⊠ 연구논문

Cell Formation Models Considering Loading, Alternative Routes, and Operations in a FMS

-FMS에 있어서 공정할당 및 대체 가공경로를 고려한 셀 형성모델*-

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요 지

본 연구에서는 다양한 종류의 공정을 수행할 수 있는 가공장비와 자동화된 물류 운반시스템으로 구성된 FMS의 효율적 운영을 위해 셀 제조방식의 개념을 도입한 다수의 유연셀 형성 방법을 제시하고 있다. 서로 독립적인 유연셀의 형성을 위해 FMS의 장점인 공정유연성을 최대한 활용할 수 있도록 기계-공정 행렬과 부품-공정 행렬에 기초한 2 종류의 새로운 유사도 계수와 예외적 요소의 수를 최소화하기 위한 셀 형성 알고리즘을 개발하였다. 두 행렬들은 다공정 수행가능기계간 유사도와 기계셀과 부품간 비처리 능력지수 산출의 기초자료로 사용된다. 알고리즘은 예외적 요소의 수를 최소로 하면서 셀의 수를 최대로 하는 정리에 기초하여 공정을 기계에 할당하며, 다수의 대체경로가 유연셀내에서 형성될 수 있도록 크게 2 단계로 구성되어 있다. 마지막으로 수치예제와 함께 예외적 요소의 발생수를 척도로 하여 기존의 방법들과 비교, 평가하고 있다.

1. Introduction

Group Technology (GT) has been well recognized as one of the most important factors in improving the productivity of manufacturing systems. Although numerous publications have appeared on the subject of GT, room for improvement in cell formation methods still exists. A number of factors including machine failure rates, resource utilization, workload balance in a cellular manufacturing system should be thoroughly investigated to take full advantage of GT. However, conventional GT tries to avoid the interactions between the cells and tends to set up permanent idealistic machine cells. Hence, the conventional GT does not consider any Exceptional Element (E.E) which was first introduced by King (1980). The basic idea of GT is the decomposition of the manufacturing system into subsystems by classifying the parts into families and machines into machining cells based on the similarity of the part manufacturing characteristics.

The success or failure of cell formation to any problem depends mainly on the identification of similarity measures. The similarity measures published in the literature have a limitation in solving the E.E related problem (Kusiak and Cho, 1992). The similarity coefficients presented in Table 1 consider neither minimizing the E.E nor machining operations in the part-machine incidence matrix.

This paper presents a cell formation model to minimize the E.Es based on the similarity of individual machines. A theorem on determining the maximum number of cells, not including any E.E, is studied. The solution procedure of cell formation using the Hungarian method is presented with illustrative examples.

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Table 1. Similarity measures

	Coefficient	Reference	Formula	Form
Coefficient without considering flow	Mincowski	Arthanari and dodge	$\left(\sum_{k=1}^{n} \mid a_{ik}-a_{jk} \mid p\right)^{1/p}$	Integer
	McAuley	McAuley	$\frac{\sum_{k=1}^{n} \delta'(a_{ik}, a_{jk})}{\sum_{k=1}^{n} \delta''(a_{ik}, a_{jk})}$	0≤s≤1
	Kusiak and Cho	Kusiak	1, if $a_{ik} \geq a_{il}$ or $a_{ik} \leq a_{il}$, for all i 0, otherwise	Integer
Coefficient with considering flow	Jaccard	Leskowsky et, al.	$\frac{n_{ij}}{(n_i+n_j-n_{ij})}$	0≤s≤1
	Dutta et al	Dutta et, al.	$n_i+n_j-2n_{ij}$	Integer
	Dice-Sorensen	Chu and Pan	$\frac{2n_{ij}}{(n_i+n_j)}$	0≤s≤1
	Dot-product	Chu and Tsai	$\frac{n_{ij}}{(n_i+n_j)}$	0≤s≤1
	Chu and Lee	Chu and Lee	$\frac{n_{ij}^2}{(n_i+n_j-n_{ij})}$	Integer

$\delta'(a_{ik}, a_{jk})=1$, if $a_{ik}=a_{jk}=1$; 0, otherwise

$\delta''(a_{ik}, a_{jk})=0$, if $a_{ik}=a_{jk}=0$; 1, otherwise

n_i : the number of machines visited by part i.

n_{ij} : the number of machines visited by parts i and j

2. Methodology

The following assumptions are made in this paper.

(1) The operations are assigned to the machines based on part manufacturing characteristics.

(2) Alternative machining routes of a part are available.

(3) Each machine has multiple functionalities in machining or assembly.

(4) But, all operations of the part types can not be completed in one machine.

(5) A machine cell includes more than one machine.

In this paper, a preliminary cell is defined as a cell used temporarily in the cell formation processes. The following notation is used throughout the paper.

M_j : The number of machines to perform operation j

DC_{ci} : The incapability index of machine cell c in processing part i, which denotes the number of non-performable operations of part i in machine cell c

NMC : $\left[\frac{\textit{the number of machines}}{2}\right]$

$$a_{ij}=\begin{cases}1, & \textit{if part i needs operation j}\\ 0, & \textit{otherwise}\end{cases}$$

$$n_{cj}=\begin{cases}1, & \textit{if machine cell c can perform operation j}\\ 0, & \textit{otherwise}\end{cases}$$

$$q_{kj}=\begin{cases}1, & \text{if machine } k \text{ can perform operation } j\\ 0, & \text{otherwise}\end{cases}$$

$$\delta(q_{kj}, q_{k'j})=\begin{cases}1, & \text{if } q_{kj}=1 \text{ and } q_{k'j}=1\\ 0, & \text{otherwise}\end{cases}$$

$$\alpha(n_{cj}, a_{ij})=\begin{cases}1, & \text{if } n_{cj}<a_{ij}\\ 0, & \text{otherwise}\end{cases}$$

$$\beta(n_{cj}, a_{ij})=\begin{cases}1, & \text{if } n_{cj}=1 \text{ and } q_{kj}=1\\ 0, & \text{otherwise}\end{cases}$$

2.1 Similarity coefficient

We can reduce the intercell movement by improving the capability of a cell to perform the operations of a part. One way to improve the capability of the cells, without adding any machine, is to form the cells based on the following similarity coefficients:

- The similarity coefficient of individual machines, which represents the number of operations available by a pair of machines (k, k′), $S_{kk'}$ is formulated next:

$$S_{kk'} = \sum_{j=1}^{n}\delta(q_{kj}, q_{k'j}) \ , \qquad k=1,2,\ldots,l \ , \ k'=1,2,\ldots,l$$

- The similarity coefficient between machine k and preliminary cell c, which represents the number of operations available by the machine k and the cell c, CS_{ck} is formulated next:

$$CS_{ck}= \sum_{j=1}^{n}\beta(n_{cj}, q_{kj}) \ , \qquad c=1,2,\ldots,C \ , \ k=1,2,\ldots,l$$

2.2 Basic Theory

To date, a study on the determination of the maximum number of cell without an E.E has not been performed yet. We use the following lemma and theorem to determine the range of the maximum number of cells.

[Lemma]

$NPC \le \min_{1\le j\le n} M_j$. where NPC is the number of cells required to perform the operations of all part types.

Proof: Let the number of cells, NC be $\min_{1\le j\le n} M_j + 1$. If every machine according to $\min_{1\le j\le n} M_j$ can perform operation k, the number of machines to process the operation k is NC-1. At least, one of NC-machine cells does not perform the operation k since the operation k can not be assigned to one of the NC-machine cells. If the NC-cells are formed, the number of the formed cells exceeds NPC. Therefore, $NPC \le \min_{1\le j\le n} M_j$.

The theorem, which determines the maximum number of cells not including any E.E, is derived from the above lemma.

[Theorem]

Suppose that MAXC is the maximum number of cells not including any E.E.

Case I; If $NMC = NPC$, then $MAXC = NPC$.
Case II; If $NMC > NPC$, then $NPC \le MAXC \le NMC$.
Case III; The case of $NMC < NPC$ does not exist.

Proof:

1. Case I ; $NMC = NPC$:

   One can see that NPC is the number of cells not including any E.E in any case from the above lemma. NMC is the maximum number of cells which may not have any E.E. Therefore, $MAXC = NPC$.

2. Case II ; For $NMC > NPC$:

   NPC denotes the number of cells not including any E.E. NMC represents the maximum number of cells which may not include any E.E. Hence, $NPC \leq MAXC \leq NMC$.

3. Case III; The case of $NMC < NPC$ does not exist.

   Since NMC = [the number of machines / 2], and a machine cell includes at least two machines from the assumptions, the case of $NMC < NPC$ will not occur

3. The solution procedure

In this paper, the solution procedure for the cell formation has two algorithms. Algorithms I determines the maximum number of cells, and algorithm II clusters the part families and the cells by using Hungarian method, respectively.

Algorithm I

Step 1. Calculate the inter-machine similarity coefficients from the machine-operation incidence matrix. Set the number of preliminary cells,

$$PC = \min_{1 \leq j \leq n} M_j.$$

Step 2. Find the pairs of machines from the inter-machine similarity matrix by using the Hungarian method. Determine the pairs of machines as many as PC, to minimize inter-machine similarity. If there are pairs of machines with the same similarity coefficient, choose a pair with the largest number of operations performable in the pair.

Step 3. Calculate the similarity coefficients between the unassigned machine and the preliminary cells and fill in the preliminary cell - machine similarity matrix.

Step 4. Apply the Hungarian method to the matrix of the preliminary cell - machine similarity, and assign machines to the preliminary cells.
If there is an unassigned machine, go to step 3.
Otherwise, if the cells formed could perform all operations to be completed, set the number of cells, NC = NPC and go to step 5. Otherwise, go to step 2.

Step 5. Based on the theorem in section 2, determine the range of the possible number of cells or the maximum number of cells which does not include any E.E If the maximum number of cells is, in Case II, set MAXC = the MAXC - 1 and NC = MAXC. If an E.E exists, go to step 1. Otherwise, stop.

Algorithm II

Step 1. Find the pairs of machines from the inter-machine similarity matrix by using the Hungarian method. Determine the pairs of machines, which minimizes the inter-machine similarity, by as many as the number of preliminary cells.
If there are pairs of machines with an identical similarity coefficient, choose a pair with the largest number of operations performable in the pair.

Step 2. Calculate the similarity coefficients between the unassigned machine and the preliminary cells and fill in the preliminary cell - machine similarity matrix.

Step 3. Apply the Hungarian method to the preliminary cell - machine similarity matrix, and assign a machine to the preliminary cells.

If all machines are included in the cells, go to step 4. Otherwise, go to step 2.

Step 4. Find the incapability indices of the machine cells for the parts, and write the matrix of part-machine cell incapability index.

If DC_{ci} > 0 and the E.E exists,

set NC = NC - 1, and go to step 1. Otherwise, go to step 5.

Step 5. Apply the Hungarian method to the matrix of the part-machine cell incapability index, and assign parts to each machine cell and remove the assigned parts from the matrix.

If there is a part to be assigned, go to step 5, Otherwise, stop.

4. Illustrative examples

To illustrate the solution procedure of the two algorithms, consider the incidence matrix of 22-parts and 15-operations in Table 2 and the incidence matrix of 13-machines and 15-operations in Table 3, respectively.

Algorithm I

Step 1. The inter-machine similarity coefficients are calculated in Table 4 from the machine-operation incidence matrix in Table 3. The number of preliminary cells is set to three, since $\min_{1 \le j \le n} M_j$ = 3.

Step 2. As shown in Table 5, one can obtain the machine pairs by applying the Hungarian method to the machine similarity matrix in Table 4. Table 6 represents the lists of performable operations the preliminary cell which is determined to maximum number of operations in the cells.

Table 2. Part-operation incidence matrix

	O1	O2	O3	O4	O5	O6	O7	O8	O9	O10	O11	O12	O13	O14	O15
P1	1			1			1			1		1			1
P2		1			1				1				1		
P3			1			1				1		1			
P4	1							1					1		1
P5		1	1		1	1			1		1				
P6	1			1			1			1		1			1
P7					1			1				1		1	
P8	1		1			1			1	1				1	
P9		1			1		1								1
P10			1					1			1	1		1	
P11	1	1		1											
P12							1		1	1		1			1
P13				1					1				1		
P14	1	1	1		1					1					
P15			1			1					1			1	
P16		1		1		1			1	1	1	1		1	
P17				1											1
P18						1	1				1				1
P19	1	1							1				1		1
P20				1			1					1			1
P21	1				1			1							
P22		1		1					1	1		1	1		

Table 3. Machine-operation incidence matrix.

	O1	O2	O3	O4	O5	O6	O7	O8	O9	O10	O11	O12	O13	O14	O15
M1	1	1		1			1		1			1		1	1
M2			1		1			1					1		
M3				1			1				1	1			
M4	1	1				1			1				1		1
M5			1	1	1			1			1			1	
M6	1						1			1			1	1	
M7			1		1			1						1	
M8	1					1	1	1			1			1	1
M9		1	1	1					1			1			
M10	1				1	1	1			1		1	1	1	
M11			1						1					1	1
M12		1				1		1				1			
M13				1		1				1			1		

Table 4. Machine similarity matrix

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13
M1	0	0	3	4	2	3	1	4	4	4	3	2	1
M2	0	0	0	1	3	1	3	1	1	2	1	1	1
M3	3	0	0	0	2	1	0	2	2	2	0	1	1
M4	4	1	0	0	0	2	0	3	2	3	2	2	2
M5	2	3	2	0	0	1	4	3	2	2	2	1	1
M6	3	1	1	2	1	0	1	3	0	5	1	0	2
M7	1	3	0	0	4	1	0	2	1	2	2	1	0
M8	4	1	2	3	3	3	2	0	0	4	2	2	1
M9	4	1	2	2	2	0	1	0	0	1	2	2	1
M10	4	2	2	3	2	5	2	4	1	0	1	2	3
M11	3	1	0	2	2	1	2	2	2	1	0	0	0
M12	2	1	1	2	1	0	1	2	2	2	0	0	1
M13	1	1	1	2	1	2	0	1	1	3	0	1	0

Table 5. The number of performable operations on each pairs

Machine pair	No. of performable operations	Machine pair	No. of performable operations
Machine 1,13	11	Machine 8,9	12
Machine 2,1	12	Machine 9,8	12
Machine 3,2	8	Machine 10,11	11
Machine 4,5	12	Machine 11,10	11
Machine 5,4	12	Machine 12,6	9
Machine 6,12	9	Machine 13,7	8
Machine 7,3	8		

Table 6. Performable operations in preliminary cell

	Performable operation in each machine	Performable operation in each machine pair
Machine 1, 2	Machine 1: 1, 2, 4, 7, 9, 12, 14, 15	1,2,3,4,5,7,8,9,12,13,14,15
	Machine 2 : 3, 5, 8, 13	
Machine 4, 5	Machine 4 : 1, 2, 6, 9, 13, 15	1,2,3,4,5,6,8,9,11,13,14,15
	Machine 5 : 3, 4, 5, 8, 11, 14	
Machine 8, 9	Machine 8 : 1, 6, 7, 8, 11, 14, 15	1,2,3,4,6,7,8,9,11,12,14,15
	Machine 9 : 2, 3, 4, 9, 12	

Step 3. The similarity coefficients between the preliminary cell and the unassigned machines are obtained in Table 7.

Table 7. Similarity coefficients between the preliminary cell and the unassigned machines

	M3	M6	M7	M10	M11	M12	M13
Preliminary cell 1	3	4	4	6	4	3	2
Preliminary cell 2	2	3	4	5	4	3	3
Preliminary cell 3	4	3	3	5	4	4	2

Step 4. The Hungarian method is applied to the preliminary cell-machine similarity matrix in Table 7. The available operations in each preliminary cell are listed in Table 8.

Table 8. Available operations in a preliminary cell

	The assigned machine	Performable operation
Preliminary 1	Machine 1,2,13	1,2,3,4,5,***6***,7,8,9,***10***,12,13,14,15
Preliminary 2	Machine 4,5,3	1,2,3,4,5,6,7,8,9,11,***12***,13,14,15
Preliminary 3	Machine 8,9,6	1,2,3,4,6,7,8,9,***10***,11,12,***13***,14,15

Step 3. The similarity coefficients between the preliminary cell and the unassigned machines are calculated in Table 9.

Table 9. Similarity between the preliminary cell and the unassigned machines

	M7	M10	M11	M12
Preliminary cell 1	4	8	4	4
Preliminary cell 2	4	7	4	4
Preliminary cell 3	3	7	4	4

Since machine cell 1 can not perform operation 11. We set NPC = 2 and go to step 5.

Table 10. Available operations in final preliminary cell

	The assigned machine	Available operation
Machine cell 1	Machine 1,2,13,12	1,2,3,4,5,6,7,8,9,10,12,13,14,15
Machine cell 2	Machine 4,5,3,11,10	1,2,3,4,5,6,7,8,9,***10***,11,12,13,14,15
Machine cell 3	Machine 8,9,6,7	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15

Step 5. Since $NPC = 2$ and $NMC = \left\lceil \frac{13}{2} \right\rceil = 6$. The range of the maximum number of cells is obtained as $2 \leq MAXC \leq 6$ from the theorem presented in section 2.

Algorithm II

We formed six cells in table 11 by steps 1, 2, and 3.

Table 11. Available operations in a preliminary cell

Machine cell	The assigned machine	Available operation
Machine cell 1	Machine 1,2	1,2,3,4,5,7,8,9,12,13,14,15
Machine cell 2	Machine 4,5	1,2,3,4,5,6,8,9,11,13,14,15
Machine cell 3	Machine 6,12	1,2,6,7,8,10,12,13,14
Machine cell 4	Machine 8,9	1,2,3,4,6,7,8,9,11,12,14,15
Machine cell 5	Machine 3,7,13	3,4,5,6,7,8,10,11,12,13,14
Machine cell 6	Machine 10,11	1,3,5,6,7,9,10,12,13,14,15

Step 4. The incapability index is calculated in the part-machine cell incidence matrix in Table 12. It present that if number of cell is 6, E.E(Part 1, 6, 14, 16, 22) is included in cell formation.

Table 12. Part-machine cell incidence matrix of an incapability index

	M/C Cell 1	M/C Cell 2	M/C Cell 3	M/C Cell 4	M/C Cell 5	M/C Cell 6
Part 1	1	3	2	1	2	1
Part 2	0	0	2	2	2	1
Part 3	2	2	1	1	0	0
Part 4	0	0	1	1	2	1
Part 5	2	0	4	1	2	2
Part 6	1	3	2	1	2	1
Part 7	0	1	1	1	0	1
Part 8	2	1	2	1	2	0
Part 9	0	1	2	1	2	1
Part 10	1	1	2	0	0	2
Part 11	0	0	1	0	2	2
Part 12	1	3	2	1	2	0
Part 13	0	0	2	1	1	1
Part 14	1	1	2	2	2	1
Part 15	2	0	2	0	0	1
Part 16	2	2	2	1	2	2
Part 17	0	0	2	0	1	1
Part 18	2	1	2	0	1	1
Part 19	0	0	2	1	4	1
Part 20	0	2	2	0	1	1
Part 21	0	0	1	1	1	1
Part 22	1	2	2	2	2	2

So, we formed five cells in table 13 by steps 1, 2, and 3.

Table 13. Available operations in a preliminary cell

	The assigned machine	Available operation
Machine cell 1	M/C 1,2,13	1,2,3,4,5,6,7,8,9,10,12,13,14,15
Machine cell 2	M/C 4,5,3	1,2,3,4,5,6,7,8,9,11,12,13,14,15
Machine cell 3	M/C 6,12,7	1,2,3,5,6,7,8,10,12,13,14
Machine cell 4	M/C 8,9	1,2,3,4,6,7,8,9,11,12,14,15
Machine cell 5	M/C 10,11	1,3,5,6,7,9,10,12,13,14,15

Step 4. The incapability index is calculated in the part-machine cell incidence matrix in Table 14.

Step 5. The final cell formation is completed in Table 15.

Table 14. Part-machine cell incidence matrix of an incapability index

Part	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Cell1	1	0	2	0	2	1	0	2	0	1	0	1	0	1	2	3	0	2	0	0	1	1
Cell2	3	0	2	0	0	3	1	1	1	1	0	3	0	1	0	2	0	1	0	2	0	2
Cell3	1	1	0	0	2	1	1	0	1	1	1	0	1	1	1	2	1	1	0	1	0	1
Cell4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Cell5	2	2	0	2	2	2	0	2	2	0	2	2	1	2	0	2	1	1	4	1	1	2

Table 15. Final machine cell and part family

Machine / Part	M1	M2	M4	M5	M6	M11	M12	M8	M9	M10	M3	M7	M13
P2	1	1											
P9	1	1											
P11	1	1											
P17	1	1											
P20	1	1											
P4			1	1									
P5			1	1									
P13			1	1									
P19			1	1									
P3					1	1	1						
P8					1	1	1						
P12					1	1	1						
P21					1	1	1						
P1								1	1	1			
P6								1	1	1			
P14								1	1	1			
P16								1	1	1			
P22								1	1	1			
P7											1	1	1
P10											1	1	1
P15											1	1	1
P18											1	1	1

5. Computational Analysis

In this section, we compare and analyze the outcomes of the p-median method of Kusiak (1987), the algorithm of Srinivasan et, al. (1990), and the solution procedure in this paper. As shown in Table 16, one can see that the machine cells formed by the solution procedure in this paper produced less E.Es than the p-median method by maximizing the number of operations available in each cell.

Table 16. Not available operation in each machine cell

	Machine cell	Not available operation in machine cell
This paper	machine cell 1 : 1,2	operation 6,10,11
	machine cell 2 : 4,5	operation 7,10,12
	machine cell 3 : 6,11,12	operation 4,5,11
	machine cell 4 : 8,9,10	•
	machine cell 5 : 3,7,13	operation 1,2,9,15
P-median method	machine cell 1 : 1,2,3	operation 6,10
	machine cell 2 : 4,5,7	operation 7,10,12
	machine cell 3 : 6,8,9	operation 5
	machine cell 4 : 10	operation 2,3,4,8,9,15
	machine cell 5 : 11,12,13	operation 1,5,7,11

Table 17. The summary of cell formations obtained by the three approaches

No. of cells	Method	Machine cell	Not available operation	No. of unavailable operations
2	This paper	cell 1 : 1,2,7,8,10,12,13	•	0
		cell 2 : 3,4,5,6,9,11	•	
	p-median method	cell 1 : 1,5,7,8,9,10,11	•	0
		cell 2 : 2,3,4,6,12	•	
	Algorithm of Srinivasan et. al	Impossible to form		
3	This paper	cell 1 : 1, 2, 13, 12	Op. 11	1
		cell 2 : 3, 4, 5, 10 ,11	•	
		cell 3 : 6, 7, 8, 9	•	
	p-median method	cell 1 : 1, 2, 3, 4, 12	Op. 10	5
		cell 2 : 5, 7, 11, 13	Op. 1,2,7,12	
		cell 3 : 6, 8, 9, 10	•	
	Algorithm of Srinivasan et. al	Impossible to form		
4	This paper	cell 1 : 1, 2, 13	Op. 11	5
		cell 2 : 3, 4, 5	Op. 10	
		cell 3 : 6, 11, 12	Op. 4,5,11	
		cell 4 : 7, 8, 9, 10	•	
	p-median method	cell 1 : 1, 2	Op. 6,10,11	8
		cell 2 : 3, 4, 5, 7	Op. 10	
		cell 3 : 6, 8, 9, 10	•	
		cell 4 : 11, 12, 13	Op. 1,5,7,11	
	Algorithm of Srinivasan et. al	Impossible to form		
5	This paper	cell 1 : 1, 2	Op. 6,10,11	13
		cell 2 : 4, 5	Op. 7,10,12	
		cell 3 : 6, 11, 12	Op. 4,5,11	
		cell 4 : 8, 9, 10	•	
		cell 5 : 3, 7, 13	Op. 1,2,9,15	
	p-median method	cell 1 : 1, 2, 3	Op. 6,10	16
		cell 2 : 4, 5, 7	Op. 7,10,12	
		cell 3 : 6, 8, 9	Op. 5	
		cell 4 : 10	Op. 2,3,4,8,9,15	
		cell 5 : 11, 12, 13	Op. 1,5,7,11	
	Algoritm of Srinivasan et. al	cell 1 : 1, 2, 3, 7, 13	•	15
		cell 2 : 4, 5	Op. 7,10,12	
		cell 3 : 8, 9	Op. 5,10,13	
		cell 4 : 10, 11	Op. 2,4,8,11	
		cell 5 : 6 ,12	Op. 3,4,5,9,11,15	

Tables 17 shows the summary of various cell formations obtained by the solution procedure in this paper, the p-median method, and the algorithm of Srinivasan et. al. The proposed approach efficiently minimizes the number of E.Es at the cell formation stages as presented in Table 17. Table 18 presents part families and machine cells formed by the two approaches and the solution procedure in this paper, respectively. The proposed approach does not contain any E.E through cells 1, 2, 3, 4, and 5.

Table 18. The number of E.E

No. of cells	Method	Part family	EE	No.of EE
2	This paper	cell 1 : 1,3,5,7,9,11,13,15,17,19,21	•	0
		cell 2 : 2,4,6,8,10,12,14,16,18,20,22	•	
	p-median method	cell 1 : 1,3,5,7,9,11,13,15,17,19,21	•	0
		cell 2 : 2,4,6,8,10,12,14,16,18,20,22	•	
	Srinivasan et. al.	Impossible to form.		
3	This paper	cell 1 : 1,4,7,11,13,16,19,22	•	0
		cell 2 : 2,5,8,10,14,17,20	•	
		cell 3 : 3,6,9,12,15,18,21	•	
	p-median method	cell 1 : 2,4,5,7,10,19,20	•	5
		cell 2 : 8,13,14,15,17,18,21	P8(1),P(1,2),P18(7),P21(1)	
		cell 3 : 1,3,6,9,11,12,16,22	•	
	Srinivasan et. al.	Impossible to form		
4	This paper	cell 1 : 1,6,9,13,17,20	•	0
		cell 2 : 2,5,10,15,18	•	
		cell 3 : 3,8,12,19,21	•	
		cell 4 : 4,7,11,14,16,22	•	
	p-median method	cell 1 : 2,7,9,11,19,20	•	1
		cell 2 : 4,5,10,15,18,21	•	
		cell 3 : 1,6,8,12,14	•	
		cell 4 : 3,13,16,17,22	P16(11)	
	Srinivasan et. al.	Impossible to form		
5	This paper	cell 1 : 2,9,11,17,20	•	0
		cell 2 : 4,5,13,19	•	
		cell 3 : 3,8,12,21	•	
		cell 4 : 1,6,14,16,22	•	
		cell 5 : 7,10,15,18	•	
	p-median method	cell 1 : 2,7,9,10,14	P14(10)	8
		cell 2 : 4,5,11,19	•	
		cell 3 : 1,6,8,12,20	•	
		cell 4 : 3,15,18,21	P3(3),P15(13),P18(15),P21(1,8)	
		cell 5 : 13,17,16,22	P16(5,11)	
	Srinivasan et. al.	cell 1 : 1,4,9,14,22	•	7
		cell 2 : 2,5,13,17	•	
		cell 3 : 10,11,15,18,16	P16(10)	
		cell 4 : 3,8,12,19	P19(2)	
		cell 5 : 21,7,6,20	P7(5),P6(4,15),P20(4,15)	

We further evaluate and compare the three approaches by solving ten problems taken from Boctor(1991). The proposed approach in this paper considers the operations as the parts to form the machine cells since the only machine-part matrix was not given in Boctor's problems. The total number of performable operations (TPNO) in the formed cells is used as an evaluation criteria. The proposed solution approach obtained the largest TPNO among the three approaches at every cell in each problem in Table 19. The entry (-) in Table 19 implies that the approach of Srinivasan et. al. (1990) can not solve the problem when the specific cell number is given. The approach presented in this paper shows the superiority of itself to the other methods in the each cell in maximizing the TPNO.

Table 19. Comparison of three approaches with Bocktor's problem numbers 1-4 (continued)

Problem No.	No. of Cell	This paper	Srinivasan et. al.	P-median
Boctor's problem 1	3	27	14	23
		28	22	11
		26	29	27
	TPNO	81	65	61
	4	23	-	27
		22	-	11
		26	-	4
		23	-	27
	TPNO	94	-	69
	5	22	-	27
		22	-	11
		21	-	4
		23	-	27
		18	-	4
	TPNO	106	-	73
Boctor's problem 2	3	25	24	29
		28	9	1
		26	28	21
	TPNO	79	61	51
	4	25	-	29
		26	-	1
		22	-	21
		20	-	3
	TPNO	93	-	54
	5	23	-	29
		20	-	1
		20	-	21
		19	-	3
		20	-	3
	TPNO	102	-	57
Boctor's problem 3	3	26	30	3
		26	9	14
		24	12	30
	TPNO	76	51	47
	4	26	-	3
		22	-	8
		22	-	30
		18	-	16
	TPNO	88	-	57
	5	21	-	3
		20	-	8
		22	-	29
		14	-	3
		14	-	17
	TPNO	91	-	60
Boctor's problem 4	3	26	-	26
		25	-	24
		26	-	4
	TPNO	77	-	54
	4	22	14	26
		25	28	24
		22	12	4
		21	15	3
	TPNO	90	69	57
	5	22	-	26
		23	-	24
		22	-	4
		17	-	3
		15	-	5
	TPNO	99	-	62

Table 19. Comparison of three approaches with Bocktor's problem numbers 5-10

Problem No.	No. of Cell	This paper	Srinivasan et. al.	P-median
Boctor's problem 5	3	26	×	3
		28	×	21
		27	×	29
	TPNO	81	×	53
	4	22	×	3
		27	×	21
		24	×	29
		21	×	6
	TPNO	94	×	59
	5	21	×	4
		21	×	3
		20	×	21
		21	×	29
		21	×	6
	TPNO	104	×	63
Boctor's problem 6	3	28	30	29
		28	17	23
		21	8	2
	TPNO	77	55	54
	4	26	-	4
		20	-	28
		19	-	24
		23	-	2
	TPNO	88	-	58
	5	23	-	4
		20	-	4
		17	-	28
		18	-	24
		17	-	2
	TPNO	95	-	62
Boctor's problem 7	3	26	-	21
		28	-	28
		27	-	3
	TPNO	81	-	52
	4	22	-	21
		22	-	28
		22	-	4
		23	-	3
	TPNO	89	-	56
	5	20	-	21
		19	-	4
		19	-	27
		19	-	4
		22	-	3
	TPNO	99	-	59
	8	17	17	20
		15	15	6
		15	15	4
		14	14	24
		13	13	4
		13	13	3
		13	13	5
		12	12	5
	TPNO	112	112	71
Boctor's problem 8	3	28	-	29
		27	-	24
		27	-	5
	TPNO	82	-	58
	4	25	-	29
		26	-	24
		24	-	4
		22	-	5
	TPNO	97	-	62
	5	24	-	29
		22	-	4
		20	-	24
		22	-	4
		21	-	5
	TPNO	109	-	66
	6	20	20	28
		22	15	4
		20	17	5
		20	9	24
		18	21	4
		11	23	5
	TPNO	111	105	70
Boctor's problem 9	3	27	30	5
		29	17	29
		25	15	24
	TNO	81	62	58
	4	25	-	5
		24		29
		23	-	24
		23	-	7
	TPNO	95	-	65
	5	22	-	5
		22	-	29
		20	-	24
		22	-	7
		21	-	9
	TPNO	107	-	74
Boctor's problem 10	3	25	-	3
		22	-	26
		28	-	23
	TPNO	75	-	52
	4	24	-	3
		20	-	24
		26		26
		19	-	3
	TPNO	89	-	56
	5	26	27	3
		16	16	24
		18	12	4
		19	15	24
		20	14	3
	TPNO	99	84	58

6. Summary and Conclusion

This paper minimizes the inter-machine similarity to reduce the E.Es at the cell formation stage, which in turn minimizes the intercell movements of the parts. Algorithms I and II are consecutively used to form machine cells. The similarity coefficients are developed and used to minimize the E.Es.

In Algorithm I, the pairs of machines, equal to the maximum number of cells arranged to a descending order of the inter-machine similarity coefficient values are selected in order to form the preliminary cells. If there is a tie, i.e., the pairs of machines have an identical similarity coefficient value, the pair of machines which perform an operation placed first in the sequence of operations is selected. Algorithm II improves the preliminary cells by assigning the unassigned machines ultimately to minimize the number of E.Es. The incapability of the machine cell for the parts is used as a criteria to form the part families where the number operations of a part, which can not be performed in the cell, uses the incapability index. It is desirable to assign the parts to the machine cells in order to minimize the incapability of the machine cells. Then, the performance of the machine cells can be improved.

The cell formation model in this paper is viewed as a typical assignment problem and is solved by using the Hungarian method. The two existing cell formation approaches and the approach in this paper solved various sizes of problems. The number of operations which can not be performed in each cell and the number of E.Es are used as the criterion to evaluate the approaches. The computational experience showed that the cells formed by the solution procedure in this paper were of good quality.

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