◈ Research Papers

Warranty Cost Models for a Second-Hand Products

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Abstract

A warranty cost analysis for new products have received a lot of attention. In contrast, there is hardly any literature on similar analysis for second-hand products. The market of second-hand products has been increasing and along with that the importance of warranties for second-hand products has also been increasing. However, warranty policies similar to new products are not economically acceptable to dealers. One needs to formulate new warranty policies and models to estimate expected warranty costs for second-hand products. This paper proposes one-dimensional cost sharing warranty policies and develops models at system level to analysis warranty cost for second-hand products sold with these policies.

Key words: Warranty cost, Second-hand, Cost sharing, Lump-sum cost sharing.

1. Introduction

A warranty is a contractual obligation incurred by a manufacturer(vendor/seller) in connection with the sale of a product. The purse of warranty is to establish liability in the event of a premature failure of an item or the inability of the item to perform its intended function. The contract specifies the promised product performance and, when it is not met then, the redress available to the buyer as compensation. Warranty serves many roles.

1.1 Protective role: It protects manufacturers by putting certain responsibilities on the consumer such as proper use of the product, adequate care, etc. In the process it limits the manufacturer's liability for product failure. For the buyer, it acts as an insurance against early failures of an item due to poor quality of design and/or manufacturing. This is achieved by the manufacturer repairing or replacing items that fail during the warranty period often at no cost the buyer.

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1.2 Advertising role: It acts as an effective advertising tool for a manufacturer to compete effectively against other manufacturers.

1.3 Informative role: It indicates the quality and reliability of the product. This is especially important in the context of complex or innovative new products where the consumer is not able to evaluate performance because of lack of knowledge or experience.

Warranty for new products have received a lot of attention. Many different types of warranty policies have been studied from both manufacturer and buyer perspectives. The study deals with many different issues ranging from technical (warranty and product design or manufacturing) to commercial (warranty and marketing, warranty serving) and the effective management of the warranty process.

In contrast, there is hardly any literature on similar analysis for second-hand products. Second-hand products are products which have been used before. The market of second-hand products has been increasing. The importance of warranties for second-hand products has also been increasing due to consumer pressure and competition. Better warranty terms for an item imply greater protection to the buyer against failures or problems with the item. Warranty also leads to an increase in the probability of selling the item, but this is achieved at the expense of increased warranty serving cost. As a result, it has an impact on the sale price which, in turn, has an impact on the probability of selling the item. So warranty terms must be selected in a manner which takes into account these interactions and their impact on the overall profit for the dealer. Moreover, each second-hand item is unique in terms of age and condition. The condition is influenced by the past usage and maintenance history.

The dealer might not have complete knowledge about the condition of a second-hand item. Therefore, warranty policies similar to new products are not economically acceptable to dealers. One needs to formulate new warranty policies and models to estimate expected warranty costs for second-hand products.

In this paper, we define one-dimensional cost sharing warranty policies and develop models at system level to analyse warranty cost for second-hand products sold individually. The outline of this paper is as follows. In Section 2 and 3, we briefly review the modelling and analysis of warranties for new products and second-hand products, respectively. In section 4 we propose cost sharing warranty policies. In section 5 we develop models for failures and costs and section 6 discusses expected warranty cost for lump-sum cost sharing. Finally, we conclude with a brief discussion of probable topics for future research.

2. Warranties for New Products

A variety of warranty policies have been developed for new products. The two commonly offered warranties are free replacement warranty (FRW) and pro-rata warranty (PRW). Blischke and Murthy (1992) proposed a taxonomy for classification of warranties. Claims over warranty period results from failure. The first failure can be characterized in terms of the failure distribution function $F(t)$, failure density function $f(t)$ or the failure rate $r(t)=f(t)/(1-F(t))$. In the case of a non-repairable item, a failed item needs to be replaced by a new (or used) one. In the case of a repairable item, different types of repair action result in different characterization. One such repair action is the minimal repair, where the failure rate of the item after repair is the same as that just before failure. If the time to rectify a failed item is less than mean time between failures then we can treat the time to rectification as being nearly zero. In this case, failures over the warranty period can be modeled by renewal process if the rectification involves replacing failed item by new ones. In the case of minimal repair, the failures are characterized by a non-stationary Poisson process. A variety of models for warranty cost analysis based on these formulations can be found in Blischke and Murthy (1993). The warranty cost can be reduced through better design and manufacturing. Reliability design in the context of warranties can be found in Nguyen and Murthy (1988) and Murthy and Hussian(1995). Effect of quality variation and control on warranty cost can be found in Murthy, Wilson and Djamaludin(1993). Better warranty term results in greater sales. However, this increases the warranty cost. Optimum pricing and duration of warranty is studied in Glickman and Berger (1976)

3. Warranties for Second-Hand Products

The products sold are statistically similar in the case of new products. In contrast, the case of second hand products, each product needs to be treated differently as it is dependent on various factors such as age, usage and maintenance history. These factors define the state of the second hand product. In the case of used products, the dealer can either have the same warranty terms for all products sold or they can vary from product to product. The former case is similar to the new product warranties where the terms are set by the dealer. In the latter case, the consumer and dealer can negotiate the warranty for each product separately. In either case, the warranty can either cover the product as a whole or exclude one or more of the components.

The state of the used product often can be improved through an upgrade action where the product is either overhauled and/or some of the deteriorated components are replaced by better components. Such an action results in additional cost to the dealer. However, such actions allow for higher sale price and reduce the warranty cost. Decisions with regard to upgrades must be done by doing a proper cost benefit analysis. An additional complication is with regards the information relating to the state of the used product. This aspect is critical for both the dealer as well as the consumer. In the case of the consumer, uncertainty about the state can seriously influence the purchase decision and warranty can serve as an effective assurance tool. Finally, warranties need to be studied from two perspectives: dealer and consumer perspectives. From the consumer's point of view, the analysis is to decide on whether to buy the product or not and the type of warranty. From the dealer's point of view, the analysis is needed to answer one or more of the following questions: 1. What is the expected cost of warranty as a function of warranty duration? 2. Should the product be subjected to an upgrade? 3. How should the product be priced taking into account the warranty costs.

3.1 Taxonomy for Second Hand Warranties

A taxonomy for the different types of one-dimensional warranty policies for second-hand products is shown in Figure 1. The policies can be divided into two groups based on whether the policies offer a buy-back option or not. Policies without buy-back options imply that the seller has no obligation to take back an item sold. As a result, the warranty expires after the duration indicated in the warranty policy. Any failures within the warranty period are rectified according to the items off the warranty policy. Theses policies can be further subdivided into two subgroups-Group A: Non-renewing policies and Group B: renewing policies.

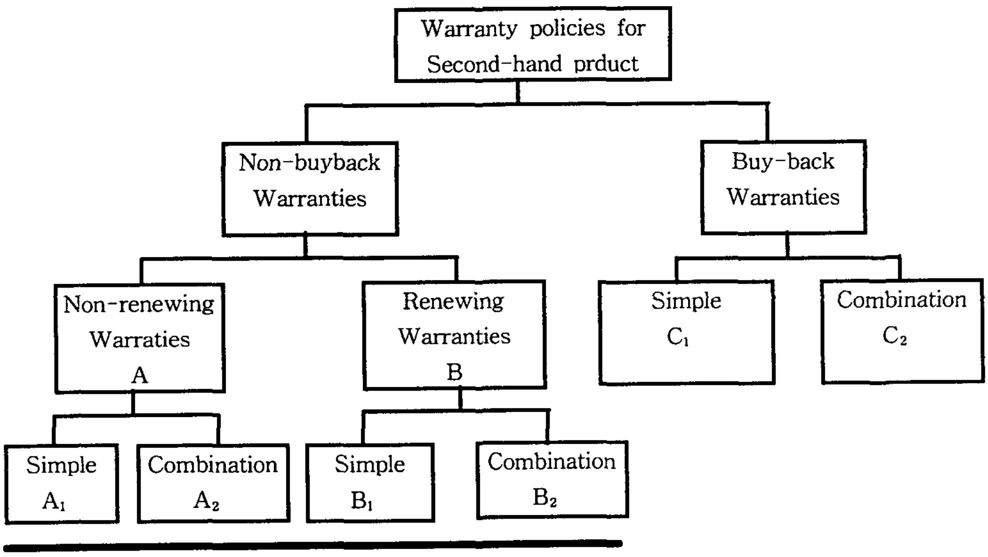

Fig. 1 Taxonomy for Second Hand Warranties

Under a non-renewing warranty, the items of the warranty do not change during the warranty period. As a result, if an item fails during the warranty period, it is rectified by the dealer and returned to the buyer without any changes to the original warranty terms. Under a renewing warranty, the warranty items change, for example, after failure, the item is returned with a new warranty either identical to, or different from, the original warranty terms. Each of theses can be further subdivided into two subgroups - Group A1(B1): Simple policies and Group A2(B2): Combination policies. Under the buy-back option, the buyer can get a monetary refund(either full or fraction of the sale price)by returning the purchased item any time within the warranty period. The warranty terminates when this occurs. We group such policies under Group C. All failures before the termination of the warranty are rectified according to the terms of the warranty.

3.2 Type A [Non-renewing] Policies

3.2.1 FRW policies

No cost to buyer: The dealer agrees to repair or provide replacements for failed items free of charge up to a time W from the time of the initial purchase. The warranty expires at time W after purchase.
Cost Sharing Warranty[CSW] Policies: Under the cost sharing warranty the buyer and the dealer share the repair cost. The basis for the sharing vary as indicated below.

Policy 1 - Specified Parts Excluded[SPE]: Under the policy, the dealer rectifies failures of components belong to the set I at no cost to the buyer over the warranty period. The costs of rectifying failures of components belong to the set E are borne by the buyer.

Cost Limit Warranty[CLW] Policies: Under the cost limit warranty policy, the dealer's obligations are determined by cost limits on either individual claims or total claims over the warranty period.

Policy 2 - Limit on Individual Cost[LIC]: Under this policy, all claims under warranty are rectified by the dealer. If the cost of a rectification is below the limit c_I, then it is borne completely by the dealer and the buyer pays nothing. If the cost of a rectification exceeds c_I, then the buyer pays the excess.

Policy 3 - Limit on Total Cost[LTC]: Under this policy the dealer's obligation ceases when the total repair cost over the warranty period exceeds c_T. As a result the warranty ceases at W or earlier if the total repair cost, at any time during the warranty period, exceeds c_T.

3.2.2 PRW Policy. The manufacturer agrees to refund a fraction of the purchase price should the item fail before time W from the time of the initial purchase. The buyer is not constrained to buy a replacement item. The function characterizing the fraction refunded can be either a linear or non-linear function of the age of the item at failure.

3.2.3 FRW-PRW combination policies. These are as follows:
No Cost to Buyer: The manufacturer agrees to provide a replacement or repair free of charge up to time W_1 from initial purchase; any failure in the interval W_1 to W results in a pro-rated refund. The warranty does not renew.

Combination Policies with Limits[CLW]. Two policies in this group as follows:

Policy 4 - Limits on individual and total cost[LITC]: Under this policy, the cost to the dealer has an upper limit(c_I) for each rectification and the warranty ceases when the total cost to the dealer exceeds c_T or at time W whichever occurs first. The difference in the actual cost of rectification and the cost borne by the dealer is paid by the buyer.
Policy 5 - [FRW-LIC]: Under this policy the dealer repairs all failures in the interval $[0, W_1)$ at no cost to the buyer. The rectification of a failure in the interval $[W_1, W)$ results in no cost to the buyer if it is below c_I and an amount which is the excess if it exceeds c_I

3.3 Type B[Renewing] Policies

Policy 6 - Partial renewing free replacement warranty[PR-FRW]: Under this policy the dealer rectifies all failures in the interval $[0, W_1)$ at no cost to the buyer. The warranty period after rectification is the remaining period of the original warranty. If a failure occurs in the interval $[W_1, W)$ then the item is rectified by the dealer at no cost to the buyer and returned with a new warranty of duration $(W-W_1)$. This implies that the warranty ceases only when there is no failure in the period $[W_1, W)$ under the original warranty or there is no failure under the new warranty.

3.4 Type C[Buy-Back] Policies

3.4.1 Simple buy-back warranty[BBW] Policies. Under a buy-back warranty policy, the buyer has the option to return an item within the warranty period with the dealer refunding either a fraction or the full sale price. The refund can also include payment for associated costs incurred by the buyer. This option can be either unconditional(money back guarantee) or conditional on certain events, if the number of failures over the warranty period exceed some specified limit.

Policy 7 - Money Back Guarantee[MBG]: Under the money-back guarantee policy all failures over warranty period $[0, W)$ are rectified at no cost to the buyer. If the number of failures over $[0, W)$ exceed a specified value k $(k \geq 1)$, then at the $(k+1)$ failure, the buyer has the option of returning the item for 100% money-back and warranty ceases when the buyer exercises this option. If the number of failures over $[0, W)$ is either $\leq k$ or the buyer does not exercise the buy-back option when the $(k+1)$ failure occurs, then the item is covered for all failure till W.

3.4.2 Combination buy-back warranty[BBW] policies. A combination buy-back policy is one where the buy-back option is combined with one or more of the non-buyback policies as indicated below.

Policy 8 - [MBG-FRW]: Under this policy all failures over warranty period $[0, W)$ are rectified at no cost to the buyer. If the number of failures over $[0, W_1)$, for $W_1 < W$, exceeds a specified value $k(k \geq 1)$, then at the $(k+1)$ failure the buyer has the option of returning the item for 100% money-back and the warranty ceases when the buyer exercise the buy-back option when the $(k+1)$ failure occurs, the item is covered for all failures till W

4. Warranty Cost Analysis for Second-Hand Products

We will confine our attention to expected warranty cost per item. In the new product case all items are statistically similar. In contrast, in the case of second hand product each item is different so that they are not similar. The failures over the warranty period depend on item condition. This in turn depends on the age and prior maintenance history. In the simplest case, the condition is a function of the age of the item and effect of maintenance is ignored. One needs to examine two situations:

In this section we discuss the modelling of failures at the system level. To simplify the analysis, we make the following assumptions: 1) The failures are statistically independent. 2) The failure of a used item is only a function of its age and not its past usage and/or maintenance history. 3) The item to carry out the rectification action(either repair or replace) is relatively small compared to the mean time between failures so that it can be treated as being nearly zero. This is not too unrealistic in the real world. 4) Every item failure under warranty period results in a claim and all claims are valid.

4.1 Model 1

The age of the item at sale, A, is known. The item failure is modelled by a point process with intensity function $r(t)$ where t represents the age of the item. $r(t)$ is an increasing function of t indicating that the number of failures(in a statistical sense) increases with age. The failures over the warranty period $[A, A+W)$ occur according to a non-stationary Poisson process with intensity function $r(t)$. This implies that $N(W;A)$, the number of failures over the warranty period W for an item of age A at the time sale, is a random variable with

$$P\{N(W,A)=n\}=\frac{\left[\int_A^{A+W} r(t)dt\right]^n exp\left[-\int_A^{A+W} r(t)dt\right]}{n!} \tag{1}$$

Then the expected number of failures over the warranty period is given by

$$E[N(W,A)]=\int_A^{A+W} r(t)dt \tag{2}$$

This type of characterization is appropriate when an item failure occurs due to one or more component failures and item is made operational through repair or replacement of the failed components and no action being taken with regards the remaining workings. Since the number of failed components at each failure is a very small relative to the number of components in the item, the rectification action can be viewed as having negligible impact on the failure rate of the item as a whole. In other words, the failure rate after a repair is nearly the same as that just before failure. Such a repair action is called minimal repair(Barlow and Hunter[1960]). In this case $r(t)$ is the failure rate associated with the failure distribution for the item. A simple form for $r(t)$ is as follows;

$$r(t)=\lambda\beta(\lambda t)^{\beta-1}, \quad \beta>1,\ \lambda>0 \tag{3}$$

This is an increasing function of t. Note that this corresponds to the failure rate for a Weibull distribution. This implies that item failure distribution is given by a two parameter Weibull distribution. The expected number of failures over the warranty period is given by

$$E[N(W,A)]=\lambda^\beta[(A+W)^\beta-A^\beta] \tag{4}$$

A slightly more complex formulation for $r(t)$ is the following:

$$r(t)=\sum_{i=0}^{K} p_i\, r_i(t) \tag{5}$$

with $p_i>0$, and

$$r_i(t)=\lambda_i\beta_i(\lambda_i t)^{\beta_i-1}, \quad \beta_i>1,\ \lambda_i>0,\ 1\le k\le K \tag{6}$$

If $\sum_{i=0}^{K} p_i=1$, then one can interpret the model as being decomposed into K disjointed subsystem, with failure of each subsystem being characterized by a point process with intensity function $r_i(t)$.

4.2 Model 2

Here the age of the item at sale, A, is unknown. We model this as a random variable with a distribution function $H(a)$ with $H(L)=0$ and $H(U)=1$. The L and U represent the lower and upper limits for A. Let $h(a)=dH(a)/da$ denote the density function associated with $H(a)$. The values for L and U vary with the type of product. A form of $H(a)$ which is analytically tractable is the following ;

$$H(a)=\frac{e^{-\rho L}-e^{-\rho a}}{e^{-\rho L}-e^{-\rho U}} \tag{7}$$

This implies that the age is given by a truncated exponential distribution with the mean value of the age given by

$$E[A]=[(Le^{-\rho L}-Ue^{-\rho U})+(e^{-\rho L}-e^{-\rho U})/\rho]/[e^{-\rho L}-e^{-\rho U}] \tag{8}$$

As in Model 1, the failures occur according to a point process with an intensity function $r(t)$. Conditional on $A=a$, the number of failures over the warranty period, N(W), is a random variable with

$$P\{N(W)=n \mid A=a\}=\frac{\left[\int_a^{a+W} r(t)dt\right]^n exp\left[-\int_a^{a+W} r(t)dt\right]}{n!} \tag{9}$$

Using the conditional expectation argument, we have

$$P\{N(W)=n\}=\int_L^U \frac{\left[\int_a^{a+W} r(t)dt\right]^n exp\left[-\int_a^{a+W} r(t)dt\right]}{n!} dH(a) \tag{10}$$

and from this we have the expected number of failures over the warranty period given by

$$E[N(W)]=\int_L^U \int_a^{a+W} r(t)dt\, dH(a) \tag{11}$$

5. Cost Analysis for Repairable Items

In this section, the dealer's and buyer's expected warranty cost per item sold will be obtained for Model 1 and 2, respectively. Examples for these will be given in this section.

5.1 Model 1

In this subsection we carry out an analysis to obtain expected warranty costs. Since the cost is shared by the dealer and the buyer, we have for the *ith* failure, $D_i=\alpha C_i$ and

$B_i=(1-\alpha)C_i$, where $\alpha(0<\alpha<1)$ is the fraction of cost shared by the dealer. In this case, the expected number of failures is given by (2). Then the expected warranty cost to the dealer, $E[C_d(W,A)]$, is given by

$$E[C_d(W,A)]=\alpha\bar{c}\int_A^{A+W} r(t)dt \tag{13}$$

The expected cost to the buyer over the warranty period, $E[C_b(W,A)]$, is given by

$$E[C_b(W,A)]=(1-\alpha)\bar{c}\int_A^{A+W} r(t)dt \tag{14}$$

Numerical Example 1

Here the failures are modelled by (3) with $\beta=2$ and $\lambda=0.443$. . This implies that the mean time to failure, $\mu=2$ years. The age distribution(when item age not known) is given by (7) with $\rho=0.2$ and we consider different values for L and U. Based on data set with $\alpha=0.75$, the ratios, $E[C_d(W,A)]/\bar{c}$ and $E[C_b(W,A)]/\bar{c}$ for different combinations of A and W are shown in Tables[1] and [2] respectively.

Table[1] $E[C_d(W,A)]/\bar{c}$

W(warranty)	A(age)				
	1	2	3	4	5
0.5	0.1840	0.3312	0.4784	0.6256	0.7727
1.0	0.4416	0.7360	1.0304	1.3247	1.6191
1.5	0.7727	1.2143	1.6559	2.0975	2.5390
2.0	1.1775	1.7663	2.3550	2.9438	3.5325

Table[1] $E[C_b(W,A)]/\bar{c}$

W(warranty)	A(age)				
	1	2	3	4	5
0.5	0.0613	0.1104	0.1595	0.2085	0.2576
1.0	0.1472	0.2453	0.3435	0.4416	0.5397
1.5	0.2576	0.4048	0.5520	0.6992	0.8463
2.0	0.3925	0.5888	0.7850	0.9813	1.1775

5.2 Model 2

Here the expected number of failures is given by (11). Then the expected warranty cost to the dealer, $E[C_d(W,A)]$ is given by

$$E[C_d(W)] = \alpha\bar{c}\int_L^U\int_a^{a+W} r(t)dt\,dH(a) \tag{15}$$

The expected cost to the buyer over the warranty period, $E[C_b(W,A)]$, is given by

$$E[C_b(W)] = (1-\alpha)\bar{c}\int_L^U\int_a^{a+W} r(t)dt\,dH(a) \tag{16}$$

Numerical Example 2

Based on data set with $\alpha = 0.75$, the ratios, $E[C_d(W)]/\bar{c}$ and $E[C_b(W)]/\bar{c}$ for different combinations of $L=1$, U and W are shown in Tables[3] and [4] respectively.

Table[3] $E[C_d(W,A)]/\bar{c}$

W(warranty)	$L=1$				
	$U=3$	$U=4$	$U=5$	$U=6$	$U=7$
0.5	0.3214	0.3828	0.4395	0.4916	0.5393
1.0	0.7164	0.8393	0.9527	1.0568	1.1522
1.5	1.1849	1.3693	1.5394	1.6957	1.8386
2.0	1.7271	1.9728	2.1997	2.4040	2.5989

Table[4] $E[C_b(W,A)]/\bar{c}$

W(warranty)	$L=1$				
	$U=3$	$U=4$	$U=5$	$U=6$	$U=7$
0.5	0.1071	0.1276	0.1465	0.1639	0.1798
1.0	0.2388	0.2798	0.3176	0.3523	0.3841
1.5	0.3950	0.4564	0.5131	0.5652	0.6129
2.0	0.5757	0.6776	0.7332	0.8027	0.8662

6. Conclusions

A warranty cost analysis for new products have received a lot of attention. But there is hardly any literature on similar analysis for us second-hand products. The expected warranty cost for a second-hand products depends on the age of the product and/or its earlier usage history. As a result, the model formulations for warranty cost analysis for second-hand products are more complex than that for new products. we discusses the stochastic framework for warranty cost modeling for second products. In this paper we have discussed cost sharing warranty policies and system level modelling for determining the expected warranty cost for second hand products sold with these policies. Some extensions are as follows; 1) To carry out similar cost analyse for other type of policies such as cost limit warranty and buy-back warranty. 2) To build models incorporating past usage and maintenance. 3)To incorporate upgrade of second hand products before sale. 4) To build models for price warranty package and probability of sale.

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