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Effect of Rotary Inertia of Concentrated Masses on the Natural Vibration of Fluid Conveying Pipes

Myeong-Gie Kang
Andong National University
388 Songchun-dong, Andong-city, Kyungbuk 760-749, Korea

mgkang@andong.ac.kr

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Abstract

Effects of the rotary inertia of concentrated masses on the natural vibrations of fluid conveying pipes have been studied by theoretical modeling and computer simulation. For analysis, two boundary conditions for pipe ends, simply supported and clamped-clamped, are assumed and Galerkin's method is used for transformation of the governing equation to the eigenvalues problem and the natural frequencies and mode shapes for the system have been calculated by using the newly developed computer code. Moreover, the critical velocities related to a system instability have been investigated. The main conclusions for the present study are (1) Rotary inertia gives much change on the higher natural frequencies and mode shapes and its effect is visible when it has small value, (2) The number and location of nodes can be changed by rotary inertia, (3) By introducing rotary inertia, the second natural frequency approaches to the first as the location of the concentrated mass approaches to the midspan of the pipe, and (4) The critical fluid velocities to initiate the system unstable are unchanged by introduction of rotary inertia and the first three velocities are π, 2π, and 3π for the simply supported pipe and 2π, 8.99, and 12.57 for the clamped-clamped pipe.

Key Words : rotaryinertia, concentrated mass, flow induced vibration

1. Introduction

The flow induced vibration in industry fields has been studied for a long time since it always contain a possibility of severe accidents by the several types of vibrations related to a fluid and structure interaction. A fluid flowing through a pipe can impose pressure on the pipe walls and deflect the pipe. Special interest is on a nuclear power plant which consists of many pipe lines containing high velocity fluid. The study on the flow-induced vibrations in nuclear power plants has been focused both on design and maintenance. The main purposes are (1) to supply proper supports to reduce deflection of a system and (2) to find out the fluid velocities which causing system instability. The selection of the proper support locations is closely related to the analysis on the natural frequencies and mode shapes of a system. Once proper points are selected, this can reduce fatigue-related pipe failures due to vibration and, of course, preclude anticipated consecutive failures due to broken parts. While the large deflection of a pipe is related to the fatigue failures, system instability is directly related to the abrupt pipe break. If a piping system gets a critical fluid velocity, pipe gets broken suddenly and discharges fluid of high pressure and high temperature into the environment. Moreover, broken pipes can cause secondary pipe failure due to pipe whipping. Such that, these kinds of failures always have the possibility of damaging staffs and relevant piping systems.

Since nuclear power plants consist of many pipe lines having heavy valves, connections, and flow regulatory parts which can be modeled as the concentrated masses, to investigate the effect of the concentrated masses on the vibrations is very important. Above all the studies, knowledge on the natural frequencies and mode shapes of a system is the most important and is the basis of vibration and noise analyses. Moreover, the stability of the fluid conveying pipes is of practical importance since the natural frequencies of a pipe generally decrease with increasing the fluid velocity. Such that, to study about the correlation between concentrated masses and natural vibrations (including system instability) is very important in the nuclear power plants which having many pipe lines containing high velocity fluid.

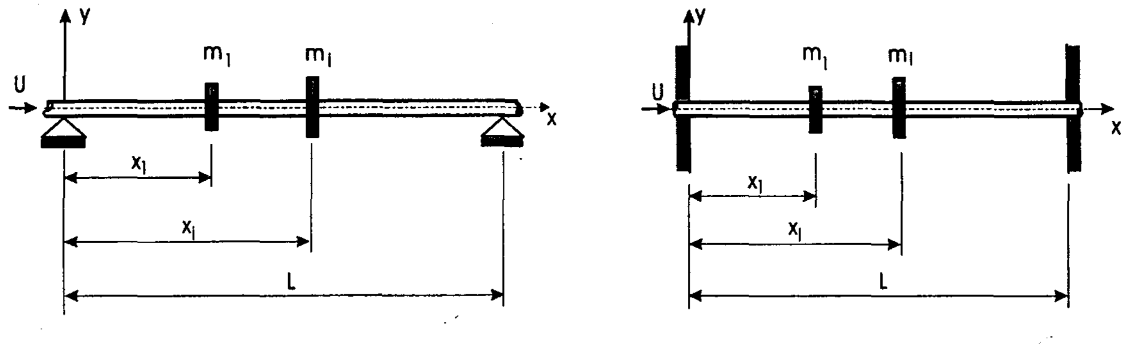
Housner [1] was the first who had derived the correct governing equation for the motion of a fluid conveying pipe. For several decades, many investigators have studied about this problem by assuming several boundary conditions. Although the vibration analysis of a pipe which having some concentrated masses without a fluid flow had been studied early [2-5], the vibration analysis of that case when fluid flows through the pipe was not until 1970 when Hill and Swanson [6] published their paper on ASME. Hill and Swanson [6] investigated the effect of concentrated masses on the instability of the fluid conveying cantilever pipe. According to their results, concentrated masses can reduce the critical velocities occurring system instability. Since then, effects of concentrated masses have been studied in the flow-induced vibration field. Chen and Jendrzejczyk [7] experimentally studied the natural frequencies, mode shapes, and critical velocities for the fluid conveying cantilever pipe with a concentrated mass at the end of the pipe. Wu and Raju [8] proposed that a concentrated mass installed at the midspan of the simply supported pipe could change natural frequencies and mode shapes. Although some interesting papers [4,5,9] regarding the rotary inertia effect of the concentrated masses were published, no one introduced it into the flow-induced vibration field until Kang et al. [10] published some preliminary results on the natural vibrations for the simply supported pipe.

Therefore, the main objectives of the present paper are (1) to develope a new governing equation which containing the effect of the rotary inertia of concentrated masses attached on the fluid conveying pipe, (2) to develope a new computer code which can supply proper ways to solve eigenvalues problem, and (3) to analysis the effect of the rotary inertia of concentrated masses on the vibrations (natural frequencies and mode shapes) and instability of the given system.

For the analysis simply supported and clamped-clamped pipe boundary conditions and incompressible fluid flow have been selected. Moreover, the approach according to Hill and Swanson [6] and Meirovitch [11] is used to develope a new computer code.

2. Theory and Mathematical Development

The well known governing equation [1,12] for



(a) Simply Supported Pipe (b) Clamped-Clamped Pipe

Fig. 1. Schematic Diagram of the Fluid Conveying Pipes

the pipes conveying incompressible fluid becomes

$$EI\frac{\partial^4 y}{\partial x^4} + 2m_f U\frac{\partial^2 y}{\partial t\partial x} + m_f U^2\frac{\partial^2 y}{\partial x^2} + (m_f + m_t)\frac{\partial^2 y}{\partial t^2} = 0 \quad (1)$$

According to Pan [4] and Sato et al. [5], the effect of concentrated masses placed at $x=x_i$ can be written as follows :

$$\sum_{i=1}^{M} m_i\delta(x-x_i)\frac{\partial^2 y}{\partial t^2} - \frac{\partial}{\partial x}\left\{\sum_{i=1}^{M} J_i\delta(x-x_i)\frac{\partial^3 y}{\partial x\partial t^2}\right\} \quad (2)$$

The first equation containing m_i represents the inertia force due to lateral acceleration of the concentrated masses while the second equation containing J_i represents the rotary inertia force due to the angular acceleration of the concentrated mass. If a mass m_i is located at a distance x_i, the rotary inertia term J_i can be expressed as $m_i r_i^2$. Here, r_i is the radius of gyration of the concentrated mass, m_i, about an axis through its center of mass [13].

Finally, the governing equation for the pipe conveying incompressible fluid and having several concentrated masses can be expressed as followings;

$$EI\frac{\partial^4 y}{\partial x^4} + 2m_f U\frac{\partial^2 y}{\partial t\partial x} + m_f U^2\frac{\partial^2 y}{\partial x^2} + \left\{m_f + m_t + \sum_{i=1}^{M} m_i\delta(x-x_i)\right\}\frac{\partial^2 y}{\partial t^2}$$

$$-\frac{\partial}{\partial x}\left\{\sum_{i=1}^{M} J_i\delta(x-x_i)\frac{\partial^3 y}{\partial x\partial t^2}\right\} = 0 \quad (3)$$

The boundary conditions for the two fluid conveying pipes shown in Fig. 1 are as follows [14]:

- simply supported pipe; y(0, t) = y(L, t) = 0 and

$$\frac{\partial^2 y}{\partial x^2}(0, t) = \frac{\partial^2 y}{\partial x^2}(L, t) = 0$$

- clamped-clamped pipe; y(0, t) = y(L, t) = 0 and

$$\frac{\partial y}{\partial x}(0, t) = \frac{\partial y}{\partial x}(L, t) = 0$$

Here, y is the pipe displacement, x the axial coordinate, t the time, m_f and m_t the mass per unit length of the fluid and pipe, respectively ; U the constant flow velocity, m_i and J_i the concentrated mass and its rotary inertia placed at $x=x_i$, respectively ; M number of the concentrated masses, δ Dirac delta function, and EI the flexural rigidity of the uniform pipe. To derive the governing equation Euler-Bernoulli type pipe, small lateral motion about the equilibrium position, neglect the effect of the gravity, uniform pipe except concentrated masses, neglect rotary inertia and shear force of the pipe itself, and steady state uniform flow are assumed.

Introducing dimensionless parameters, eq. (3) becomes

$$\frac{\partial^4\eta}{\partial\xi^4}+2u\beta^{\frac{1}{2}}\frac{\partial^2\eta}{\partial\xi\partial\tau}+u^2\frac{\partial^2\eta}{\partial\xi^2}+\left\{1+\sum_{i=1}^{M}a_i\delta(\xi-\xi_i)\right\}\frac{\partial^2\eta}{\partial\tau^2}$$
$$-\frac{\partial}{\partial\xi}\left\{\sum_{i=1}^{M}\mu_i\delta(\xi-\xi_i)\frac{\partial^3\eta}{\partial\xi\partial\tau^2}\right\}=0 \tag{4}$$

where the dimensionless parameters are

$$a_i=\frac{m_i}{L(m_f+m_t)}\quad \beta=\frac{m_f}{m_f+m_t}\quad \eta=\frac{y}{L}\quad \mu_i=\frac{J_i}{(m_f+m_t)L^3}$$

$$\tau=\left[\frac{EI}{m_f+m_t}\right]^{\frac{1}{2}}\frac{t}{L^2}\quad u=\left[\frac{m_f}{EI}\right]^{\frac{1}{2}}UL\quad \xi=\frac{x}{L}\quad \xi_i=\frac{x_i}{L}$$

$$\omega=\left[\frac{m_f+m_t}{EI}\right]^{\frac{1}{2}}L^2\Omega$$

The motion $\eta(\xi,\tau)$ can be written as follows

$$\eta(\xi,\tau)=a_m(\tau)\Phi_m(\xi) \tag{5}$$

By inserting eq. (5) into eq. (4), the governing equation becomes

$$\left[\left\{1+\sum_{i=1}^{M}a_i\delta(\xi-\xi_i)\right\}\Phi_m(\xi)-\sum_{i=1}^{M}\mu_i\delta'(\xi-\xi_i)\Phi_m'(\xi)\right.$$
$$\left.-\sum_{i=1}^{M}\mu_i\delta(\xi-\xi_i)\Phi_m''(\xi)\right]\ddot{a}_m(\tau)+2u\beta^{\frac{1}{2}}\Phi_m'(\xi)\,\dot{a}_m(\tau)$$
$$+\{\Phi_m''''(\xi)+u^2\Phi_m''(\xi)\}a_m(\tau)=0 \tag{6}$$

3. Computational Analysis Procedure

Galerkin' s method [6,11] says that $\Phi_m(\xi)$ can be represented by the superposition of $\phi_m(\xi)$ which is the mode shape function of the pipe without fluid and the concentrated masses. Then eq. (5) becomes

$$\eta(\xi,\tau)=\sum_{m=1}^{\infty}a_m(\tau)\phi_m(\xi) \tag{7}$$

Introducing the orthogonality of the functions, multiplying eq. (6) by $\Phi_m(\xi)$, and integrating it about ξ from $\xi=0$ to $\xi=1$, we finally obtain the governing equation in the matrix form

$$[A_{mn}]\ddot{a}_m(\tau)+[B_{mn}]\dot{a}_m(\tau)+[C_{mn}]a_m(\tau)=0 \tag{8}$$

The equations of $\phi_m(\xi)$ [14] and matrices of [A], [B], and [C] are shown in Table 1 and 2, respectively.

Rearranging eq. (8) we get

$$[M]\{\dot{p}(\tau)\}+[K]\{p(\tau)\}=\{0\} \tag{9}$$

where,

$$\{p(\tau)\}=\begin{Bmatrix}\{\dot{a}_m(\tau)\}\\ \{a_m(\tau)\}\end{Bmatrix}\quad [M]=\begin{bmatrix}[0]&[A]\\ [A]&[B]\end{bmatrix}\quad [K]=\begin{bmatrix}-[A]&[0]\\ [0]&[C]\end{bmatrix}$$

Multiplying eq (9) by $[M]^{-1}$, we get

$$[I]\{\dot{p}(\tau)\}+[M]^{-1}[K]\{p(\tau)\}=\{0\} \tag{10}$$

Since $a_m(\tau)=e^{j\omega\tau}\Psi$ (where, $j=\sqrt{-1}$ and Ψ are constants),

$$\{p(\tau)\}=\begin{Bmatrix}\{\dot{a}_m(\tau)\}\\ \{a_m(\tau)\}\end{Bmatrix}=e^{j\omega\tau}\{\Psi\} \tag{11}$$

Introducing eq. (11) into eq. (10),

$$j\omega[I]\{\Psi\}+[M]^{-1}[K]\{\Psi\}=\{0\} \tag{12}$$

$$[D]\{\Psi\}-\nu[I]\{\Psi\}=\{0\} \tag{13}$$

where,

$$\nu=-j\omega\quad [D]=[M]^{-1}[K]=\begin{bmatrix}[A]^{-1}[B]&[A]^{-1}[C]\\ -[I]&[0]\end{bmatrix}$$

Table 1. Mode Shapes, $\phi_m(\xi)$ for the Two Pipe Boundary Conditions [14]

Ends condition	λ_m ; m=1,2,3,…	Mode shape $\phi_m(\xi)$	σ_m ; m=1,2,3,…
simply supported	$m\pi$	$\sin(\lambda_m\xi)$	-
clamped-clamped	m=1, λ_m=4.73004074 m=2, λ_m=7.85320462 m=3, λ_m=10.9956079 m=4, λ_m=14.1371655 m=5, λ_m=17.2787597 m>5, $\lambda_m=(2m+1)\frac{\pi}{2}$	$\cosh(\lambda_m\xi) - \cos(\lambda_m\xi)$ $- \sigma_m\{\sinh(\lambda_m\xi) - \sin(\lambda_m\xi)\}$	m=1, σ_m=0.982502215 m=2, σ_m=1.000777312 m=3, σ_m=0.999966450 m=4, σ_m=1.000001450 m=5, σ_m=0.999999937 m>5, σ_m= 1.0

Table 2. Matrices of [A], [B], and [C]

Matrix	Simply supported pipe	Clamped-clamped pipe
$[A_{mn}]$	$\delta_{mn} + \sum_{i=1}^{M}\{\alpha_i\phi_m(\xi_i)\phi_n(\xi_i) + \mu_i\phi_m'(\xi_i)\phi_n'(\xi_i)\}$	$\delta_{mn} + \sum_{i=1}^{M}\{\alpha_i\phi_m(\xi_i)\phi_n(\xi_i) + \mu_i\phi_m'(\xi_i)\phi_n'(\xi_i)\}$
$[B_{mn}]$	$2u\beta^{\frac{1}{2}}\frac{\lambda_m\lambda_n[1-(-1)^{m+n}]}{\lambda_n^2-\lambda_m^2}$	$8u\beta^{\frac{1}{2}}\frac{\lambda_n^2\lambda_m^2\{1-(-1)^{m+n}\}}{\lambda_n^4-\lambda_m^4}$
$[C_{mn}]$	$(\lambda_m^4 - u^2\lambda_m^2)\delta_{mn}$	$4u^2\frac{\lambda_n^2\lambda_m^2(\sigma_n\lambda_n-\sigma_m\lambda_m)\{1+(-1)^{m+n}\}}{\lambda_n^4-\lambda_m^4}$ $+ \{\lambda_m^4 + u^2\lambda_m\sigma_m(2-\lambda_m\sigma_m)\}\delta_{mn}$

Equation (13) can, therefore, be written as

$$[f(\nu)]\{\Psi\} = \{0\} \qquad (14)$$

where,

$$[f(\nu)] = [D] - \nu[I]$$

The eigenvalues problem (14) has a nontrivial solution only if the characteristic determinant, i.e. the determinant of the matrix $[f(\nu)]$, vanishes [11]:

$$|f(\nu)| = 0 \qquad (15)$$

Generally, eigenvalues of the determinant have its real and imaginary parts as follows:

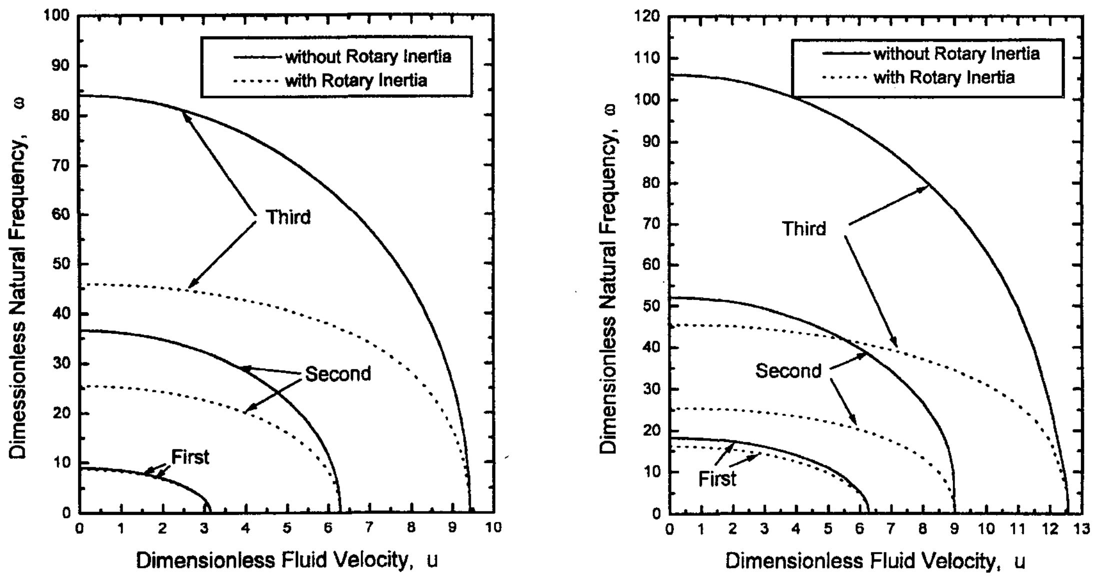
$$\omega = \mathrm{Re}(\omega) + j\mathrm{Im}(\omega) \qquad (16)$$

The real component, Re (ω), corresponds to the frequency of oscillation, whereas the imaginary component, Im (ω), is associated with stability of the system. Im (ω) > 0 signifies damping motions, while Im (ω) < 0 amplifying motions. Hence, the point of crossing the Re(ω) axis to negative Im (ω) represents the threshold value of flow velocity for the onset of unstable motions. It is seen that the system is subject to a large number of stabilities, in the regions over which Im (ω) < 0, by buckling if Re(ω)=0, and by flutter if Re(ω) $\neq$ 0 [12]. The condition of neutral stability is one of dynamic equilibrium where, in the course of one cycle of oscillation, the energy transfer from fluid to pipe and vice versa exactly balance. When the energy of the fluid stream exceeds that of the pipe, the amplitude increases without limit: in the opposite case oscillations are damped.

Table 3. The Effect of m on the Results*

Number of m	Natural frequencies					
	Simply supported pipe			Clamped-clamped πpe		
	First	Second	Third	First	Second	Third
3	7.6176	25.5972	56.8443	15.8445	28.1210	63.0836
5	7.6162	25.1329	49.8033	15.7346	26.3881	49.6429
7	7.6156	24.9385	45.7688	15.6929	25.6035	45.6103
8	7.6156	24.7748	45.7421	15.6878	25.1641	45.3679
9	7.6155	24.7730	45.4885	15.6871	25.1641	45.2096

* (conditions; $u=1.5$, $\alpha_1=0.2$, $\alpha_2=0.1$, $\beta=0.2$, $\xi_1=0.3$, $\xi_2=0.5$, $\mu_1=0.018$, $\mu_2=0.025$)

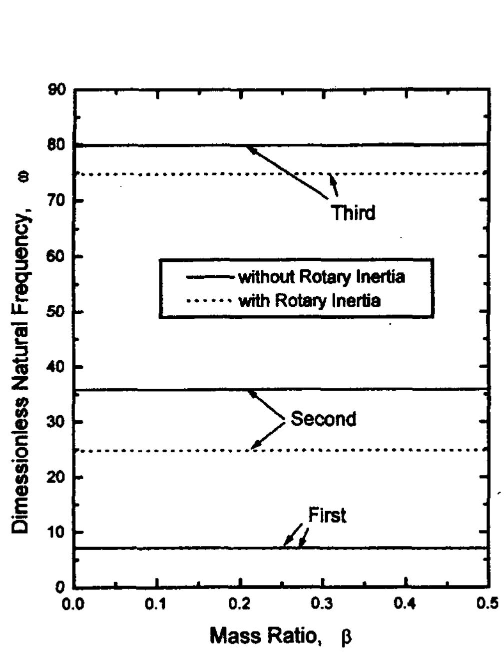


(a) Simply Supported Pipe (b) Clamped-Clamped Pipe

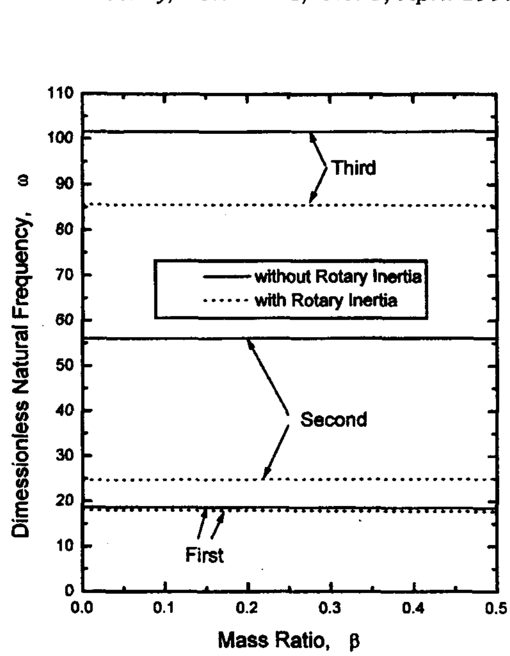
Fig. 2. Natural Frequency Variation Due to Fluid Velocity Change; $\alpha_1=0.2$, $\alpha_2=0.1$, $\beta=0.2$, $\mu_1=0.018$, $\mu_2=0.025$, $\xi_1=0.3$, $\xi_2=0.5$

4. Results and Discussion

The natural frequencies, mode shapes, and instability of the system have been calculated by using the developed computer code. The needed values can be achieved by superposing the mode shape $\phi_m(\xi)$ more than 7. As shown in Table 3, values of the first, second, and third natural frequencies have similar values, respectively, if m is larger than 7. The present paper will discuss some results obtained by assuming m as 9.



(a) Simply Supported Pipe



(b) Clamped-Clamped Pipe

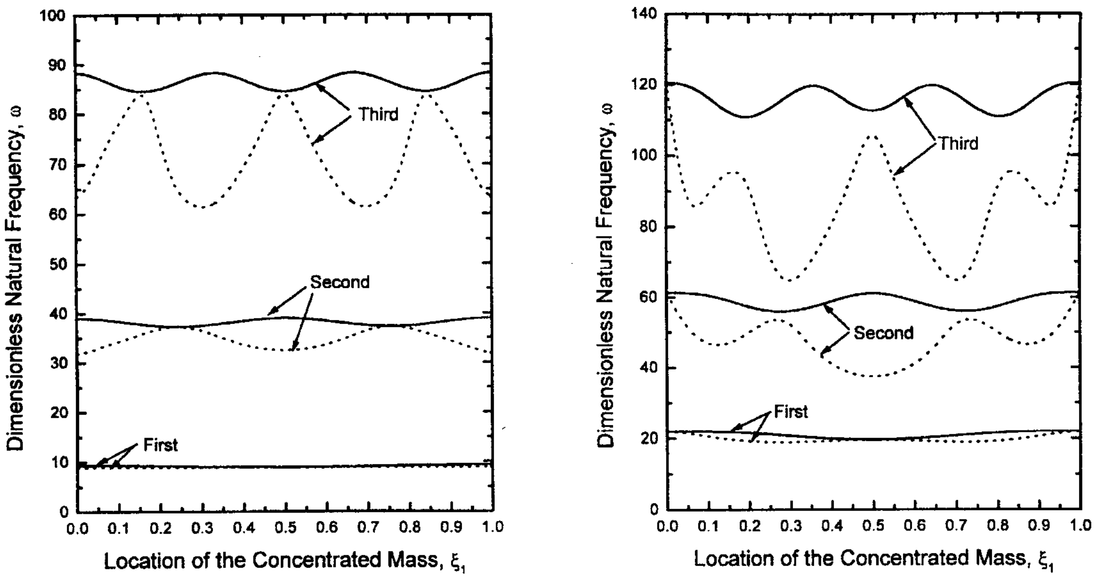
Fig. 3. Natural Frequency Variation Due to Mass Ratio Change; u=2.0, α_1=0.1, α_2=0.1, μ_1=0.004, μ_2=0.025, ξ_1=0.2, ξ_2=0.5

4.1. Natural Frequency

Figure 2 shows variation of the natural frequencies of fluid conveying pipes with two concentrated masses. Through the analysis the first three natural frequencies have been found as a function of the fluid velocity. The dotted lines are results of the system with rotary inertia while solid lines are without rotary inertia. Introduction of rotary inertia gives much change for the second and third natural frequencies while it gives relatively small effect on the first natural frequency. As shown in the figure, the natural frequencies are decreased and become zero with increasing the fluid velocity. When the dimensionless fluid velocity gets π, 2π, and 3π for the simply supported pipe and 2π, 8.99, and 12.57 for the clamped-clamped pipe the first three natural frequencies have zero value. Once a natural frequency arrives in zero value, instability of the system begins.

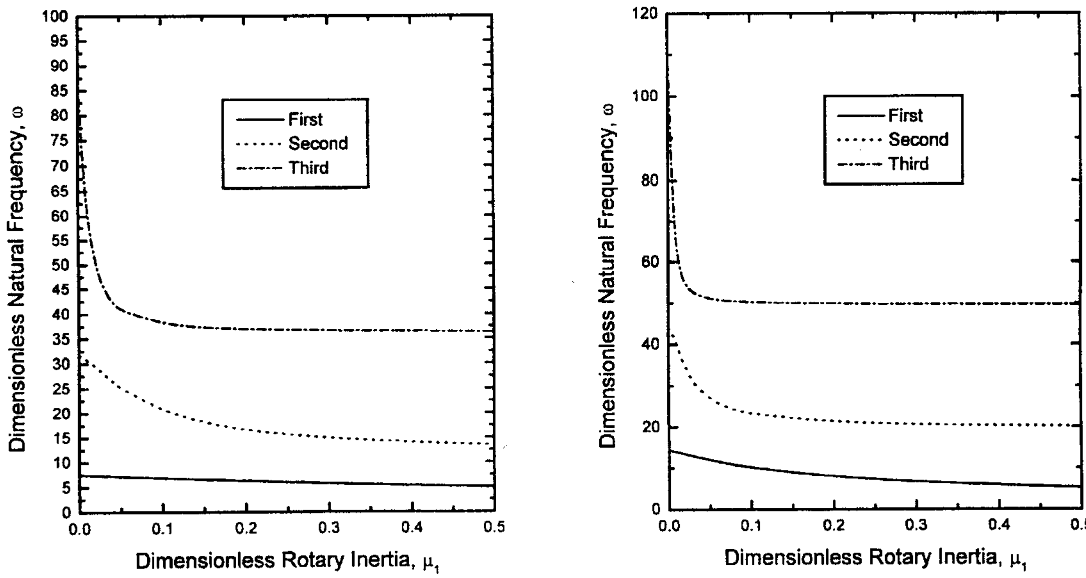
The natural frequencies with or without rotary inertia are shown in Fig. 3 as a function of the mass ratio, β. With assuming two concentrated masses at 0.2L and 0.5L of the pipe, respectively, the mass ratio β between pipe and fluid has been changed from 0.0 to 0.5 to find out variations in the natural frequencies. According to the results, the mass ratio gives no visible change to the natural frequencies. For the case, inclusion of the rotary inertia into the analysis results in much change on the higher natural frequencies.

The change on the location of a concentrated mass, ξ_i, gives much variation on the natural frequencies. The effect of ξ_i change on the first three natural frequencies for the two pipe boundary conditions is shown in Fig. 4. For the analysis, one concentrated mass (α_1=0.1) is



(a) Simply Supported Pipe (b) Clamped-Clamped Pipe

Fig. 4. Natural Frequency Variation Due to Change of the Location of the Concentrated Mass; u=1.0, α_1=0.1, β=0.5, μ_1=0.01(— ; without rotary intertia, … ; with rotary inertia)



(a) Simply Supported Pipe (b) Clamped-Clamped Pipe

Fig. 5. Natural Frequency Variation Due to Change of the Rotary Inertia; u=0.5, α_1=1.0, β=0.2, ξ_1=0.3

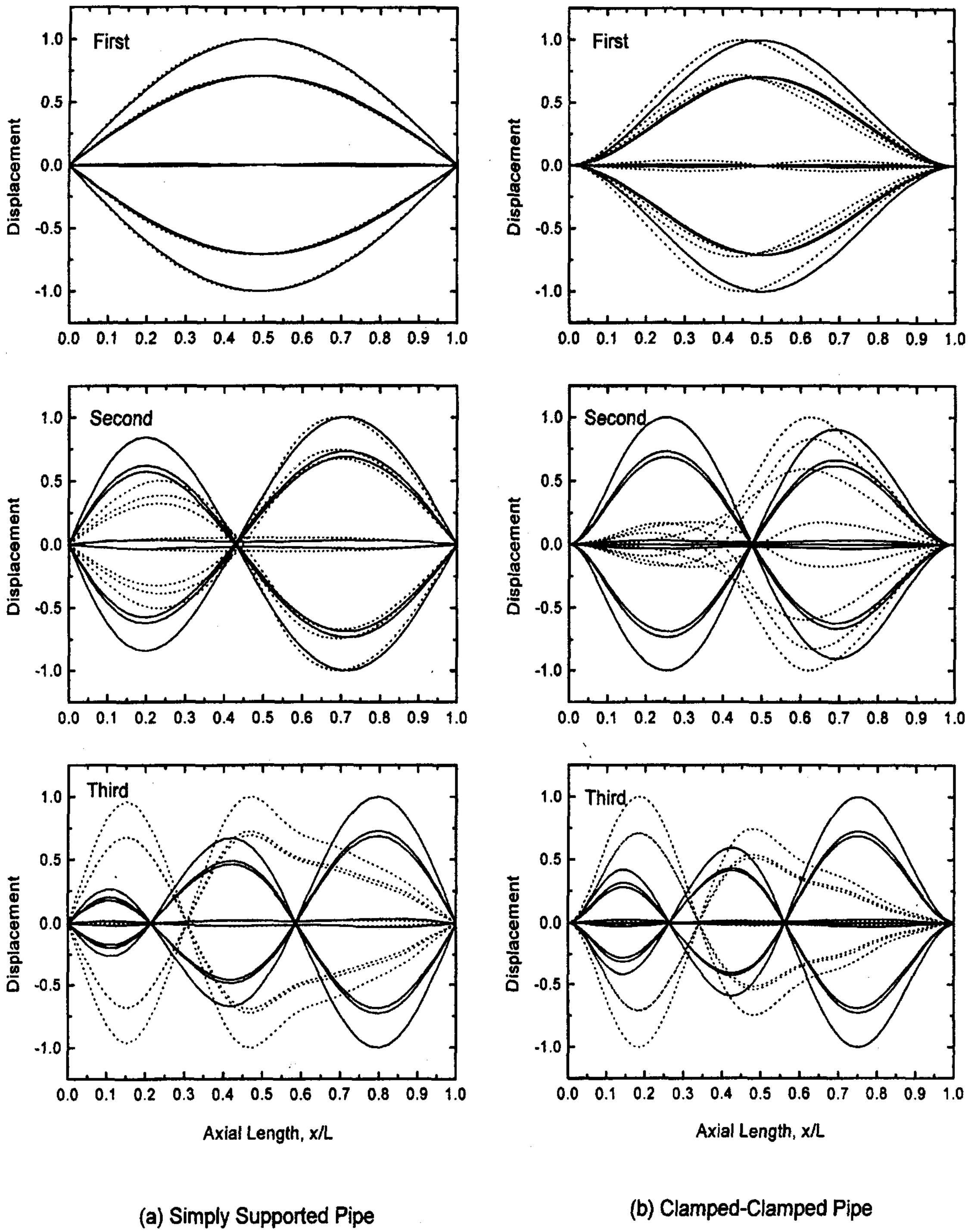


Fig. 6. Natural Mode Shapes; u=0.5, α_1=1.0, α_2=0.2, β=0.4, μ_1=0.01, μ_2=0.05, ξ_1=0.1, ξ_2=0.5 (— ; without rotary intertia, ··· ; with rotary inertia)

assumed and its location has been changed from $\xi_1=0.0$ to 1.0. To verify the effect of the rotary inertia of the concentrated mass, μ_1, on the natural frequencies $\mu_1=0$ and 0.01 are assumed for the cases of without and with rotary inertia, respectively. According to Fig. 4, it is verified that rotary inertia greatly change natural frequencies. When α_1 is located at $\xi_1=0.5$ the second natural frequencies for the simply supported and clamped-clamped pipes have been reduced to 83.2 % and 60.6 % of the natural frequencies without rotary inertia, respectively. By considering rotary inertia into the analysis the second natural frequency approaches to the first natural frequency as α_1 approaches to the midspan of the pipe.

Figure 5 shows the correlation between the value of the rotary inertia, μ_1, and the first three natural frequencies. For the analysis, a concentrated mass ($\alpha_1=1.0$) is assumed to be at 0.3L (i.e., $\xi_1=0.3$) of the pipe and the rotary inertia of it is changed from $\mu_1=0.0$ to 0.5. As shown in the figure, the consideration of the rotary inertia gives much change on the natural frequencies. Its effect on the change of the natural frequencies is visible as μ_1 has small value (i.e., less than 0.1 for the present case). Further increase in μ_1 gives relatively small effect on the natural frequencies in comparison with the former case which having small μ_1 value.

4.2. Natural Mode Shape

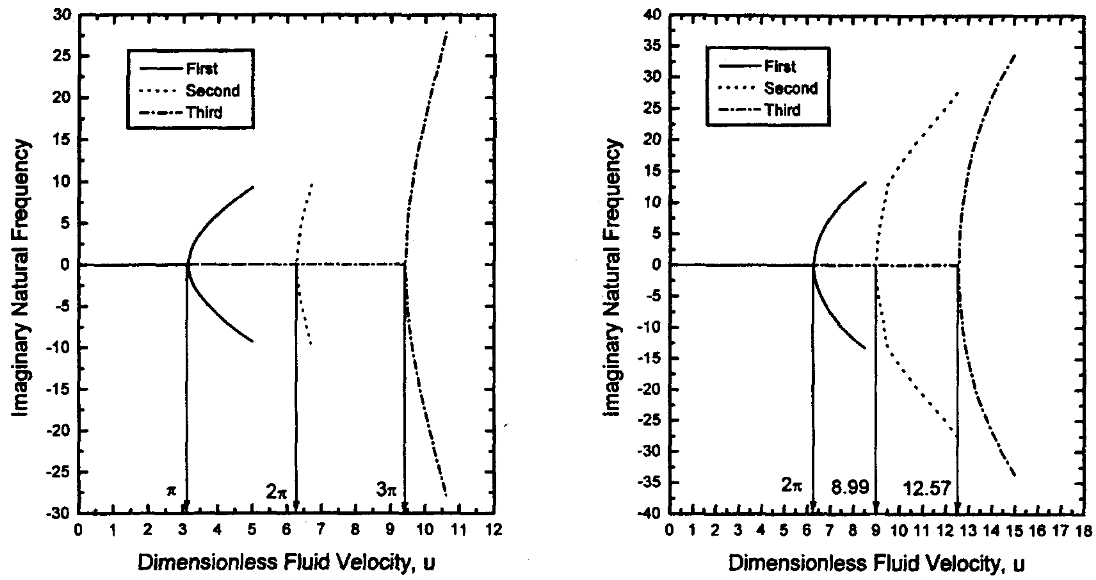
Figure 6 is the natural mode shapes of the system for the case of having two concentrated masses ($\alpha_1=1.0$ and $\alpha_2=0.2$). The heavier one is located at 0.1L and the lighter one is at 0.5L. In the figure, solid lines are mode shapes of the system without rotary inertia and dotted lines are with rotary inertia. As shown in the figure, rotary inertia gives small change on the first mode shape while it gives much change on the second and third mode shapes. By introducing rotary inertia, the number of nodes and its location can be changed. There is no fixed node at the second mode shape of the clamped-clamped pipe when the rotary inertia effect is included into the analysis. The number of the node is also changed at the third mode shape with considering rotary inertia.

4.3. Stability Criteria

A steady, high-velocity flow through a thin walled pipe can buckle or deform the pipe and these deflections are called instabilities of fluid conveying pipes. The stability of fluid conveying pipes is of practical importance because the natural frequency of a pipe generally decreases with fluid velocity increase. The type of instability depends on the end conditions of the pipe. Pipes supported or clamped at both ends bow out and buckle when the flow velocity exceeds the critical velocity.

Figure 7 shows the variation of the imaginary natural frequencies due to the fluid velocity variation. The instability begins if the imaginary natural frequency gets negative. According to the figure, the fluid velocities for the three natural frequencies to occur system instability are π, 2π, and 3π for the simply supported pipe and 2π, 8.99, and 12.57 for the clamped-clamped pipe. Through the analysis it is identified that the consideration of the rotary inertia can not change the critical fluid velocities for the system instability as suggested by Paidoussis and Issid [15] and Laura et al. [16] for the fluid conveying pipe without concentrated masses.

Mathematically, the instability arises from the mixed derivative terms. The mixed derivative terms represent forces imposed on the pipe by the flowing fluid that is always 90° of phase with the displacement of the pipe, and always in phase



(a) Simply Supported Pipe

(b) Clamped-Clamped Pipe

Fig. 7. Critical Velocity for System Instability; α_1=0.3, α_2=0.1, β=0.2, μ_1=0.018, μ_2=0.025, ξ_1=0.3, ξ_2 =0.5

with the velocity of the pipe. Theses forces are essentially a negative damping mechanism which extracts energy from the fluid flow and inputs energy into the bending pipe to encourage, initially, vibration, and ultimately, instability [12,15]. However, the rotary inertia considered in the present paper expressed as the second derivative with time. Such that, this gives similar effect to the system like mass addition. Accordingly, although rotary inertia gives much change on the natural vibration no change is identified in the system instability.

5. Conclusions

The effects of the rotary inertia of concentrated masses on natural vibrations and system stability of the simply supported and clamped-clamped pipes conveying incompressible fluid have been analyzed by proposing a new governing equation and by developing a new computer code. The major conclusions for the present study are as follows:

1) Rotary inertia gives very much change on the higher natural frequencies and mode shapes.
2) The number and location of the nodes can be changed by considering the effect of the rotary inertia of concentrated masses.
3) The rate of decrease of the natural frequencies due to the effect of the rotary inertia of the concentrated mass increases when μ_1 has small value (e.g., less than 0.1 for the present case).
4) By introducing rotary inertia, the second natural frequency approaches to the first natural frequency as the location of the concentrated mass approaches to the midspan of the pipe.
5) The increase of the mass ratio between fluid and pipe gives no visible change on natural frequencies whereas the fluid velocity gives

much change.

6) The first three critical fluid velocities for the system instability are unchanged by introduction of the rotary inertia and have the values of π, 2π, and 3π for the simply supported pipe and 2π, 8.99, and 12.57 for the clamped-clamped pipe.

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