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# A New Method to Calculate Pseudoskin Factor of a Partially-Penetrating Well

# 부분관통정의 유사표피인자 계산을 위한 새로운 방법

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**Abstract :** This study considers pseudosteady-state flow to a restricted-entry well in a single or multilayer aquifer with crossflow. A simple method for calculating the pseudoskin factor caused by partial penetration is presented to overcome a limited applicability in geometrical or computational aspects of previous methods. The computation is based on the solution of a simplified pseudosteady-state equation that describes the long-time behavior of the closed radial system. We illustrate the applicability of this method to various types of cylindrical systems and provide the results graphically. Comparisons with previously published results have indicated that this method yields highly accurate estimates of pseudoskin factor with minimum computational effort. This method has also shown to be particularly useful for geometrically-complicated systems. Greatly improved computational efficiency of pseudosteady-state approach permits the engineer to easily account for the effect of partial penetration on the late-time performance of a well.

**요 약 :** 층간 교차유동이 일어나는 다층 대수층에 존재하는 부분 관통정에서의 유사정상상태 유동에 대하여 연구하였다. 본 논문에서는 부분관통에 의해 발생하는 압력 감소를 나타내는 유사표피인자를 계산할 수 있는 간단한 해법을 제시하였다. 이 해법은 기존에 제시되었던 방법들이 가진 기하학적 또는 계산시간 상의 한계성을 극복한 것이다. 본 계산을 위하여 폐쇄 대수층에서의 장기 압력 거동을 나타내는 간략화된 유사정상상태 확산방정식을 방사형 시스템에 적용하고 그 해를 이용하였다. 다양한 형태의 원통좌표계 시스템에서 본 해법의 적용성을 검토하고 그 결과를 그래프로 제시하였다. 기존의 결과와 비교해볼 때 본 해법을 이용할 경우 최소의 계산량으로 매우 정확한 유사표피인자 값을 계산할 수 있음을 나타내었다. 특히 본 해법은 기하학적으로 복잡한 시스템에 매우 유용함을 보여주었다.

## Introduction

In many cases, the open hole or well screen of a pumping well does not coincide with the full thickness of the aquifer. This situation is referred to as restricted-entry or partial penetration. When the interval open to flow is smaller than the entire aquifer thickness, deformation of the flow pattern owing to vertical flow components causes a resistance superimposed on that of a uniform radial flow. Therefore, pressure responses show an additional pressure drop in comparison to the pressure drop for a fully-penetrating well. At late times, the additional pressure loss or productivity decrease due to partial penetration can be accounted for by a lumped quantity called a pseudoskin. The pseudoskin factor is a time-independent quantity for times exceeding the start of pseudoradial flow which is detected from the second straight line on a semi-log plot of pressure vs. time (Streltsova, 1988).

This type of well completion has received considerable attention in both the petroleum and groundwater hydrology literature. Odeh (1968) presented a correlation for pseudoskin based on steady-state flow in a finite reservoir. Seth (1968) presented a general expression to calculate the unsteady state pressure distribution in a finite reservoir with partial well penetration. Odeh (1977) used a finite cosine transform to derive an analytical solution for the pseudosteady-state flow of a well with limited entry and with an altered zone. Streltsova-Adams (1978) used Laplace and Henkel transformations to solve partial-penetration problems and derived an expression for a pseudoskin factor in terms of infinite sine and cosine series.

In Odeh's (1980) article, pseudoskin factor as a function of sand thickness, location of the open interval, and the wellbore radius was given in a general equation form. Using a 2D finite-difference simulator, Reynolds *et al.* (1984) graphically presented pressure transient responses of a partially-penetrated, two-layered reservoir where only one layer is open to flow. Analyzing the steady-state analytical solution, they identified the correlating parameters and then obtained a correlation for skin factor by regression analysis. Papatzacos (1987) used the method of images, which uses an infinite number of image wells to generate no-flow condition, to solve partial-penetration problems. Using a numerical simula-

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tor, Yeh and Reynolds (1989) graphically presented pseudoskin factors, and obtained an expression for the pseudoskin factor. Olarewaju and Lee (1989) studied the pressure buildup behavior of a partially-penetrating well in a two-layered reservoir with closed top and bottom boundaries. By regression analysis, they developed a series of expressions correlating the pseudoskin factor with the penetration ratio for various values of permeability ratios.

Vrbik (1986, 1991) derived a simple formula for the pseudoskin factor due to partial well completion using equations for steady-state flow of an incompressible fluid. Under the assumptions of a pseudosteady-state interlayer crossflow, Gomes and Ambastha (1993) developed analytical expressions for partially-penetrating wells in multilayered reservoirs with both closed top and bottom boundaries, and with bottom-water zones and/or gas caps. Ding and Reynolds (1994) extended Papatzacos' (1987) expression for pseudoskin for a single-layered reservoir to that of a multilayered reservoir and reported a good match with simulated results.

All of preceding methods, however, have a limited applicability in geometrical or computational aspects. They are either insufficiently accurate due to assumptions made during the mathematical derivation or require the extensive use of a transient numerical simulator. The main objective of this study is to present a simple method for estimating pseudoskin factors in various types of cylindrical reservoirs with minimum computational effort. This method focuses on the application of the pseudosteady-state equation to flow in a cylindrical system with a restricted-entry well completed in a single or multilayer reservoir.

## Mathematical Formulation

To compute the pseudoskin factor, we consider a vertical, single or multilayer porous cylinder of uniform thickness $h$ and a finite system outer radius $r_e$. Each layer is assumed to be horizontally continuous, homogeneous, of uniform thickness and either isotropic or anisotropic, but the permeability is different from that of other layers. The average horizontal and vertical permeabilities $\bar{k}$ are and $k_z$, respectively. The vertical permeability of each layer is nonzero and the layers are not entirely separated by impervious layers. Therefore, interlayer crossflow can occur. A single well of radius $r_w$ produces at a constant rate q at the center of the cylinder, but may extend only through a limited height $h_w$ at a specified level. The surface of the cylinder is impermeable to the flow and thus represents a no-flow boundary.

Since the pressure exhibits radial symmetry, the mathematical model is a 2D r-z model. Rewriting dimensionless diffusivity equation for the flow through an anisotropic aquifer in radial geometry results in

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(k_D r_D \frac{\partial p_D}{\partial r_D}\right) + \frac{\partial}{\partial z_D}\left(k_{zD}\frac{\partial p_D}{\partial z_D}\right) = \frac{\partial p_D}{\partial t_D} \tag{1}$$

The dimensionless variables are defined by

$$p_D = \frac{2\pi \bar{k} h}{q\mu}(p_w - p) \tag{2}$$

$$t_D = \frac{\bar{k}t}{\phi \pi c_t r_w^2} \tag{3}$$

$$r_D = \frac{r}{r_w} \tag{4}$$

$$k_D = \frac{k}{\bar{k}} \tag{5}$$

$$k_{zD} = \frac{k_z}{\bar{k}} \tag{6}$$

$$z_D = \frac{z}{r_w} \tag{7}$$

where, $c_t$ = total compressibility
$k$ = permeability
$p_w$ = wellbore pressure
$t$ = time
$\mu$ = viscosity
$\phi$ = porosity

The boundary condition along the z axis at the wells open interval is to be a fixed one to incorporate the assumptions of both constant flowrate and infinite-conductivity wellbore.

$$p_D = 0 \tag{8}$$

For the other boundaries, no flow conditions are assigned.

$$(\nabla p)\cdot \mathbf{n} = 0 \tag{9}$$

Solving Eq. 1 for the case of a fully penetrating well, we can obtain the pressure distribution in the aquifer.

$$p_D(r_D, t_D) - p_{wD}(t_D) = \frac{1}{r_{eD}^2 - 1}\left(\frac{r_D^2 - 1}{2} - r_{eD}^2 \ln r_D\right) \tag{10}$$

where

$$r_{eD} = \frac{r_e}{r_w} \tag{11}$$

We let $(\bar{p}_D - p_{wD})_c$ denote the dimensionless pressure drawdown for the case where the entire interval is open to flow. Integrating Eq. 10, we have

$$(\bar{p}_D - p_{wD})_c = -\frac{r_{eD}^2}{r_{eD}^2 - 1}\left[\ln r_{eD} + \frac{1}{r_{eD}^2 - 1}\ln r_{eD} - \frac{r_{eD}^2 + 1}{4r_{eD}^2(r_{eD}^2 - 1)} - \frac{r_{eD}^2 - 1}{2r_{eD}^2}\right]. \tag{12}$$

Assuming that an aquifer is sufficiently large ($r_{eD}^2 \gg 1$) and neglecting the term of $O[\ln r_{eD}/(r_{eD}^2-1)]$ in Eq. 12, we obtain the usual inflow equation.

$$(\bar{p}_D - p_{wD})_c = -\left(\ln r_{eD} - \frac{3}{4}\right) \tag{13}$$

Long-time behavior of a closed aquifer can be described as a pseudosteady-state equation (Lee *et al.*, 1998). With a single calculation, the pseudosteady-state approach yields complete pressure and flux distributions for all time after onset of pseudosteady state. Applying the approach to Eq. 1 transforms it into

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(k_D r_D \frac{\partial p_D}{\partial r_D}\right) + \frac{\partial}{\partial z_D}\left(k_{zD} \frac{\partial p_D}{\partial z_D}\right) = \frac{2\pi h}{\pi(r_{eD}^2 - 1^2)h}. \tag{14}$$

During the pseudosteady-state flow period, the pressure drawdown for the case of partial penetration, which can be easily obtained by integrating the solution of Eq. 14 computed from a numerical model, is given by

$$\bar{p}_D - p_{wD} = (\bar{p}_D - p_{wD})_c - s_p. \tag{15}$$

The pseudoskin factor due to partial penetration $s_p$ is defined as an additional dimensionless pressure drawdown. From the Eq. 13, one can evaluate the pseudoskin factor as

$$s_p = (\bar{p}_D - p_{wD})_c - (\bar{p}_D - p_{wD}). \tag{16}$$

That is, the pseudoskin factor represents the additional dimensionless pressure drop necessary to produce fluid at a given rate because of partial penetration.

## Numerical Examples

The computation of pseudoskin factor is based on the numerical results generated from a finite-element program (Sewell, 1993) that yields highly accurate solutions. In this study, we first consider the single layer problem. Next, several example cases of multilayered problems are considered. For each case, our solutions are compared with results available in the literature.

### Single-Layer Cases

Two possible single-layer cases are considered. The first case assumes that the open interval is adjacent to the top (or by symmetry the bottom) of the aquifer and the second case allows an arbitrary location of the open interval.

First, we consider the case where the top of the open interval is adjacent to the top boundary, as shown in Figure 1. Here h is the total aquifer thickness and $h_w$ is the length of the open interval, as designated before. From the figure, we can define dimensionless thickness and height of open interval and penetration ratio.

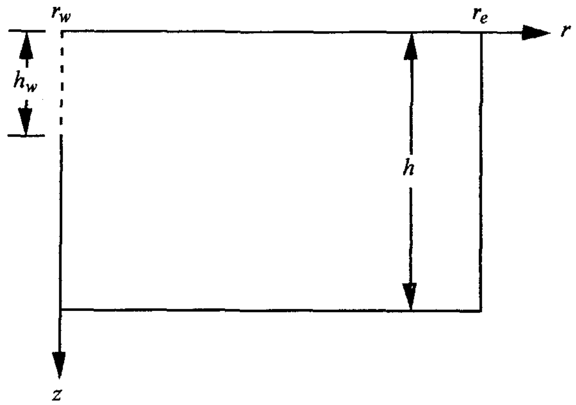


**Figure 1.** Schematic representation of a single-layer cylindrical system with open interval at the top of the aquifer.

$$h_D = \frac{h}{r_w}\sqrt{\frac{k}{k_z}} \tag{17}$$

$$h_{wD} = \frac{h_w}{r_w} \tag{18}$$

$$b = \frac{h_w}{h} \tag{19}$$

Computations were performed for a wide range of $h_{wD}$ between 50 and 1000. For a single-layer aquifer with b = 0.1, Figure 2 presents a comparison of pseudoskin factor computed from our pseudosteady-state model and values estimated from earlier studies. As shown in the figure, for a given penetration ratio of 0.1, all results essentially agree. The actual value of pseudoskin factor increases as aquifer thickness increases.

The aquifer configuration of the single-layer case, in which the location of the open interval is arbitrary, is depicted in Figure 3. Here, $h_1$ is equal to the height of the open interval. The ratio of $h_2/h$ represents the dimensionless distance between the top of the open interval and the top of the aquifer. Figure 4 presents the results for one set of computations which was conducted to investigate the effect of the location of the open interval. For all cases considered, b = 0.1 and $h_D$ = 200. The location of the open interval ($h_2/h$) varies from

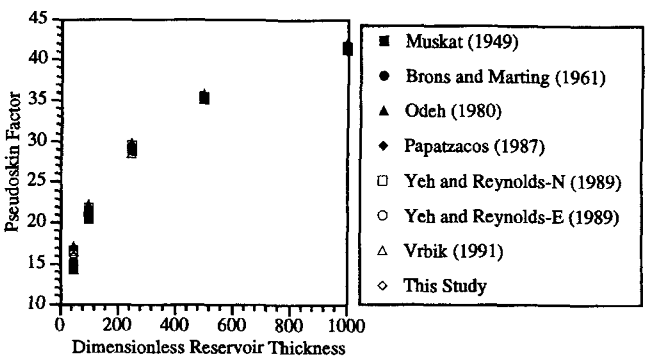


**Figure 2.** Pseudoskin factor of a single-layer cylindrical system with open interval at the top of the aquifer ($h_w/h$ = 0.1).

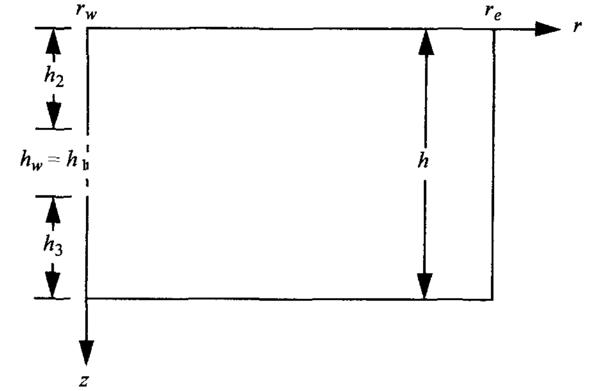


**Figure 3.** Schematic representation of a single-layer cylindrical system with open interval at arbitrary vertical location.

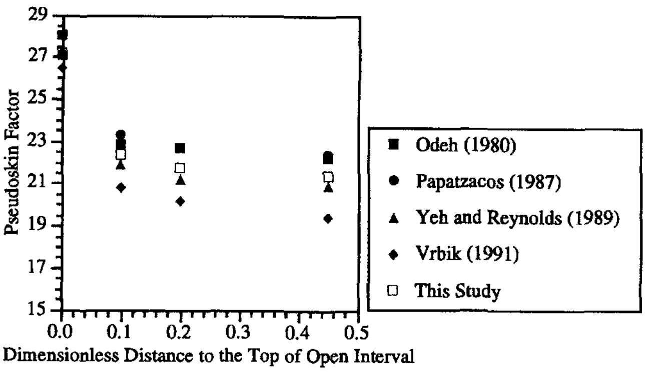


**Figure 4.** Pseudoskin factor of a single-layer cylindrical system with open interval at arbitrary vertical location ($h_D = 200$, $h_w/h = 0.1$).

case to case. Differences between the sets of results are not great, but the discrepancy is more significant than the first case due to the scale of the y axis and relatively complex geometry of the problem. Our solutions are closest to the Yeh and Reynolds (1989) results, which we believe to be the most accurate because of fewest assumptions made during numerical simulations. Figure 4 indicates that as $h_2/h$ increases, the pseudoskin factor decreases. In other words, as the producing interval moves away from the boundary, the pseudoskin value decreases, having a minimum value when the producing interval is centrally located ($h_2/h = 0.45$).

## Multilayered Cases

The computation of pseudoskin factor resulting from restricted entry is also extended to the multilayered cases. We first consider a two-layered case where one layer is open to flow. In other words, the open interval is adjacent to only one layer and the length of this interval is equal to the thickness of this layer. Second, we consider a five-layered case where the open interval is at the top or middle of the aquifer.

For the layered problem considered here, it is appropriate to define the thickness-averaged horizontal permeability by

$$\bar{k} = \frac{1}{n}\sum_{j=1}^{n} k_j h_j \tag{20}$$

where j is a layer index and n is the number of layers. Accordingly, the dimensionless pressure and time should be redefined on the basis of average properties.

The two-layer problem shown in Figure 5 is considered first. For all cases, the top of the open interval is adjacent to the top of the aquifer. The open interval is always designated as layer 1, and thus, our assumptions imply that $h_w = h_1$. Three different cases, which cover a wide range of practical interest, are chosen from Yeh and Reynolds' (1989) paper. Table 1 compares geometry and permeability data for each system. Figure 6 compares values from our study with values from earlier studies. To obtain a numerical value from Gomes and Ambastha's (1993) analytical expression containing modified Bessel functions, we use a polynomial approximation (Abramowitz and Stegun, 1964). The origin of the relating large discrepancies with Gomes and Ambastha's solution comes from the facts that they assumed pseudosteady-state crossflow and their solution always shows the highest value. Our results are sufficiently close to Yeh and Reynold's numerical solutions which are considered to be the most reliable because they are based on fine-grid simulation study.

Results from the equal-thickness, five-layered problem are given in Table 2. Each layer has a different permeability and is anisotropic. Cases 1 and 2 of Table 2 pertain to five-layered cases where the top layer is open to flow (Figure 7a). Figure 7b shows a five-layered aquifer with the middle layer

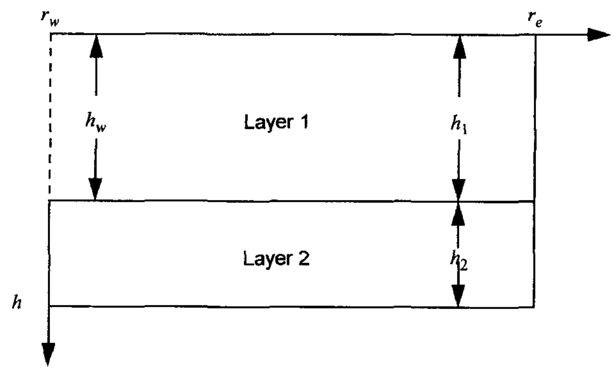


**Figure 5.** Two-layered system with top layer open to flow.

**Table 1.** Description of two-layered aquifers chosen for pseudoskin calculation

| | Case 1 | Case 2 | Case 3 |
|---|---|---|---|
| $h_1/r_w$ | 100 | 75 | 50 |
| $h_1/h$ | 0.4 | 0.3 | 0.2 |
| $k_1/k_2$ | 0.167 | 0.259 | 0.444 |
| $k_1/k_{z1}$ | 0.25 | 0.444 | 1.0 |
| $k_2/k_{z2}$ | 0.9 | 6.514 | 50.625 |

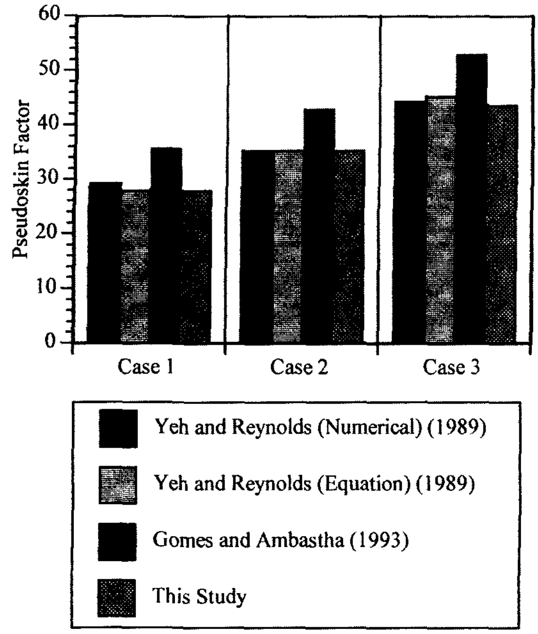


**Figure 6.** Comparison of pseudoskin factors for a two-layered aquifer.

**Table 2.** Description of five-layered aquifers chosen for pseudoskin calculation

| | Case 1 | Case 2 | Case 3 | Case 4 |
|---|---|---|---|---|
| $h/r_w$ | 250 | 250 | 250 | 250 |
| $k_1/k_2$ | 0.50 | 0.50 | 0.80 | 0.80 |
| $k_1/k_3$ | 0.25 | 0.25 | 0.32 | 0.32 |
| $k_1/k_4$ | 5.00 | 5.00 | 0.80 | 0.80 |
| $k_1/k_5$ | 0.50 | 0.50 | 0.32 | 0.32 |
| $k_1/k_{z1}$ | 1.00 | 1.00 | 1.00 | 1.00 |
| $k_2/k_{z2}$ | 1.00 | 1.00 | 5.00 | 5.00 |
| $k_3/k_{z3}$ | 6.67 | 1.00 | 5.00 | 1.00 |
| $k_4/k_{z4}$ | 5.00 | 1.00 | 5.00 | 5.00 |
| $k_5/k_{z5}$ | 70.0 | 1.00 | 5.00 | 1.00 |

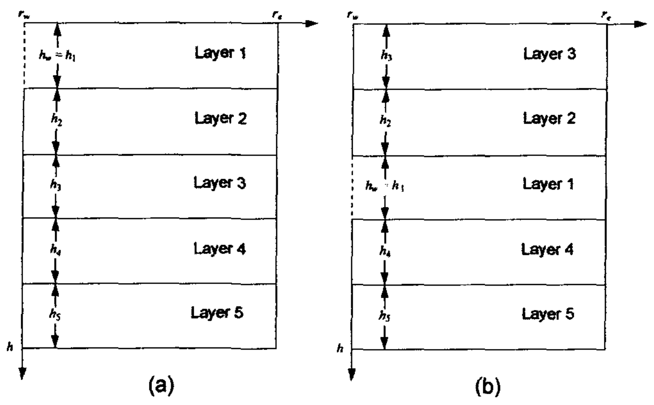


**Figure 7.** Five-layered system with (a) top layer open to flow and (b) middle layer open to flow.

perforated (Cases 3 and 4). We have denoted the layer open to the flow as Layer 1. In this problem, few numerical values

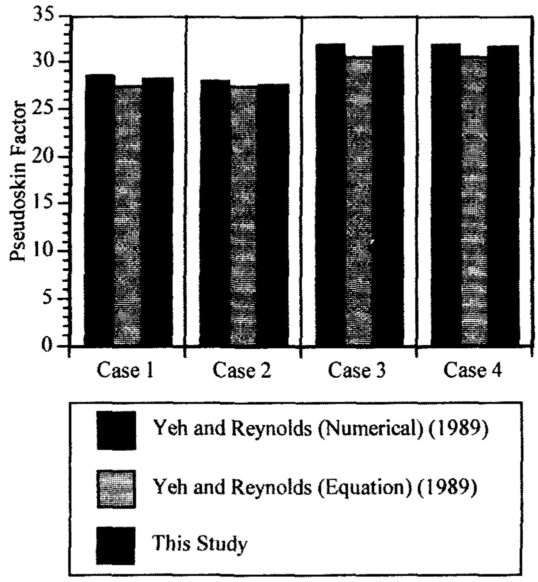


**Figure 8.** Comparison of pseudoskin factors for a five-layered aquifer.

for pseudoskin have been published due to its geometrical complexity. As presented in Figure 8, a comparison is, therefore, made with values only from Yeh and Reynolds (1989). Again, the agreement is seen to be quite good for all cases considered.

## Conclusions

A simple method was presented to generate the pseudoskin factor resulting from a partial penetration in a single or multilayered aquifer where crossflow occurs between layers. The development of this method consists of applying a pseudosteady-state diffusivity equation stating a long-time pressure distribution of a closed aquifer. With a single application of the pseudosteady-state approach, we can obtain the correct value of pseudoskin factor directly for any geometrically-complicated radial systems normally encountered in practice. A detailed numerical investigation in various situations shows that the pseudosteady-state approach gives a simple and reliable means to calculate pseudoskin factor. The method overcomes a limited applicability in geometrical or computational aspects of previous methods.

## References

Abramowitz, M. and Stegun, I. A., 1964, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, Washington, DC, National Bureau of Standards Applied Mathematics Series, US Government Printing Office, p. 378-379.

Ding, W. and Reynolds, A. C., 1994, Computation of the Pseudoskin Factor for a Restricted-Entry Well, SPE Form. Eval., p. 9-14.

Gomes, E. and Ambastha, A. K., 1993, Analytical Expressions for

Pseudoskin for Partially Penetrating Wells Under Various Reservoir Conditions, paper SPE 26484 presented at 1993 Annual Technical Conference and Exhibition, Houston, TX, October 3-6.

Lee, K. S., Miller, M. A. and Sepehrnoori, K., 1998, Succession-of-States Model for Calculating Long-Time Performance of Depletion Reservoirs, Soc. Pet. Eng. J., p. 279-284.

Odeh, A. S., 1968, Steady-State Flow Capacity of Wells with Limited Entry to Flow, Soc. Pet. Eng. J., p. 43-51.

Odeh, A. S., 1977, Pseudosteady-state Flow Capacity of Oil Wells with Limited Entry and an Altered Zone around the Wellbore, Soc. Pet. Eng. J., p. 271-280.

Odeh, A. S., 1980, An Equation for Calculating Skin Factor Due to Restricted Entry, J. Pet. Tech., p. 964-965.

Olarewaju, J. S. and Lee, W. J., 1989, Pressure Buildup Behavior of Partially Completed Wells in Layered Reservoirs, paper SPE 18876 presented at 1989 Production Operations Symposium, Oklahoma City, OK, March 13-14.

Papatzacos, P., 1987, Approximate Partial-Penetration Pseudoskin for Infinite-Conductvity Wells, SPE Reser. Eng., p. 227-234.

Reynolds, A. C., Chen, J. C. and Raghavan, R., 1984, Pseudoskin Factor Caused by Partial Penetration, J. Pet. Tech., p. 2197-2210.

Sewell, G., 1993, PDE2D: Easy-to-use Software for General Two-Dimensional Partial Differential Equations, Advances in Engineering Software, p. 105-112.

Streltsova-Adams, T. D., 1979, Pressure Drawdown in a Well with Limited Flow Entry, J. Pet. Tech., p. 1469-1476.

Streltsova, T. D., 1988, Well Testing in Heterogeneous Formations, An Exxon Monograph, New York, NY, John Wiley & Sons

Vrbik, J., 1986, Calculating the Pseudo-skin Factor due to Partial Well Completion, J. Canadian Pet. Tech., p. 57-61.

Vrbik, J., 1991, A Simple Approximation to the Pseudoskin Factor Resulting from Restricted Entry, SPE Form. Eval., p. 444-446.

Yeh, N. S. and Reynolds, A. C., 1989, Computation of Pseudoskin Factor Caused by a Restricted-Entry Well Completed in a Multilayer Reservoir, SPE Form. Eval., p. 253-263.