Modified Circulant Feedback Delay Networks (MCFDN's) for Artificial Reverberator Using a General Recursive Filter and CFDN's

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Abstract

Circulant Feedback Delay Networks (CFDN's), whose feedback matrix is circulant to control the stability of system and time-frequency response easier than unitary one, were recently proposed. However, the drawback of this structure is that the flatness of the frequency response of CFDN's is not enough and it is difficult to adjust the placement of zeros to decrease this problem.

Therefore, we propose Modified CFDN's (MCFDN's) consisted of a general recursive filter and CFDN's to maintain maximally the impulse response of CFDN's and improve the flatness of frequency response without adjusting the placement of zeros. The delay unit of a general recursive filter's feedback loop is replaced by CFDN's, are omitted the direct path. We represent the usefulness of MCFDN's to build artificial reverberators and the main parameter to determine characteristics of MCFDN's in this paper.

I. Introducion

There exist two basic kinds of sound. One is direct sound that is defined as the wavefront that reaches the ears first by a linear paths, without having bounced off a surrounding surface. And the other, is reflected sound (reverberation). This refers to the energy of a sound source that reaches the listener indirectly, by reflecting from surfaces within the space surrounding the sound source and the listener. Late reflected sounds in particular defined as reverberation, play an important role in perceiving characteristics of sound source's space, and is a core element to implement spatial impression and an additional cue for direction and distance impression [1,2,3]. Therefore, artificial reverberator to simulate reverberation is essential to realize 3-D sound.

The artificial reverberator has been developed into having both rich echo density impulse response and nearly flat frequency response. However, if frequency response is perfect flat, there is a metallic sound. Otherwise, coloration occurs in the reverberated sound [4,5]. The first stage artificial reverberator to satisfy these conditions was proposed, consisting of comb and allpass filters [6,7]. The main drawback of these structures is that they are difficult to parameterize because of the poor relationship between the system parameters and the physical quantities of a real room [8,9,10]. Recently, it was proposed to minimize those problems by using Digital Waveguide Networks (DWN's) and Feedback Delay Networks (FDN's), whose feedback matrix is unitary [11,12,13]. More recently, Circulant Feedback Delay Networks (CFDN's), whose feedback matrix is circulant.

Circulant FDN's can control the stability and time-frequency response of system more easier than unitary one [6,8]. But there is a problem in that the flatness of the frequency response of CFDN's is insufficient, and it is difficult to adjust position of zeros to increase it.

We propose Modified CFDN's (MCFDN's), which is built a general recursive filter inserted CFDN's in feedback loop in order to maintain maximally the impulse response of CFDN's and get more the flatness of the frequency response without adjusting the placement of zeros, and verify usefulness for artificial reverberator.

II. Circulant Feedback Delay Networks (CFDN's)

2.1. Feedback Delay Networks (FDN's)

FDN's are built using feedback matrix and N delay lines, each having a length in seconds given by $\tau_i = m_i T$, where $T = 1/F_s$ is the sampling period.

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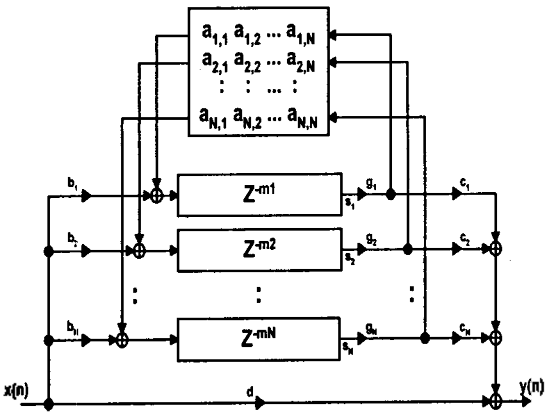


Figure 1. General structure of Feedback Delay Networks.

The complete FDN's (see Fig. 1) are given by the following relations:

$$y(n)=\sum_{i=1}^{N} c_i s_i(n)+dx(n)$$
$$s_i(n+m_i)=\sum_{j=1}^{N} a_{i,j} s_j(n)+b_i x(n) \quad (1)$$

where, $s_i(n)$, $1\le i\le N$, are the delay line outputs at time sample n. Using Z transform, assuming zero initial conditions, we can rewrite (1) in the frequency domain as

$$Y(z)=c^T S(z)+dX(z)$$
$$S(z)=D(z)[AS(z)+bX(z)], \quad (2)$$

where, $S^T=[s_1(z),\cdots,s_N(z)]$, $b^T=[b_1,\cdots,b_N]$ and $c^T=[c_1,\cdots,c_N]$. The diagonal matrix $D(z)=diag(z^{-m_1},\cdots,z^{-m_N})$ is called the delay matrix and $A=[a_{i,j}]_{N\times N}$ is called the feedback matrix [8,12,14].

From (2), the transfer function is easily found to be

$$H(z)=c^T[D(z^{-1})-A]^{-1}b+d. \quad (3)$$

The poles of the FDN's are the solution of

$$det[A-D(z^{-1})]=0. \quad (4)$$

The system zeros are found as the solution of

$$det[A-b\frac{1}{d}c^T-D(z^{-1})]=0, \quad (5)$$

where, feedback matrix A is generally unitary matrix, which has all eigenvalues on the unit circle. So, it is possible to design a reverberator having infinity reverberation time at all frequency mode [8,14].

Frequency density D_f can be derived from the order of the system (3), assuming that all the poles are distinct, and no cancellation occurs;

$$D_f=\frac{1}{F_s}\sum_{i=1}^{N} m_i \quad (6)$$

Time Density D_t is defined as the number of nonzero samples per second in the impulse response. In actual rooms, D_t is an increasing function of time. In order to obtain dense reverberation after the early reflections (e.g., after 80ms), it helps to use different delay lengths.

When the total delay length is sufficient, the prototype (reference) FDN's can be compared to a pseudo-random noise generator. Therefore, in practice, we must insert attenuation coefficients and filters in the feedback loop. For example, one may insert a gain

$$g_i=\alpha^{m_i} \quad (7)$$

at the out of each delay line in the FDN's. With this choice of the attenuation coefficients, all the system poles are uniformly contracted by a factor α, thus ensuring a uniform decay of all the modes, multiplying the reference impulse response by a decaying exponential envelope. Frequency-dependent decay characteristics, specified by the reverberation time vs. frequency $RT(\omega)$, are obtain by use of absorptive filters making each attenuation g_i frequency-dependent:

$$20\cdot\log_{10}|g_i(\omega)|=-60\cdot\tau_i/RT(\omega) \quad (8)$$

where, $i=1,\cdots,N$, $\tau_i=m_iT$ is the delay length expressed in seconds and T is the sample period [8,12].

2.2. Circulant Feedback Delay Networks (CFDN's)

Feedback matrix plays an important role in determining characteristics of FDN's. FDN's whose feedback matrix are replaced by circulant matrix are called CFDN's. Some merits of CFDN's are that: they maintain the same position of the poles of the system as FDN's; it is easier to analyze the stability of a system than with FDN's; and it is possible to process in real-time.

Consider the class of circulant feedback matrices having the form

$$A=\begin{bmatrix} v(0) & v(1) & \cdots & v(N-1) \\ v(N-1) & v(0) & \cdots & v(N-2) \\ \vdots & \vdots & \cdots & \vdots \\ v(1) & \cdots & v(N-1) & v(0) \end{bmatrix}. \qquad (9)$$

The circulant matrix is normal. It is unitary if a matrix is lossless. In particular, the circulant matrix is diagonalized by the Discrete Fourier Transform (DFT) matrix. This implies that the eigenvalues of A can be computed by means of the DFT of the first row:

$$\{\lambda(A)\} = \{A(k)\} \\ = DFT\{[v(0)\cdots v(N-1)]^T\} \qquad (10)$$

where $\{\lambda(A)\}$ denotes the set of all eigenvalues of A, and a matrix that is both unitary and circulant has all eigenvalues on the unit circle. The advantage of choosing circulant FDN's over other kinds of FDN's is the possibility of computing A from its eigenvalues very efficiently by means of a single inverse FFT [8,15,16].

When feedback matrix is

$$A=\begin{bmatrix} -1/3 & 2/3 & 2/3 \\ 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -1/3 \end{bmatrix} \qquad (11)$$

in Fig. 1, $a=0.99$, $b=[1\ 1\ 1]^T$, $c=[1\ -0.8\ 0]^T$ and $m=[16\ 17\ 15]^T$ from (1). Fig. 2 depicts the time and frequency response of CFDN's when $a=0.99$ and sampling frequency is 22050Hz [17].

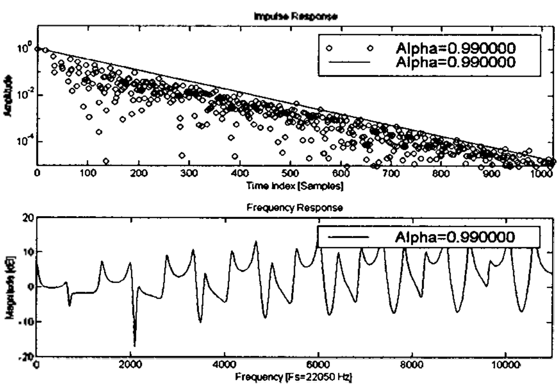


Figure 2. Time and frequency response for circulant feedback delay networks, sampling frequency is 22050Hz.

From Fig. 2, we know that impulse response decreases by exponentially and that Echo density has very good characteristic but frequency response is insufficient (it has very poor flatness). Therefore, to be a better reverberator, It is necessary to improve the frequency response.

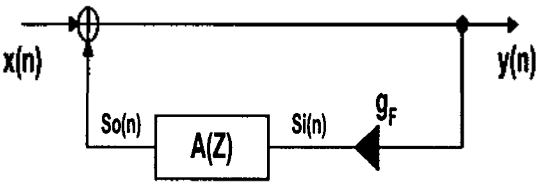
In order to improve frequency response, we must finding the value of b and c by choosing pertinently zeros of system. However, it is difficult to find the optimum value of b and c, and the more reverberation time increases, the more time it takes to compute it.

In Section III, Modified CFDN's (MCFDN's) are proposed as good method to maximally maintain the time characteristics of CFDN's and improve frequency response in real time.

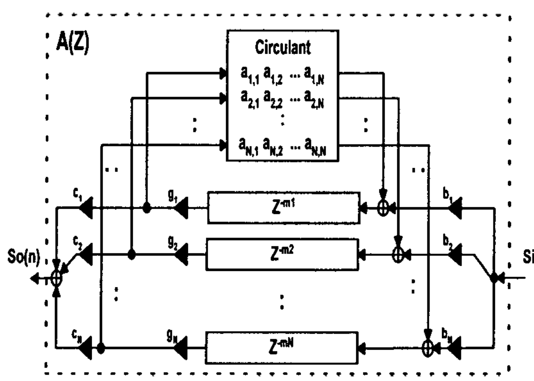
III. Modified Circulant Feedback Delay Networks (MCFDN's)

Modified CFDN's (MCFDN's) consist of a general recursive filter and CFDN's to maintain maximally the impulse response of CFDN's and improve the flatness of frequency response without adjusting the placement of zeros in real time.

Fig. 3 shows proposed MCFDN's. Fig. 3(b) shows the delay unit in $A(z)$ is replaced by CFDN's, the direct path has been omitted in Fig. 1.



(a)



(b)

Figure 3. (a) Simplified structure of modified circulant feedback networks,
(b) Detailed structure of A(z), is omitted the direct path in Fig. 1.

Using similar procedures as for (3), system response of MCFDN's is

$$H_M(z) = \frac{A - D(z^{-1})}{A + g_F c^T b - D(z^{-1})}, \quad (12)$$

the poles of the system are the solution of

$$det[A + g_F c^T b - D(z^{-1})] = 0, \quad (13)$$

the system zeros are found as the solution of

$$det[A - D(z^{-1})] = 0. \quad (14)$$

From (12), if $g_F = -1/d$, MCFDN's can be analyzed as the inverse system of CFDN's. But if $g_F < 0$, there is a great difference between CFDN's and MCFDN's. So, we will apply the case of $g_F > 0$ at (12).

Stability of MCFDN's is decided by the placement of the zeros of CFDN's, because MCFDN's have all poles on the unit circle and the zeros and poles of CFDN's are the poles and zeros in MCFDN's. And b and c are determined by the specific of CFDN's in (13), the poles of MCFDN's can be finally determined by g_F. However, it is difficult to numerically prove stability as it is with CFDN's, and so we intuitively discuss the stability of MCFDN's.

For Fig. 3(a), system response is

$$H_M'(z) = \frac{1}{1 - A(z) g_F}, \quad (15)$$

if we replace $A(z)$ by general delay units z^{-m}, system response of (15) is converted to

$$H_M''(z) = \frac{1}{1 - g_F z^{-m}}. \quad (16)$$

Consequently, it is expected results that (16) corresponds with the system response of a general comb filter. If g_F is smaller than one, the system of (16) is stable. When $A(z)$ is stable, MCFDN's will be stable too. Therefore, stability of MCFDN's is intimately associated with $g_F c^T b$ in (12). When $c^T b$ is decided by specifics of CFDN's at first, then stability of MCFDN's can be decided by only g_F.

Also g_F may be very small, frequency response of (16) becomes nearly flat, due to the characteristics of a general comb filter. And then, frequency response of MCFDN's is able to be nearly flat under $0 < g_F < 1$.

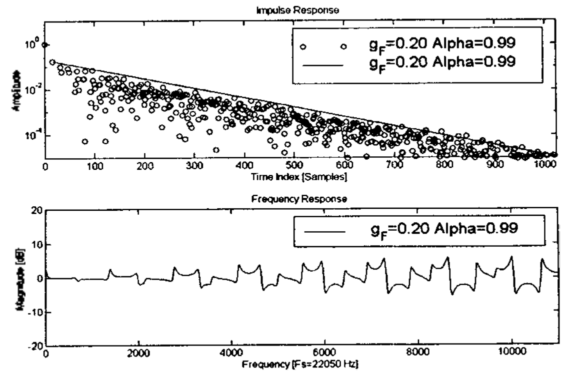


Figure 4. Time and frequency behaviors of modified circulant feedback networks, with same conditions of fig 2. and $g_F = 0.2$.

Fig. 4 depicts time and frequency behaviors of MCFDN's with same condition of Fig. 2 and $g_F = 0.2$. From Fig. 4, time density have similar views on orignal CFDN's as Fig. 2. However, we know that frequency response improved considerably. So, we will obtain the improvement of the flatness of frequency response while maximally maintaining characteristics of the impulse response of CFDN's by appropriately adjusting g_F of MCFDN's. It is the main issue finding optimum g_F for each α.

In next section, we are going to simulate and discuss characteristics of MCFDN's with varying values of α and g_F.

IV. Simulations and Discussion

We are going to simulate MCFDN's by varying value of α and g_F in order to examine and discuss results of simulation in this section.

In this paper, it is main core to analyze validity for improving frequency response of MCFDN's while maintaining maximally the impulse response of CFDN's. Therefore, it is necessary to examine the characteristics of MCFDN's with varying g_F. And the α is a main parameter for affecting reverberation time in CFDN's. It is necessary to discuss the correlation α between g_F, is very important parameter in MCFDN's.

However, we can't objectively choose it, because the standard of objective selection don't exist. And then, we will investigate it by choosing optimum g_F for each α after comparing and analyzing much data, obtain simulating a number of times.

To choose optimum g_F and verify the usefulness of

MCFDN's, we put the limitation of Condition 1, is defined by simulating many times.

Condition 1.

Each value of parameter of CFDN's is equal to case of fig 2. except α.

$0.99 \leq \alpha \leq 0.9999$, *Step* 0.0005

$0.1 \leq g_F \leq 0.2875$, *Step* 0.0125.

We put Condition 1 to simplify the procedure for simulation, and examine easily results. And in Condition 1, upper range of g_F doesn't exceed 0.3, because of insuring stability of MCFDN's.

We have simulated MCFDN's with Condition 1 to obtain the effect of g_F and α, Fig. 5 represent partially some results of simulation to compare time-frequency response of CFDN's and MCFDN's. In Fig. 5, g_F can be considered optimum value for each α. Since it is not easy to analyze the results and simplify ones, Fig. 5 are represented by different method in Fig. 2 and 4. We represent the results by line, means decay-slop of impulse response to compare easily with those of CFDN's.

From Fig. 5, we are able to get a nearly flat frequency response than CFDN's, and observe the difference of decay-slop at the same time. But this is little important than that, because of getting the more improvement at flatness than decay-slop error. Therefore we can build an artficial reverberator having a good quality for frequency response.

Also, it is very important to examine the correlation g_F between α in Fig. 5, but there are many problems to obtain the correlation because there is difficulty to numerically analyze both CFDN's and MCFDN's. Although it is possible to apply non-linear algorithms (genetic algorithm, fuzzy theory, etc), these ways include problems for real-time processing. Therefore, we can superficially define that g_F is in inverse proportion to α in Condition 1 and effect the characteristics of MCFDN's.

It is necessary to investigate reverberation time and time density, are very important cues for artificial reverberators. Although we can find that there are some errors (decay-slop error , etc) in the initial time domain, for example, the initial value of MCFDN's is smaller than CFDN's as Fig. 5. but reverberation time is similar between MCFDN's and CFDN's. It is more important than the error of initial value in time domain. And, we can get enough time-density for artificial reverberator as Fig. 2 and 4. So, MCFDN's are considered useful as a basic system for a superior artificial reverberator.

As the fact mentioned above, we have discussed the importance of g_F in MCFDN's and usefulness of MCFDN's for artificial reverberators.

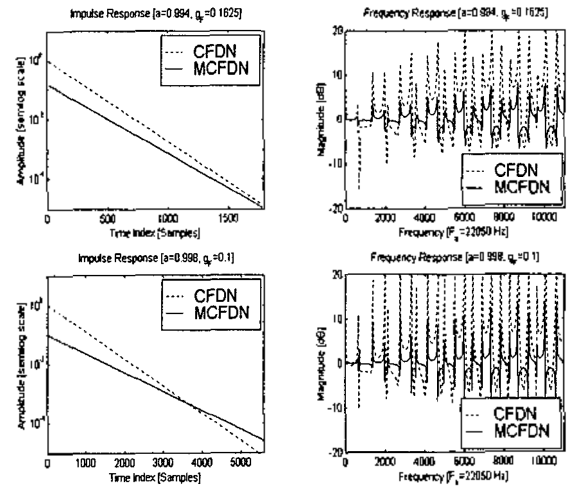


Figure 5. To compare time-frequency response of CFDN's and MCFDN's, a part of result of simulation with g_F can be considered optimum value for each α.

V. Conclusion

In this paper, We have presented Modified CFDN's (MCFDN's) to get a flatter frequency response than CFDN's while maximally maintaining the impulse response of CFDN's. Proposed MCFDN's have three merits above all: first, the structure is simple because of application of a general recursive filter defined as a comb filter; secondly, it is possible to directly use the system analysis theory of CFDN's; finally, although there is some error in the impulse response between MCFDN's and CFDN's, MCFDN's provide an improvement in that their frequency response is flatter than that of CFDN's.

This paper has presented g_F as the main factor determining the stability and characteristics of MCFDN's. Also, we describes superficially that the optimum g_F is in inverse proportion to α from simulation results for Condition 1 in section IV, although we don't induce the correlation exactly.

Since, in proposed MCFDN's, we may get the generalized correlation between α and g_F and smaller error in time domain. MCFDN's are expected to be a good basic system for implementing a good artificial reverberator. Therefore, it would be worthwhile to research this point in a future study.

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