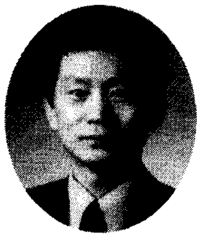
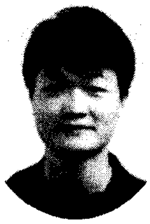
Prediction of Fracture Energy of Concrete





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ABSTRACT

A method to determine the fracture energy of concrete is investigated. The fracture energy may be calculated from the area under the complete load-deflection curve which can be obtained from a stable three-point bend test. Several series of concrete beams have been tested. The present experimental study indicates that the fracture energy decreases as the initial notch-to-beam depth ratio increases. Some problems to be observed to employ the three-point bend method are discussed. The appropriate ratio of initial notch-to-beam depth to determine the fracture energy of concrete is found to be 0.5. It is also found that the influence of the self-weight of a beam to the fracture energy is very small. A simple and accurate formula to predict the fracture energy of concrete is proposed.

Keywords : fracture energy, concrete, fracture mechanics, fracture process zone, microcracking, three-point bend tests, load-deflection curve, tensile strength, aggregate size, initial notch-to-Beam depth ratio, maximum failure loads

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1. INTRODUCTION

Fracture behavior of concrete has received considerable attention in recent years. It is becoming increasingly recognized that fracture toughness of concrete cannot be evaluated using linear elastic fracture mechanics without any appropriate modification. This is due to the fact that large size of nonlinear zone exists in front of the crack tip in concrete. This nonlinear zone in concrete, called the fracture process zone, is caused by microcracking ahead of the crack tip. The fracture process zone in concrete is known to be very large compared with those of metals or other engineering materials(2-7, 11, 29, 30). This causes difficulties to measure the fracture toughness of concrete. Another difficulty lies in measuring the crack length increments during loading process since the crack tip is blurred by a microcracking zone in concrete.

One possible remedy is to use the complete load-deflection diagram to determine the fracture energy of concrete. The fracture energy can be calculated by measuring the area under the complete load-deflection curve. This method was used recently by Petersson(22). There are, however, some problems to be resolved to employ this method. First, the energy consumption taking place outside the fracture zone must be negligible. This requirement may roughly be satisfied by selecting an appropriate value of the initial notch-to-beam depth ratio. If the initial crack is too small compared with the beam depth, the concrete material outside the fracture zone would be stressed to a great amount. This will result in a certain additional amount of energy consumption in the test beam, in addition to the energy consumed in the fracture zone. In this case, the calculated fracture energy may obviously be higher than the actual one. What is then the appropriate value of the notch-to-beam depth ratio to determine the fracture energy? We will discuss and answer this question in the following main section of this paper. The second problem to be considered is the stability of fracture. This means that the fracture must be stable so that the energy consumption due to dynamic effects may be eliminated. This in turn means the smooth and complete load-deflection curve during the fracture test.

The first purpose of this paper is, therefore, to investigate the method of three-point bend test to determine the fracture energy of concrete and then to propose a simple formula to predict the fracture energy of given concrete. The prediction equation for the fracture energy is important since the fracture energy is a required value to perform the fracture analysis of concrete structures. The second objective is to devise a so-called equivalent crack length concept to calculate the maximum failure loads of the concrete beams This concept will enable one to calculate the maximum loads of the beams without resort to the complete nonlinear fracture analysis or the R-curve analysis.

2. 3-POINT BEND FRACTURE TESTS

Several series of concrete beams were tested in the present study to measure the maximum failure loads of the beams

Table 1 Mix Proportions and Compressive Strengths

	Number of Specimens	Mix Proportions (C : S : G : W)	Compressive Strength(N/mm^2)	Monotonic (M) Or Cyclic(C)
Series 1	8	1:1.88:2.86:0.41	25.8	M
Series 2	4	1:1.88:2.90:0.41	27.5	M
Series 3	4	1:2.44:3.44:0.49	21.5	M, C
Series 4	8	1:3.07:4.09:0.58	20.3	C
Series 5	8	1:1.88:2.86:0.41	26.2	C

Note: C=Cement, S=Sand, G=Gravel(Coarse Aggregate), W= Water, 1 N/mm^2=145psi.

and to determine the fracture energy of concrete.

2.1 Materials and Test Specimens.

The mix proportions and compressive strengths for each series of concrete are summarized in Table 1. The maximum aggregate size was 25mm. The ordinary Portland cement was used. Note here that the mix proportion for Series-5 concrete was the same as Series-i concrete and it was used for cyclic loading test.

The dimension of the test beams was 100 x 100 x 400mm. The specimens were cast in steel moulds and stored in 100% humidity during the first two days after casting. The test beams were then cured in the curing water-tub until one day before the tests. The concrete beams for each series had four different initial notch depths, i.e., a_0/H=0.0, 0.2. 0.4, 0.6 in which a_0=initial notch depth and H=beam depth(fig. 1). The reason for the different initial notch depths was to investigate the effect of initial notch depth ratio on the fracture energy of concrete. The initial notch was made in advance by inserting a thin steel sheet in the mould before placing the concrete.

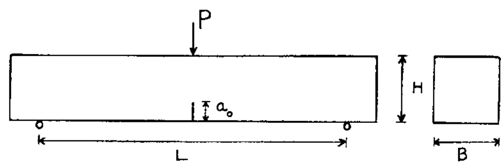

Fig.1 Schematic Diagram for the Present Three-Point

2.2 Test Method

The concrete beams were tested in three-point loading condition. The schematic diagram for the present three-point bend test is shown in Fig. 1. The test specimens were loaded in the Instron testing machine. The cross head speed (or loading velocity) was 0.05mn/min and was maintained constant throughout the tests. The load-deflection curves were generated by the plotter attached to the machine. The loading-unloading-reloading scheme was employed for some of the specimens in order to see the change of compliances for the unloading and reloading.

2.3 Test Results

The results of present experimental study are shown in Figs. 2-6. Each figure is classified according to the initial notch -to-beam depth ratios. The almost complete load-deflection curves were possible due to the stable loading arrangements.

3. DETERMINATION OF FRACTURE ENERGY OF CONCRETS

The fracture energy of concrete, G_F, can be determined as the area under the complete load-deflection diagram. The area under the curve represents the amount of energy consumed when the crack propagates through the concrete beam. The area under the curve in Fig. 2-6 was measured with a planimeter.

Now, the contribution from the self-weight of the beam must be considered to calculate the fracture energy. The fracture energy per one unit of area may then be calculated by the following equation.

$$G_F = \frac{\int_0^{\delta_0} P(\delta)\, d\delta + mg\,\delta_0/2}{B(H-a_0)} \qquad (1)$$

in which δ_0=maximum deflection[see Fig. 8(c)], m=mass of the beam, g= gravitational acceleration, B=beam width, H =beam depth, and a_0=initial crack length.

The fracture energy for each series of concrete has been calculated according to Eq. 1 and the results are shown in Fig. 7. As can be seen in Fig. 7, the fracture energy is decreased as the initial notch-to-beam depth ratio increases and approaches to an asymptotic value.

The reason having the larger fracture energy for the case of beams with a small initial crack is probably due to the fact that if the relative initial crack is small, the material outside the fracture zone would be greatly stressed and this would cause an additional energy consumption in the beam.

The beams with relatively large initial cracks would, however, yield smaller fracture energy than the actual one since the fracture process zone in front of the crack tip would not be well-developed in this case(6). It is known that the fracture energy increases somewhat as the crack propagates(6, 8, 24). Therefore, higher value of initial notch-to-beam depth ratio may not be suitable for determining the fracture energy of concrete.

To determine the fracture energy, it is, therefore, reasonable to choose the value of 0.5 for the initial notch-to-beam depth ratio as a compromise between the above-mentioned two cases. This value is reasonable in view of Fig. 7. On the other hand, this is necessary since there exists currently no exact way of evaluating experimentally the overconsumption of the energy as well as the correct length of the fracture process zone.

Table 2 shows the average values of fracture energy obtained from the present experimental study for various series of concrete. It was found in this study that the contribution of the self-weight of a beam to the fracture energy was very small and it was only about 2-6 percent of the total G_F value.

Table 2 Fracture Energy for Each Series of Concrete

	Average Fracture Energy (N/m)
Series 1	127.0
Series 2	129.6
Series 3	124.0
Series 4	112.1
Series 5	127.8

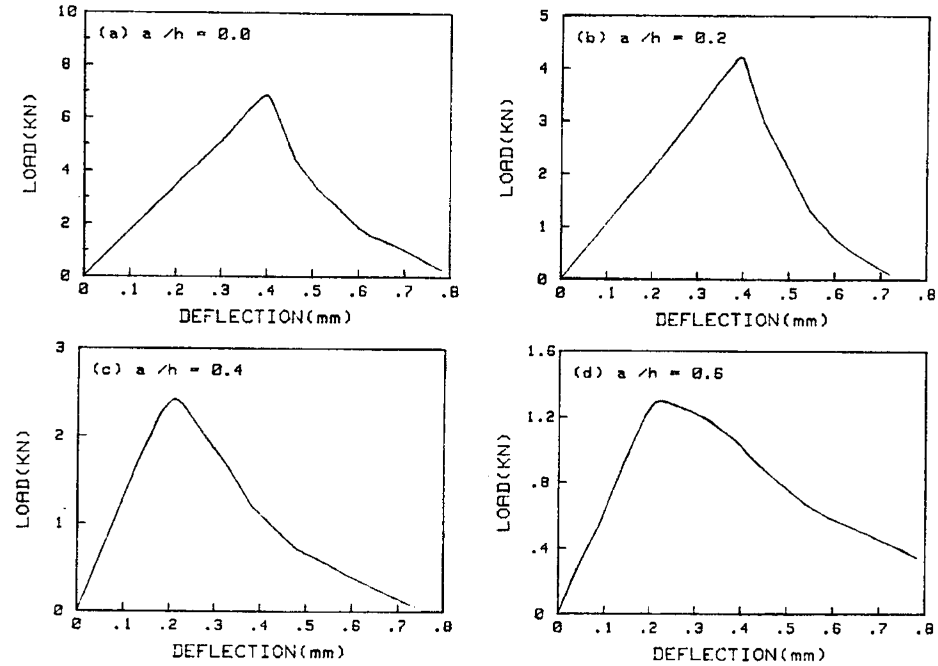

Fig.2 Bend Test Load-Deflection Curves for Series 1 Concrete

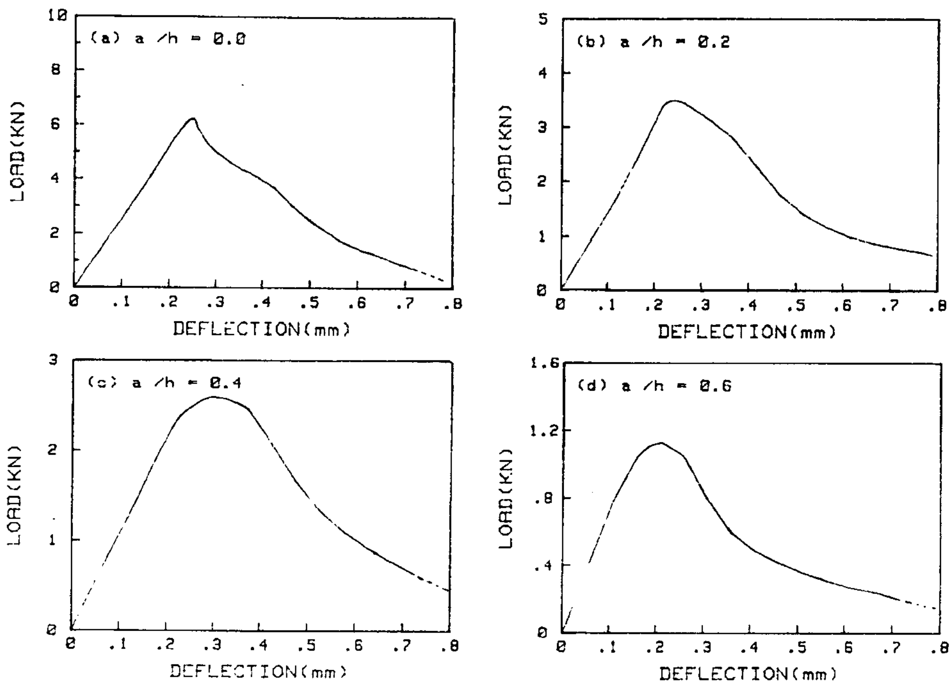

Fig.3 Load-Deflection Curves for Series-2 Concrete

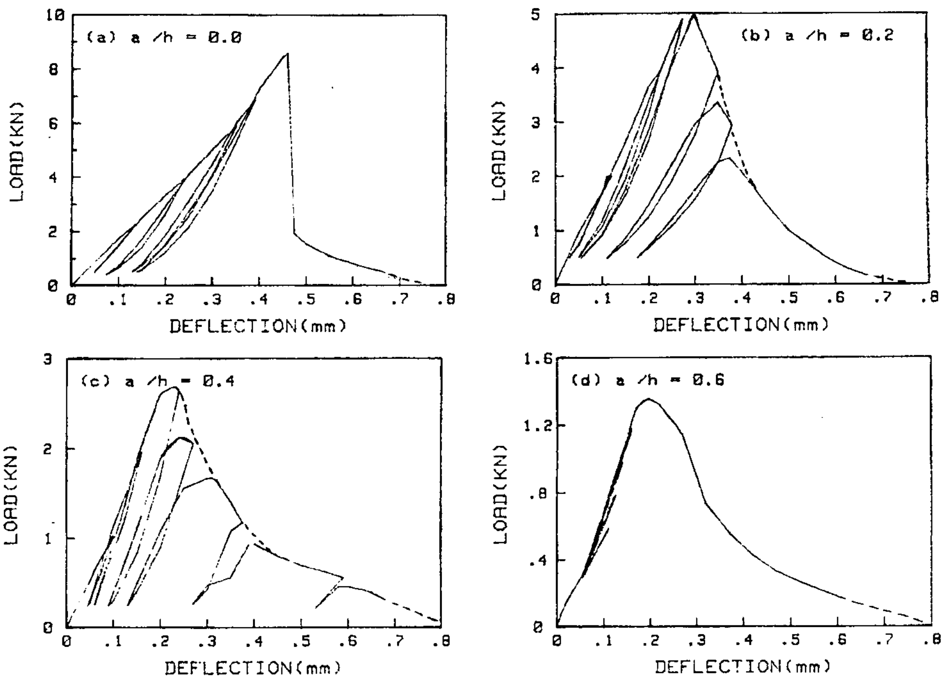

Fig.4 Load-Deflection Curves for Series-3 Concrete

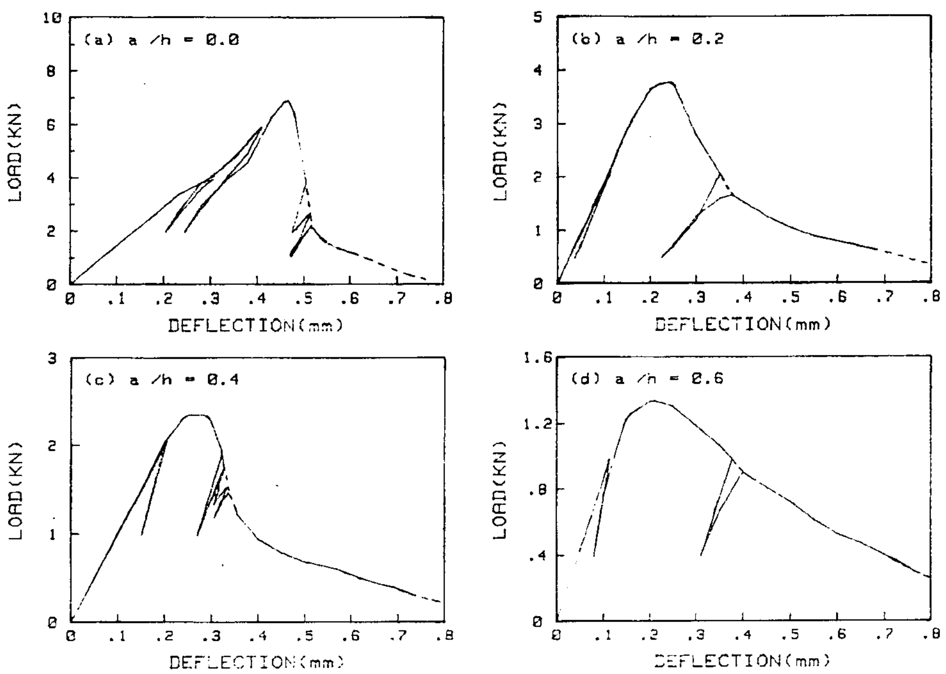

Fig.5 Load-Deflection Curves for Series-4 Concrete

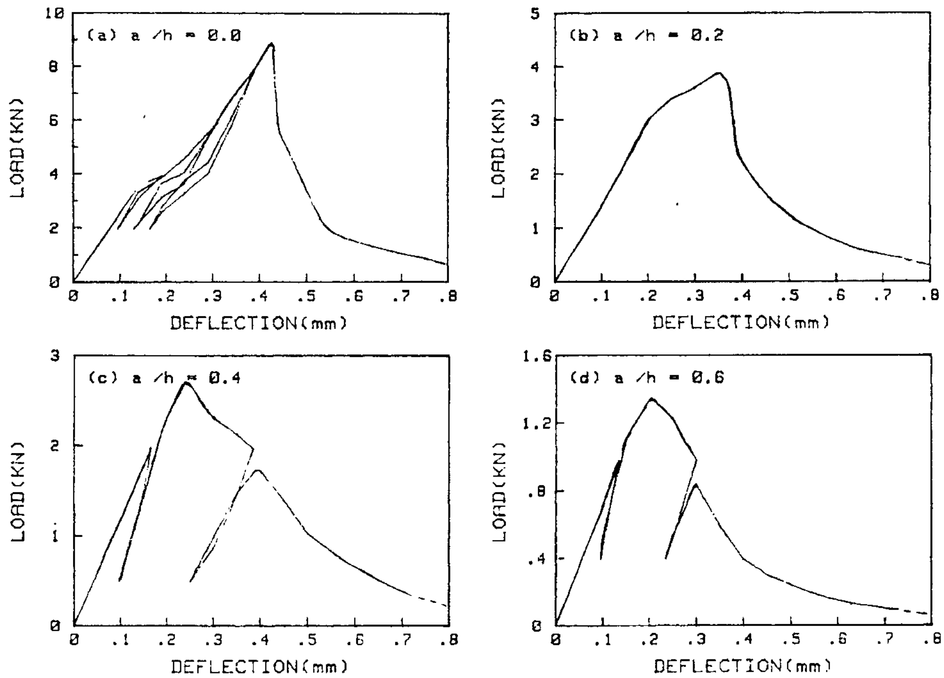

Fig.6 Load-Deflection Curves for Series-5 Concrete

3.1 Derivation of a Formula for Fracture Energy of Concrete

The fracture energy, G_F, must be known in advance as a material property in order to perform the fracture analysis of concrete structures. This requires a certain prediction formula to determine the fracture energy of concrete. The progressive microcracking in the fracture process zone may be described by a stress-strain relation that exhibits strain-softening. The strain-softening uniaxial stress-strain relation may be well-idealized as a bilinear stress-strain diagram[see Fig. 8 (a)](6). The fracture energy may then be expressed as

$$G_F = W_C A = \frac{1}{2} W_C \varepsilon_0 f_t \qquad (2)$$

in which W_C=width of microcrack band $=c_0 d_a$ (3, 6), d_a=maximum aggregate size, A=area under the stress-strain curve, ε_0=Strain at zero stress, f'_t= tensile strength of concrete, and E_C=its elastic modulus.

By assuming and $\varepsilon_0 = c_1 \varepsilon_p$ (3, 6), and noting that $\varepsilon_p = f'_t/E_C$ [Fig. 8(a)], Eq. 2 may be rewritten in the following form.

$$G_F = \frac{1}{2} c_0 c_1 f_t^2 d_a/E_C = c_2 f_t d_a/E_C \qquad (3)$$

Eq. 3 indicates that the fracture energy increases very rapidly as the tensile strength increases The present experimental study with the recent results of Gopalaratnam and Shah(14) and Petersson(22), however, indicates that the fracture energy increases slowly with the

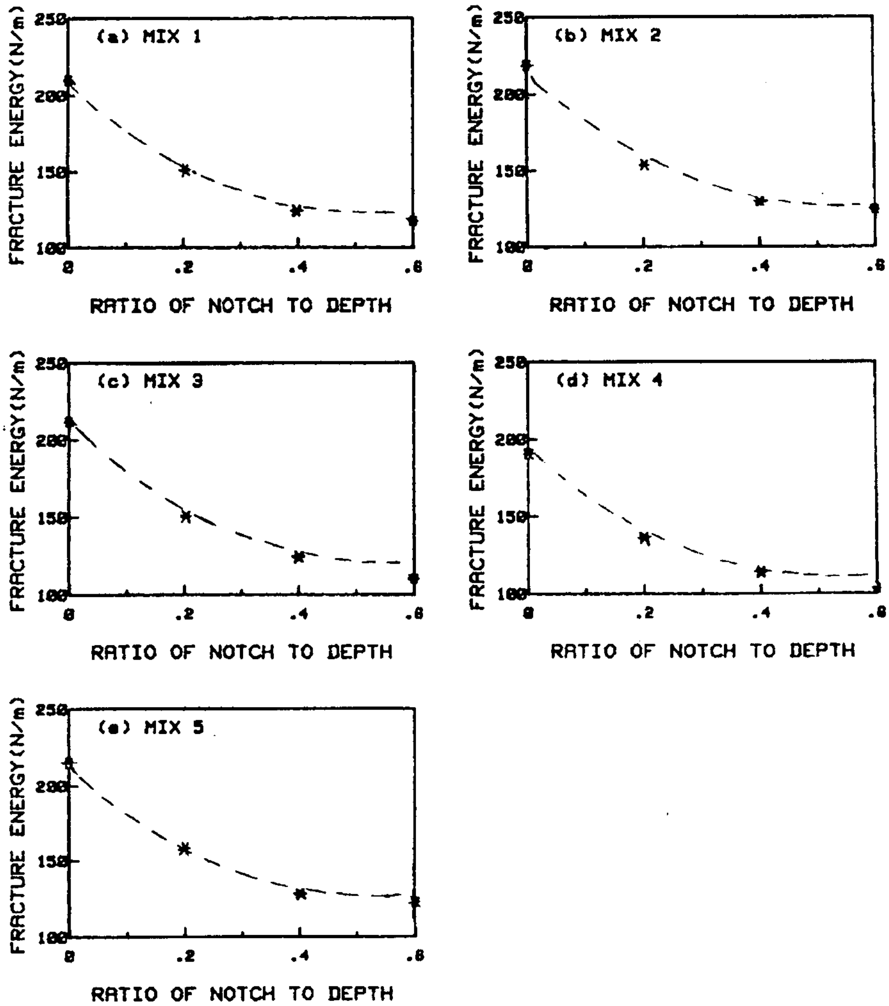

Fig.7 Variation of Fracture Energy According to the Initial Notch-to- Beam Depth Ratio.

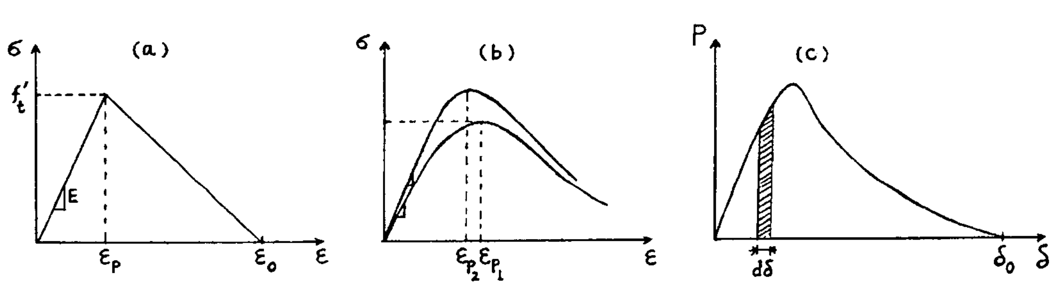

Fig.8 (a) Stress-Strain Diagram for Fracture Process Zone; (b) Stress-Strain Curves for Different Strengths; (c) Load-Deflection Curve.

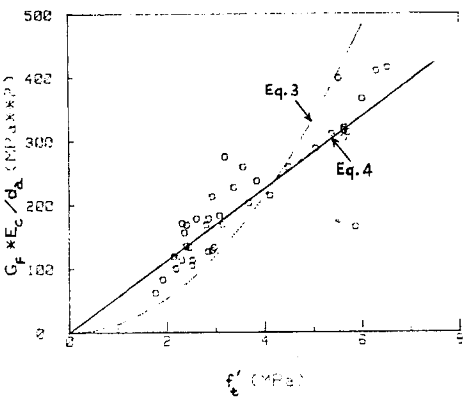

Fig.9 Fracture Energy As a Function of Tensile Strength.

increase of the tensile strength of concrete. This means that one should take the relation, which is reasonable and coincident with experimental data(14, 23) [see Fig. 8(b)]. By using this relation, Eqs. 2 and 3 may be rewritten as

$$G_F = \frac{1}{2} c_0 c_1 c_3 f_t d_a / E_C = c_4 f_t d_a / E_C \qquad (4)$$

in which c_0, c_1, c_2, c_3 and c_4 represent the empirical constants to be determined from experimental data.

The recently-published good experimental data(12, 14, 22) for the fracture energy of concrete have been used to determine the fracture energy equation. Fig. 9 shows the regression analysis of test data(12, 14, 22) and shows the dependence of fracture energy on the tensile strength. Fig. 9 clearly shows that the fracture energy varies linearly with the tensile strength. Namely, the linear form(Eq. 4) is much superior to the quadratic form(Eq 3)[see Fig.9]. The best prediction equation for the fracture energy of concrete was found from the test data as follows.

$$G_F = 56.24 f_t d_a / E_C \qquad (5)$$

in which G_F is in N/mm, f'_t and E_C in N/mm^2 (or MPa), and d_a in mm When the value of E_C is not given, the usual relation $E_C = 4.733\sqrt{f'_C}$ may be used(1) in which f'_C is the compressive strength of concrete in N/mm^2(or MPa). For the tensile strength of concrete, the following relation proposed recently by Raphael(23) on the basis of the comprehensive experimental data may be used.

$$f'_t = 0.324 f'^{2/3}_c \qquad (6)$$

$$\text{or } f'_t = 0.74 f'_r \qquad (7)$$

in which f'_t and f'_C are in N/mm^2(or MPa), f'_r=modulus of rupture=$0.438 f'_C{}^{2/3}$(N/mm^2). Here 1MPa =1MN/mm^2=1N/mm^2=145psi. The fracture energy equation without E_C was also tried, but it did not give better results. Eq. 5 may now be used to calculate the fracture energy of a given concrete.

CONCLUSIONS

1. The fracture energy of concrete can be determined from a stable three-point bend test. The fracture energy is then calculated from the area under the load-deflection curve. This method is relatively simple and accurate

2. The present experimental study indicates that the fracture energy is decreased as the initial notch-to-beam depth ratio increases. The appropriate ratio of initial notch-to-beam depth to determine the fracture energy of concrete is found to be 0.5.

3. It is found that the influence of the self-weight of a beam to the fracture energy is very small and it amounts to only several percent of the fracture energy.

4. A simple and accurate formula for the fracture energy of concrete is proposed. The fracture energy is a required value to perform the fracture analysis of concrete structures. The fracture energy of concrete depends on the aggregate size and tensile strength and is about 80-130 N/m in value for normal aggregate concrete.

ACKNOWLEDGMENT

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