⊠연구논문

상관변수를 이용한 공정 감시 절차

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A Process Monitoring Procedure Using a Correlated Variable

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Abstract

A process monitoring procedure using a correlated variable is presented when a lower specification limit is given on the performance variable. Every item is inspected with a variable correlated with the performance variable. When an item is rejected in the screening inspection, the process is checked for change using the mean and variance of measurements of the correlated variable for n preceding items including the rejected one. The performance variable is assumed to be normally distributed. A linear relationship between the performance and surrogate variables is assumed with normally distributed error term. The monitoring procedure is designed so that the prespecified outgoing quality can be attained.

1. Introduction

Due to the recent developments of many sophisticated automatic test equipments, screening (100% inspection) becomes a very attractive practice to improve the outgoing quality. These test equipments can efficiently inspect a large number of items in a short time and produce consistent and accurate results. Thus, inspection is becoming essentially an inherent part of modern manufacturing processes and hence screening is likely to be a natural practice in the future process. Screening may be based on the major quality characteristic of interest (performance variable). When inspection with the performance variable is expensive or requires destructive testing, however, a variable correlated with the performance variable is frequently used. For example, suppose that the voltage at an internal point of an electronic device, which is difficult to measure without disassembling some of its parts, is the performance variable. Instead of measuring the performance variable directly, we may measure the voltage at an external point (correlated variable). The idea of using correlated variables in screening has received much attention; see Tang and Tang(1994) for detailed literature review. For more recent works, see Bai and Kwon(1995, 1997), Bai et al.(1995), and Boys et al.(1996)

The screening procedure is usually designed so that a prespecified quality level can be achieved after screening. When the process mean or variance tends to change, however, the prespecified outgoing quality cannot be attained. And thus, screening may not be a good long-term solution for improving product quality. Implementing successful process control and quality improvement programs is essential for attaining customer satisfaction through quality assurance. As 100% inspection is becoming more popular, methods of process control under screening environment are required increasingly. There are, however, only a few studies on design of process control or monitoring procedure applicable to this environment. Bourke(1991) presented the run-length chart to monitor the lengths of runs of conforming items between successive nonconforming items when 100% inspection in the order of production is in progress. Hui(1991) studied production control with complete inspection where every finished product is subject to variable inspection assuming two possible process states - in-control and out-of-control. These studies use the performance variable to monitor the process. Currently, there seem to be no works using a correlated variable to monitor the process under 100% inspection.

By using a process monitoring procedure together with an appropriate screening scheme, it is possible to monitor the process as well as achieve a desired outgoing quality level in the long-term point of view. In this article, we present a process monitoring procedure when all the items are subject to inspection with a correlated variable. In Section 2, we suggest a process monitoring procedure assuming a linear relationship between the performance and correlated variables. In Section 3, the procedure is designed and solution methods are provided. In Section 4, the procedure is illustrated with a numerical example.

2. The Process Monitoring Procedure

Let Y be the performance variable with a lower specification limit L and X be a screening variable. Assume that Y is normally distributed with mean μ_y and variance σ_y^2 and there is a linear relationship between X and Y as follows;

$$X = \beta_0 + \beta_1 Y + \varepsilon, \tag{1}$$

where β_0 and β_1 are constants and ε is an error term. The error term ε is assumed to be independent of Y and normally distributed with mean zero and variance σ_e^2. Then

$$\begin{bmatrix} X \\ Y \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_x^2 \end{bmatrix} \right), \tag{2}$$

where $\mu_x = \beta_0 + \beta_1\mu_y$, $\sigma_x^2 = \beta_1^2\sigma_y^2 + \sigma_e^2$ and $\rho = (\sigma_y/\sigma_x)\beta_1$. We assume that β_0, β_1, σ_e^2, and ρ are positive and remain constant even if the process mean or variance changes.

The process monitoring procedure is as follows:

i) For each incoming item, take a measurement x of X. Let ω_s denote the cutoff value on X. If $x \geq \omega_s$, accept the item and reject it otherwise.

ii) If an item is rejected, calculate the sample mean $\bar{x}$ and variance $s_x^2 = \sum_{i=1}^{n}(x_i - \mu_{x0})^2/n$ for n items including the rejected one and preceding $n-1$ items where μ_{x0} is the mean of X when the process is in control.

iii) If $s_x^2 \geq \nu_p$ *or* $\bar{x} \leq \omega_p$, *investigate the process for assignable cause of variation and take a corrective action if necessary.*
Repeat from step i).

Here, (n, ν_p, ω_p) is the process monitoring scheme where ν_p and ω_p are monitoring limits for the process variance and mean, respectively.

Other cases, where Y has an upper specification limit or β_1 is negative, can be dealt with similarly.

3. Design of the Procedure

For designing the procedure, the screening limit ω_s is first determined assuming the process is in-control state. Let δ_u be the target outgoing quality which is desired to be attained through the screening procedure. The cutoff value ω_s is determined so that

$$P(Y \geq L \mid X \geq \omega_s) \geq \delta_u \tag{3}$$

when the process is in-control state. Let μ_{y0} denote the mean of Y and σ_{x0}^2 and σ_{y0}^2 denote the variances of X and Y, respectively, when the process is in control. Let γ be the proportion of conforming items in the unscreened population when the process is in-control state. Then $L = \mu_{y0} - \kappa_\gamma \sigma_{y0}$ where $\kappa_\gamma = \Phi^{-1}(\gamma)$ and $\Phi(.)$ is the standard normal distribution function. Let k be a number such that

$$\omega_s = \mu_{x0} - \kappa \sigma_{x0}. \tag{4}$$

Then, (3) is equivalent to

$$P(Y \geq L \mid X \geq \omega_s) = \frac{(P(Z_1 \geq -\kappa_\gamma, Z_2 \geq -\kappa)}{\Phi(\kappa)} = \delta_u , \tag{5}$$

where Z_1 and Z_2 have a standard bivariate normal distribution with the correlation coefficient ρ. Equation (5) is reduced to

$$\frac{\gamma + \Phi(\kappa) - 1 + \Psi(-\kappa_\gamma, -\kappa; \rho)}{\Phi(\kappa)} = \delta_u , \tag{6}$$

where $\Psi(.,.;\rho)$ is the standard bivariate normal distribution function with correlation coefficient ρ. The numerical value of κ can be found easily by a simple computer program using IMSL subroutines and the cutoff value ω_s is obtained using (4)

Note that δ_u may not be attained if the process is changed. It will be desirable for the outgoing quality to be better than a certain level $\delta_l(<\delta_u)$ even if the process changes. That is, δ_l is the poorest level of quality acceptable in the process. To guarantee the outgoing quality greater than equal to δ_l, the process should be checked for change corrected if

$$P(Y \geq L \mid X \geq \omega_s) < \delta_l \tag{7}$$

Thus, we define the out-of-control state as a state of the process under which the inequality (7) holds.

Now, the process monitoring scheme (n, ν_p, ω_p) is determined considering the prespecified type I and type II errors, α and β, respectively. We first determine (n, ω_p) so that

$$P[\overline{X} \leq \omega_p \mid in-control] = \alpha, \tag{8a}$$

$$P[\overline{X} \leq \omega_p \mid out-of-control] = 1-\beta \tag{8b}$$

And next, (n, ν_p) is determined by

$$P[s_x^2 \geq \nu_p \mid in-control] = \alpha, \tag{9a}$$

$$P[s_x^2 \geq \nu_p \mid out-of-control] = 1-\beta \tag{9b}$$

For the value of n, we choose whichever is larger among the values obtained from (8a), (8b) and (9a), (9b).

To obtain (n, ν_p, ω_p), we must first determine the tolerable amount of change in mean or variance satisfying

$$P(Y \geq L \mid X \geq \omega_s) = \delta_l. \tag{10}$$

For simplicity, we assume that the mean and variance do not change at the same time and (n, ν_p, ω_p) is obtained separately for the case where the mean or the variance changes.

Case 1. Shift in the Process Mean

Consider the case where the mean of the process is subject to shift and the variance remains constant. Let d be the tolerable amount of shift in the mean of Y, that is, $\mu_y = \mu_{y0} - d\sigma_{y0}$ and thus, $\mu_x = \mu_{x0} - d\rho\sigma_{x0}$. Based on (10), d can be determined by

$$\frac{\Phi(\kappa_\gamma - d) + \Phi(\kappa - d\rho) - 1 + \Psi(d - \kappa_\gamma, d\rho - \kappa; \rho)}{\Phi(\kappa - d\rho)} = \delta_l, \tag{11}$$

for given δ_l. Using (8a) and (8b), we obtain

$$\omega_p = \mu_{x0} - \kappa_{1-\alpha}\frac{\sigma_x}{\sqrt{n}}, \tag{12a}$$

$$n = \left(\frac{\kappa_{1-\beta} + \kappa_{1-\alpha}}{d\rho}\right)^2 \tag{12b}$$

Case 2. Change in the Process Variance

Consider the case where the variance of the process is subject to shift and the mean remains constant. Suppose the variance of Y change from σ_{y0}^2 to $(1+q)^2\sigma_{y0}^2 (q > 0)$. By a similar method previously used, for given δ_l, the tolerable amount of change in variance q can be determined by

$$\frac{\Phi\left(\frac{\kappa_\gamma}{1+q}\right)+\Phi\left(\frac{\kappa}{\sqrt{1+2q\rho^2+q^2\rho^2}}\right)-1+\Psi\left(-\frac{\kappa_\gamma}{1+q},-\frac{\kappa}{\sqrt{1+2q\rho^2+q^2\rho^2}},\rho\right)}{\Phi\left(\frac{\kappa}{\sqrt{1+2q\rho^2+q^2\rho^2}}\right)}$$

$$=\delta_l. \qquad (13)$$

Let $\chi^2_p(n)$ denote the upper 100p^th percentile of χ^2 distribution with n degrees of freedom. Based on (9a), (9b) and q, we obtain n and ν_p as

$$\nu_p=\frac{\sigma^2_{x0}\chi^2_\alpha(n)}{n}, \qquad (14a)$$

$$\frac{\chi^2_\alpha(n)}{\chi^2_{1-\beta}(n)}=1+2q\rho^2+q^2\rho^2. \qquad (14b)$$

We can first determine n using (14b) and then ν_p based on (14a).

< Table 1 > Values of k, d, q for $\rho=0.8$, $\delta_u=0.975$ and $\delta_l=0.925$

	k	d	q
0.70	-0.387	0.764	0.890
0.71	-0.345	0.761	0.882
0.72	-0.301	0.759	0.875
0.73	-0.257	0.757	0.867
0.74	-0.211	0.754	0.859
0.75	-0.165	0.751	0.850
0.76	-0.117	0.749	0.842
0.77	-0.067	0.746	0.833
0.78	-0.016	0.743	0.824
0.79	0.037	0.739	0.815
0.80	0.091	0.736	0.806
0.81	0.148	0.732	0.796
0.82	0.207	0.728	0.786
0.83	0.269	0.724	0.776
0.84	0.334	0.719	0.765
0.85	0.403	0.714	0.754

When the values of n of the two cases are different, we select whichever is the larger. To obtain the values of ω_s and (n, ν_p, ω_p), numerical calculations are necessary. By preparing tables providing the values of k, d, q and $\chi^2_{\alpha}(n)/\chi^2_{1-\beta}(n)$ for a reasonable range of the parameters, the solution procedure can be facilitated. <Table 1> provides the values of k, d, q for $\rho=0.8$, $\delta_u=0.975$ and $\delta_l=0.925$. <Table 2> provides the values of $\chi^2_{\alpha}(n)/\chi^2_{1-\beta}(n)$ for $\alpha=0.05$ and $\beta=0.10$. Using <Table 1> and <Table 2>, we can easily determine the values of ω_s and (n, ν_p, ω_p). Equations (12b), (14b) and <Table 2> show that n takes a smaller value as ρ becomes larger.

< Table 2 > Values of $\chi^2_{\alpha}(n)/\chi^2_{1-\beta}(n)$ for $\alpha=0.05$ and $\beta=0.10$

n	$\chi^2_{\alpha}(n)/\chi^2_{1-\beta}(n)$	n	$\chi^2_{\alpha}(n)/\chi^2_{1-\beta}(n)$	n	$\chi^2_{\alpha}(n)/\chi^2_{1-\beta}(n)$
1	243.272	21	2.468	41	1.904
2	28.433	22	2.416	42	1.889
3	13.373	23	2.369	43	1.875
4	8.920	24	2.326	44	1.862
5	6.875	25	2.286	45	1.849
6	5.713	26	2.249	46	1.836
7	4.965	27	2.215	47	1.824
8	4.444	28	2.183	48	1.813
9	4.059	29	2.153	49	1.802
10	3.763	30	2.125	50	1.791
11	3.527	31	2.099	51	1.781
12	3.335	32	2.074	52	1.771
13	3.176	33	2.051	53	1.761
14	3.041	34	2.029	54	1.752
15	2.925	35	2.008	55	1.743
16	2.824	36	1.989	56	1.734
17	2.735	37	1.970	57	1.726
18	2.657	38	1.952	58	1.718
19	2.587	39	1.935	59	1.710
20	2.524	40	1.919	60	1.702

4. Numerical Example

In this section, an illustrative example is provided.

Example. Consider an electronic device whose major quality characteristic Y is the voltage at an internal point. The lower specification limit of the performance variable Y is 8 volts. Instead of measuring Y, we may use the voltage X at an external point which is easy to measure. It is known that there is a linear relationship between X and Y approximately as

$$X = 2.0 + 0.5Y + \varepsilon,$$

where the error term ε is normally distributed with mean 0 and variance 0.75. When the process is in-control state, Y is known to be approximately normally distributed with $\mu_{y0} = 10$ volts and $\sigma_{y0} = 2$ volts. Suppose that the proportion of conforming items after screening is desired to be $\delta_u = 0.975$ and greater than or equal to at least $\delta_l = 0.925$ with $\alpha = 0.05$ and $\beta = 0.10$.

(Solution) We first calculate $\gamma = \Phi((10-8)/2) = 0.8413$, $\mu_{x0} = 2.0 + (0.5)(10) = 7.0$, $\sigma_{x0} = \sqrt{(0.5^2)(2^2) + (0.75^2)} = 1.25$, and $\rho = (2)(0.5)/1.25 = 0.80$. Since $\delta_u = 0.975$ and $\delta_l = 0.925$, we obtain $\kappa \cong 0.341$, $d \cong 0.718$ and $q \cong 0.764$ from <Table 1> Thus, the cutoff value of the screening variable is

$$\omega_s = 7.0 - (0.341)(1.25) \cong 6.57.$$

Since $\kappa_{1-0.05} = 1.645$ and $\kappa_{1-0.10} = 1.282$, the value of n for monitoring the process mean is $n = ((1.645 + 1.282)/((0.718)(0.80)))^2 \cong 26$. For monitoring the variance, the value of the right-hand side of (13b) is $1 + (2)(0.764)(0.8^2) + (0.764^2)(0.8^2) = 2.351$ and thus, $n \cong 24$ using <Table 2>. We choose

$$n = \max(26, 24) = 26.$$

Finally, $\chi^2_{0.05}(26) = 38.9$ and

$$\omega_p = 7.0 - (1.645)(1.25)/\sqrt{26} \cong 6.60,$$

$$\nu_p = (1.25^2)(38.9)/26 \cong 2.338.$$

5. Conclusion

We proposed a process monitoring procedure based on a correlated variable. Every item is inspected with a screening variable which is correlated with the performance variable. The cutoff value of the screening variable is determined so that the outgoing proportion of conforming items exceeds a prespecified level when the process is in-control state. The process monitoring scheme is also designed so that the outgoing quality may not be less than a lower prespecified level even under the out-of-control state. If an item is rejected in the screening inspection, the process is checked on the basis of the measured values of the screening variable of the n preceding items including the rejected one. Both of the mean and the variance are examined for any shift or change. The value of n is determined conservatively so that the lower prespecified outgoing quality can be guaranteed.

The cutoff value of the screening variable and the process monitoring scheme are not obtained in closed form solutions. The solutions, however, can be obtained numerically without any difficulty using a software such as IMSL subroutines. The solution can be further facilitated by preparing tables of the related coefficients over a reasonable range of the parameters in advance.

The study can be extended to the case where some or all of the parameters are unknown. Economic design of the process monitoring procedure under the same situation may also be studied.

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