be negligible in LMR' s (Sakurai, 1998), corrections were made only by using the fission distributions of the active core. After these calculations, we obtained the experimental values for β_{eff} shown in Table 3 with the uncertainties given in Table 4, where the total uncertainty of β_{eff} was estimated by the law of error propagation.

Table 3. Evaluation of the Experimental β_{eff}

Quantities		BFS-55-1	BFS-73-1
Measurement	$S_{Cf}/(\rho_\$ \cdot Q_f)$	9.610±0.271	24.261±0.777
Calculation	$\bar{\nu}_{core}$	2.887	2.524
	$F^*_{x,Cf}/\bar{F}^*_x$	1.830	1.743
	I_f	1504 ± 45	2238 ± 45
	I_f^{calc}	1427	2210
	f_{hetero} 1)	1.037	1.012
	$(C/E)^{2)}_{corr}$	1.017	1.001
β_{eff} (pcm)		404.9 ± 17	748.4 ± 28

1) cell heterogeneity effect
2) C/E correction by Eq. (10)

Table 4. The Sources of Uncertainty in β_{eff} Measurement (1σ)

Source of uncertainty	BFS-55-1	BFS-73-1
S_{Cf}	±1.3%	±1.5%
Q_f	±1.5%	±2.0%
$\rho_\$$	±2.0%	±2.0%
I_f 1)	±3.0%	±2.0%
Total	±4.1%	±3.8%

1) including of the uncertainties of $\bar{\nu}_{core}$ and $F^*_{x,Cf}/\bar{F}^*_x$

3. Validation of the β_{eff} Computation Code

3.1. Equation of β_{eff}

The main difference between delayed neutron fraction and effective delayed neutron fraction results from the weighting of the energy as well as the space dependent adjoint fluxes, especially mathematical adjoint fluxes. Yang (Yang, 1994) and Kim (Kim, 1996) has developed solutions of the mathematical adjoint flux in the hexagonal and rectangular NEM, respectively. Based on their results, it was possible to develop the β_{eff} computation code, BETA-K (Kim, 1998), consistent with NEM in a 3-dimension. The equation of the β_{eff} computation is given by

$$\beta_{eff} = \frac{\int_{core} \sum_{i} \sum_{l=1}^{6} <\phi^{*} \chi_{d}^{il} \times \upsilon_{d}^{il} \Sigma_{f}^{i} \phi > dV}{\int_{core} \sum_{i} <\phi^{*} \chi_{p}^{i} \times \upsilon_{p}^{i} \Sigma_{f}^{i} \phi > dV + \int_{core} \sum_{i} \sum_{l=1}^{6} <\phi^{*} \chi_{d}^{il} \times \upsilon_{d}^{il} \Sigma_{f}^{i} \phi > dV}, \quad (11)$$

where sub-script *d* and *p* denote the delayed and prompt neutrons and super-script *i* and *l* are the isotopic index and the delayed neutron group index, respectively.

3.2. Results of Computations

In these computations, we used ENDF/B-VI delayed neutron data sets and described the core as a Hex-Z geometry with one hexagon per fuel/blanket rod and 5 ~ 9 cm axial node sizes. To ensure the consistency in the computation of β_{eff} values with the code and calculation of fission integral for determining the experiment β_{eff} values, we used the same flux solver and effective cross sections. Therefore, the fluxes and fission cross sections were identical in the generation of β_{eff} values by the computation code and the experiment. Table 5 summaries the results, together with the experimental and computed results by IPPE. In these calculations, IPPE used 26-group ABBN-78 constant system, which is widely used in all neutronic computations in Russia. In the Table 5, all results are within the 1σ error bound. The computation of KAERI agrees well with experiment values. But the result from the Cf^{252} source pseudo-reactivity method of IPPE has a slight discrepancy with the others in BFS-73-1, as happened in the experiment of the uranium core of FCA (Sakurai, 1998). We believe that this discrepancy comes from the difference in the U^{235} fission cross section of between KAFAX/F22 and ABBN-78.

Table 5. Comparison of the Experimental and Computed β_{eff} Values(pcm)

Organization	Method	BFS-55-1	BFS-73-1
IPPE	Cf^{252} method	416±17	720±27
	Rossi-α	-	740±15
	Code Calculation	396	736
KAERI	Cf^{252} method	405±17	748±28
	Code Calculation	406	745

4. Conclusions

We measured β_{eff} values in the two fast critical assemblies, BFS-55-1 and BFS-73-1 by using a Cf^{252} source pseudo-reactivity method. BFS55-1 is a metallic plutonium core and BFS73-1 is a metallic uranium core. Two organizations, KAERI and IPPE, calculated fission integrals and correction factors, which were used to obtain the experimental β_{eff} values, by their own computation systems. KAERI used a LMR core design computation code system with 9 group effective cross section generated from the 80 group master library and IPPE used 22 group cross section sets, ABBN-78. The differences of β_{eff} values between the results of KAERI and IPPE are 11 and 28 pcm in plutonium and uranium core, respectively, but they are within the experimental error bound.

The β_{eff} computation code, BETA-K, has been developed based on the hexagonal nodal expansion method and validated by comparing the computed β_{eff} values with the measured ones. To have a consistency in the computation of β_{eff} values, the same flux solver, effective neutron cross sections and core geometry were used in the calculations of fission integrals and the computations with BETA-K. Within the experimental error bound, the results from BETA-

K agreed well with the experimental values, including the results of IPPE.

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