Prediction of Shear Strength of R/C Beams using Modified Compression Field Theory and ACI Code

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ABSTRACT

In recent years, the concept of the modified compression field theory (MCFT) was develped and applied to the analysis of reinforced concrete beams subjected to shear, moment, and axial load. Although too complex for regular use in the shear design of beams, the procedure has value in its ability to provide a rational method of anlysis and design for reinforced concrete members. The objective of this paper is to review the MCFT and apply it for the prediction of the response and shear strength of reinforced concrete beams. A parametric analysis was performed on a reinforced T-section concrete beam to evaluate and compare the effects of concrete strength, longitudinal reinforcement ratio, shear reinforcement ratio, and shear span to depth ratio in two different approaches, the MCFT and the ACI code. The analytical study showed that the concrete contribution to shear strength by the MCFT was higher than the one by the ACI code in beams without stirrups, while it was lower with stirrups. On the other hand, shear reinforcement contribution predicted by the MCFT was much higher than the one by the ACI code. This is because the inclination angle of shear crack is much smaller than 45° assumed in the ACI code.

Keywords : modified compression field theory, shear strength, principal strain, transverse strain, inclination angle, stirrup, shear reinforcement

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1. INTRODUCTION

It is now over twenty years since the ACI-ASCE Shear Committee concluded the introduction to its state-of-the-art report[1] with the words: "During the next decade it is hoped that the design regulations for shear strength can be integrated, simplified and given a physical significance so that designers can approach unusual design problems in a rational manner."

Although advances have been made during the past 20 years, the ACI shear design equations have not changed. Even in the ACI 318-95 Building Code[3], there is little change in Chapter 11, Shear and Torsion. The main reason for this fact is thought that the ACI shear design procedure is very simple for a designer to approach, even thought it lacks rationality and generality.

Recent years, much work has been directed toward formulating a more general and rational model. In particular, much experimental and analytical research has been conducted at the university of Toronto toward formulating a more general and rational model. From the data acquired, the modified compression field theory[9] was developed. In this theoretical model, cracked concrete is treated as a new material with its own stress-strain characteristics. Equilibrium, compatibility, and constitutive relationships are formulated to determine the load-deformation response of members subjected to shear.

The concept of the modified compression field theory can be applied to the analysis of reinforced concrete beams subjected to shear, moment, and axial load. Although too complex for regular use in the shear design of beams, the procedure has value in its ability to provide a rational method of analysis and design for members having unusual or complex geometry or loading.

The objective of this paper is to review the modified compression field theory (MCFT) and the ACI code, and apply it for the prediction of the response and the shear strength of reinforced concrete beams. The shear strengths predicted by the MCFT are compared with those by the ACI code. A parametric analysis was performed to compare the effects of concrete strength, longitudinal reinforcement ratio, shear reinforcement ratio, and shear span to depth ratio in two different approaches, the MCFT and the ACI code.

2. BACKGROUND

2.1 ACI Building Code

The current ACI design code for reinforced concrete beams in shear is based on the 45° truss analogy developed a century ago. However, many experimental tests revealed that the 45° truss analogy was quite conservative. This conservatism of the 45° truss model is attributed to the neglect of tensile stresses in concrete and the choice of 45° for compressive strut inclination.

Consequently, the ACI Building Code added an empirical correction term to the truss equation. The ACI code in 1910 accepted 40psi(0.28MPa) concrete contribution for shear. The shear resistance provided by concrete was revised into the current form for shear in 1995. This added shear capacity is taken as being equal to the shear at the commencement of diagonal cracking and is commonly referred to as the "concrete contribution."

In the current ACI code, the design shear strength, ϕV_n , should be greater than the factored shear, V_u .

$$\phi V_n \geq V_u \quad (1)$$

Where the nominal shear strength, V_n, is given by

$$V_n = V_c + V_s \quad (2)$$

where V_c is the shear carried by concrete and V_s is the shear carried by stirrups.

Typically, for reinforced concrete members, V_c, is calculated from the following equation.

$$V_c = (1.9\sqrt{f_c} + 2500\rho_w \frac{Vd}{M})b_w d \leq (3\sqrt{f_c})b_w d \quad (3)$$

As a simplification to Eq.(3) the ACI code permits the shear at flexure-shear cracking for reinforced concrete beams to be taken as

$$V_c = (2\sqrt{f_c})b_w d \quad (4)$$

The concrete contribution, V_c, is actually the sum of at least three separate components: (a) shear resistance of the compression concrete above the top of diagonal crack, (b) aggregate interlock along the diagonal crack, and (c) dowel resistance provided by the longitudinal reinforcement.

The 45° truss equation is used to calculate the steel contribution, V_s. For a beam containing stirrups perpendicular to its axis,

$$V_s = \frac{A_v f_y d}{s} \quad (5)$$

To avoid diagonal crushing of the concrete and to limit diagonal cracking at service loads, the steel contribution is limited to

$$V_s \leq (8\sqrt{f_c})b_w d \quad (6)$$

Beams that do not contain web reinforcement may fail in a relatively brittle manner immediately after the formation of the first diagonal crack. As a consequence, the shear capacity of such members can be substantially reduced by the factors such as repeated loading, tensile stresses caused by the restraint of shrinkage strains, thermal strains, or creep strains, stress concentrations due to discontinuities such as web openings, termination of flexural reinforcement. To prevent this kind of failure the current ACI Code requires a minimum amount of web reinforcement where the factored shear force, V_u, exceeds $\phi V_c/2$. The minimum amount required by the code is

$$A_v = 50\frac{b_w s}{f_{vy}} \quad (7)$$

2.2 Compression Field Theory[6]

In 1929, Wagner developed the tension field theory in analogizing the post bucking shear resistance of thin-webbed metal girder. He assumed that after buckling, the thin webs have no resistance to compression and the shear is carried by a field of diagonal tension and that the angle of inclination of the diagonal tensile stresses coincide with the angle of inclination of the principal tensile strains.

By applying the tension field theory to reinforced concrete, and assuming that after cracking, the concrete carries no tension and that the shear is carried by a field of diagonal compression, the following compatibility relationships, relating inclination angle, θ, and three strain, ε_2, ε_x and ε_t are obtained.

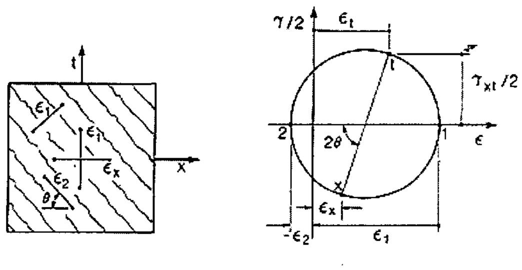
$$\tan^2\theta = \frac{\varepsilon_x - \varepsilon_2}{\varepsilon_t - \varepsilon_2} \quad (8)$$

where ε_x= longitudinal strain of web,

ε_t= transverse strain

ε_2= principal compressive strain

For cracked concrete these compatibility relationships are expressed in terms of the average strains. Fig.1 shows these strains in the Mohr's circle.



(a) Average strains in cracked element

(b) Mohr's circle for average strains

Fig.1 Compatibility conditions for cracked web element

From the Mohr's circle, the principal tensile strain in the web is

$$\varepsilon_1 = \varepsilon_x + \varepsilon_t - \varepsilon_2 \quad (9)$$

and the shear strain in the web is

$$\gamma_{xy} = 2(\varepsilon_x - \varepsilon_2)\cot\theta \quad (10)$$

If we consider a symmetrical reinforced concrete beam subjected to shear, there are four unknowns: the stress in the longitudinal bars(f_x), the stress in the stirrups(f_v), the diagonal compressive stress in the concrete(f_2), and the inclination(θ) of these diagonal compressive stresses. By using the equilibrium, compatibility, and stress-strain relationships for materials, the four unknown can be found and the load-deformation response can be determined.

Based on the tests of reinforced concrete elements in pure shear, Vecchio and Collins[7] suggested the following stress-strain relationship.

$$f_2 = f_{2\max}\left[2\left(\frac{\varepsilon_2}{\varepsilon_c'}\right) - \left(\frac{\varepsilon_2}{\varepsilon_c'}\right)^2\right] \quad (11)$$

$$\frac{f_{2\max}}{f_c'} = \frac{1}{0.8 + 170\varepsilon_1} \le 1.0$$

where ε_c' is the strain at the peak concrete stress, f_c'.

2.3 Modified Compression Field Theory[7]

Because the compression field theory neglects the contribution of tensile stresses in cracked concrete, it gives conservative estimates of shear strength. The modified compression field theory (MCFT) accounts for the contribution of the tensile stresses in the concrete between cracks.

The equilibrium conditions for the MCFT are introduced in Fig.2. The shear will be resisted by the diagonal compressive stresses, f_2, together with the diagonal tensile stresses, f_1. The tensile stresses in the diagonally cracked concrete vary in magnitude from zero at the crack locations to peak values between the cracks.

From the Mohr's stress circle shown in Fig.2, the following relationship for the principal compressive stress, f_2, can be derived:

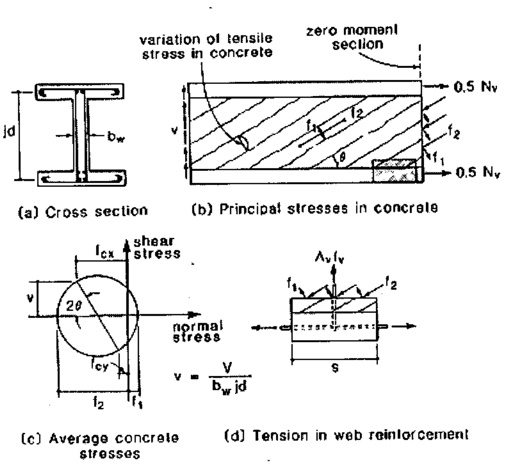


Fig.2 Equilibrium conditions of MCFT

$$f_2 = (\tan\theta + \cot\theta)v - f_1, \qquad (12)$$

where $v = \frac{V}{b_w jd}$

The vertical equilibrium requirement can be expressed as

$$A_v f_v = (f_2 \sin^2\theta - f_1 \cos^2\theta) b_w s \qquad (13)$$

Substituting for f_2 from Eq.(12) gives

$$V = f_1 b_w jd \cot\theta + \frac{A_v f_v}{s} jd \cot\theta \qquad (14)$$

The Eq.(14) has the same form as the ACI shear equation $V_c + V_s$.

The longitudinal equilibrium requires the following expression.

$$A_{sx} f_l = (f_2 \cos^2\theta - f_1 \sin^2\theta) b_w jd \qquad (15)$$

Substituting for f_2 from Eq.(12) gives

$$A_{sx} f_l = V\cot\theta - f_1 b_w jd \qquad (16)$$

Based on their tests of reinforced concrete panels in pure shear, Vecchio and Collins[9] recommended the following relationships of average tensile stress vs. average tensile strain.

$$\text{if } \varepsilon_1 \le \varepsilon_{cr} \text{ then } f_1 = E_c \varepsilon_1 \qquad (17)$$

$$\text{if } \varepsilon_1 > \varepsilon_{cr} \text{ then } f_1 = \frac{\alpha_1 \alpha_2 f_{cr}}{1 + \sqrt{500\varepsilon_1}} \qquad (18)$$

where α_1 and α_2 are factors accounting for the bond characteristics of the reinforcement and the type of loading.

The shear forces transmitting tension across the cracks will require local shear stresses, v_{ci}, on the crack surfaces as shown in Fig.3. The limiting value of v_{ci} is suggested as

$$V_{ci} = \frac{2.16\sqrt{f_c}}{0.3 + \frac{24w}{a + 0.63}} \qquad (19)$$

where w is the crack width and a is the maximum aggregate size.

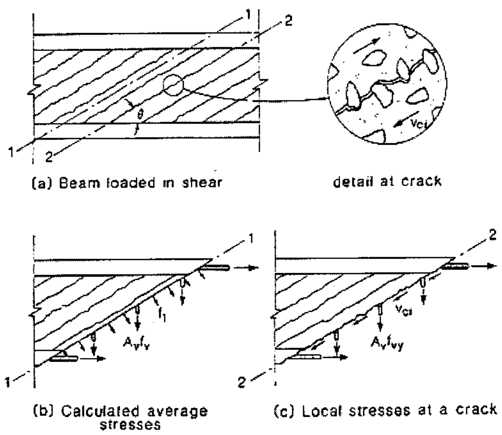


Fig.3 Transmitting forces across cracks.

Because two sets of stresses shown in Fig.5(b) and (c) must be statically equivalent, this requirement gives

$$A_v f_v\left(\frac{jd}{s\tan\theta}\right) + f_1 \frac{b_w jd}{\sin\theta}\cos\theta =$$

$$A_v f_{vy}\left(\frac{jd}{s\tan\theta}\right) + v_{ci} b_w jd \qquad (20)$$

and f_1 must be limited to

$$f_1 = v_{ci}\tan\theta + \frac{A_v}{s b_v}(f_{vy} - f_v) \qquad (21)$$

The crack width, w, to be used in Eq.(19) can be taken as the product of the principal tensile strain, ε_1, and the average spacing of the diagonal cracks.

$$w = \varepsilon_1 s_{m\theta} \qquad (22)$$

It is suggested that the spacing of the

inclined cracks be taken as

$$s_{m\theta}=\frac{1}{(\frac{\sin\theta}{s_{mx}}+\frac{\cos\theta}{s_{mv}})} \quad (23)$$

where s_{mx} and s_{mv} are the crack spacing indicative of the crack control characteristics of the longitudinal and transverse reinforcement, respectively.

According to the CEB-FIP Code crack spacing is expressed as follows:

$$s_{mx}=2(c_x+\frac{s_x}{10})0.25k_1\frac{d_{bx}}{\rho_x} \quad (24)$$

$$s_{mv}=2(c_v+\frac{s_x}{10})0.25k_1\frac{d_{bv}}{\rho_v} \quad (25)$$

where $\rho_v=A_v/(b_w s)$ and $\rho_x=A_{sx}/A_c$, and k_1 is 0.4 for deformed bars.

For the two sets of stresses in Fig.3 to be the same horizontal force the following condition should be satisfied

$$A_{sx}f_y \geq A_{sx}f_{sx}+f_1 b_w jd+ [f_1-\frac{A_v}{b_w s}(f_{vy}-f_v)]b_w jd\cot^2\theta \quad (26)$$

Above procedure can be summarized as the following steps:

(1) Choose a value of ε_1

(2) Estimate θ

(3) Calculate w from Eqs. (22), (23), (24) and (25)

(4) Estimate f_v

(5) Calculate f_1 from Eqs. (18) and (21) and take the smaller value

(6) Calculate V from Eq.(14)

(7) Calculate f_2 from Eq.(12)

(8) Calculate $f_{2\max}$ from Eq.(11)

(9) Check that $f_2 \leq f_{2\max}$. If $f_2 > f_{2\max}$, return to step (1) and choose a smaller ε_1

(10) Calculate ε_2 from Eq.(11)

(11) Calculate ε_x and ε_t from Eqs. (8) and (9)

(12) Calculate $f_v=E_s\varepsilon_t \leq f_{vy}$.

(13) Check estimate of f_v. If necessary, revise the estimate and return to step (5).

(14) Calculate $f_{sx}=E_s\varepsilon_x \leq f_y$.

(15) Calculate the axial force on the member.

$$N=A_{sx}f_{sx}-\frac{V}{\tan\theta}+f_1 b_w jd-f_c(A_c-b_w jd)$$

where f_c is the axial compressive stress outside the web.

If ε_x is tensile, then $f_c=0$, otherwise, $f_c=f'_c[2(\frac{\varepsilon_x}{\varepsilon'_c})-(\frac{\varepsilon_x}{\varepsilon'_c})^2]$

(16) Check the axial load. If N is not equal to the desired value, make new estimate of θ and return to step (2). Increasing θ increases N.

(17) Check that the longitudinal reinforcement can satisfy the condition of Eq.(26). If it does not, lower f_1 and return to step (6).

3. PRAMERTRIC ANALYSIS PROGRAM

To compare the shear strengths predicted by the MCFT with those by the ACI code, a T-section reinforced concrete beam was selected. With this test beam, a parametric analysis was performed. The parameters include concrete compressive strength, longitudinal reinforcement ratio, shear reinforcement ratio, and shear span to depth ratio. Also, the deformation charateristics, such as transverse and principal strain of concrete, inclination angle of compressive stress, and curvature, are presented for the test beam with stirrups and without stirrups. All test beams with the parameters were designed to fail in shear.

3.1 Test Beam

3.1.1 Material Properties

The following material propertis of the test beam were assumed for parametric analysis.

(1) Concrete

- Normal weight concrete
- Stress-strain relationship

. $f_c' < 6000\text{psi}(41\text{MPa})$:

$$\frac{f_c}{f_c'} = 2\frac{\varepsilon_{cf}}{\varepsilon_c'} - \left(\frac{\varepsilon_{cf}}{\varepsilon_c'}\right)^2$$

. $f_c' \geq 6000\text{psi}(41\text{MPa})$:

$$\frac{f_c}{f_c'} = \frac{n(\varepsilon_{cf}/\varepsilon_c')}{n-1+(\varepsilon_{cf}/\varepsilon_c')^{nk}}$$

where $n = 0.8 + \frac{f_c'}{2500}$, $k = 0.67 + \frac{f_c'}{9000}$

(2) Reinforcing steel :

- Deformed bar
- $f_y = 60\text{ksi}(4.14\text{GPa})$
- Perfect elastic-plastic behavior

3.1.2 Beam Details

Fig.4 shows the details of the test beam and loading set-up. The following conditions are given for a standard beam with no parameter.

- Concrete compressive strength:
  f_c' = 4000psi(28 MPa)
- Longitudinal reinforcement:
  4#10(D32), $A_s = 5.08\ in^2(3280\text{mm}^2)$,
  $\rho_w = 0.0467 = 0.87\ \rho_{max}$
- Web reinforcement: 2#3(D9.5) U-stirr
  $A_v = 0.22\text{in}^2(140\text{mm}^2)$, s=6″(150mm)
  $\rho_v = 0.0042$
- Shear span to depth ratio: $a/d = 3.5$

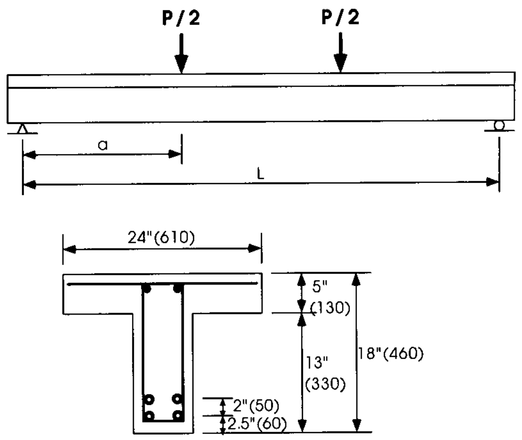


Fig.4 Details of the test beam

3.2 Parameters

3.2.1 Deformation Charateristics

- Without stirrups
- With stirrups

3.2.2 Concrete Compressive Strength

- f_c' = 3000 -7000psi(21-48MPa)

3.2.3 Longitudinal Reinforcement Ratio

- 4#7(D22), $\rho_w = 0.022$, $\rho = 0.30\ \rho_{max}$
- 4#8(D25), $\rho_w = 0.029$, $\rho = 0.41\ \rho_{max}$
- 4#9(D29), $\rho_w = 0.037$, $\rho = 0.54\ \rho_{max}$
- 4#10(D32), $\rho_w = 0.047$, $\rho = 0.87\ \rho_{max}$
- 4#11(D35), $\rho_w = 0.057$, $\rho = 1.06\ \rho_{max}$

3.2.4 Web Reinforcement Ratio

- 2#2(D6)@7″(178mm), $\rho_v = 0.0019$
- 2#2(D6)@6″(152mm), $\rho_v = 0.0022$
- 2#2(D6)@5″(127mm), $\rho_v = 0.0027$
- 2#2(D6)@4″(102mm), $\rho_v = 0.0033$
- 2#3(D9.5)@7″(178mm), $\rho_v = 0.0042$
- 2#3(D9,5)@6″(152mm), $\rho_v = 0.0049$
- 2#3(D9.5)@5″(127mm), $\rho_v = 0.0059$
- 2#3(D9.5)@4″(102mm), $\rho_v = 0.0073$

3.2.5 Shear Span to Depth Ratio
- a/d=2.0-6.0

3.3 Computer Program Response

To apply MCFT for the prediction of the response and the shear strength of a reinforced concrete beam, the computer program RESPONSE was used, because MCFT is complex and time consuming to use. The program RESPONSE was provided by Collins and Mitchell[7]. The RESPONSE can be used to predict the load-deformation response of a reinforced (or prestressed) concrete section subject to shear, moment and axial load.

Under the combined action of shear and moment, the longitudinal strains vary over the depth of the beam. Because the biaxial stresses and strains vary over the height of the beam, the inclination, θ , of the principal compressive stress changes over the height of beam, becoming larger near the flexural tension face and smaller near the flexural compression face.

To reduce the computation time, the following assumptions were made in the program:

1) The shear stress is assumed to be uniform.

2) The biaxial stresses and strains are considered at just one level of the web and are assumed that they remain constant over the depth of the web.

Considering the redistribution of shear stresses, Collins and Mitchell[7] recommended the use of the longitudinal strain at mid-depth of the web as ε_x in members with web reinforcement. Also, they commented that it is reasonable to use the highest longitudinal strain in the web as ε_x in members without web reinforcement.

4. RESULTS AND DISCUSSIONS

4.1 Deformarion Charateristics

Fig.5 shows the applied shear force and transverse and principal strains of the test beams. The strain responses of the beam with stirrups is roughly tri-linear with a change in their slopes at the level of cracking and stirrup yielding loads. On the contrary, the strain response without stirrups is curvi-linear, which is similar in shape to the stress-strain curve of concrete. At the maxium shear strength, the beam with stirrups has much larger transverse and principal strains than those without stirrups. This fact is believed to be due to the shear reinforcement provided. Also, the beam with stirrups is more ductile due to the shear reinforcement yielding after reaching its peak shear strength. On the contrary, the beam without stirrups shows sudden failure after reaching its maximum shear strength.

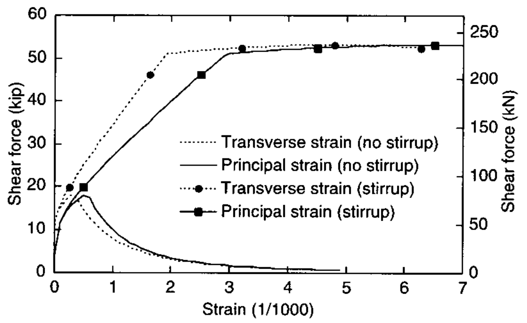


Fig.5 Shear force-strain curves

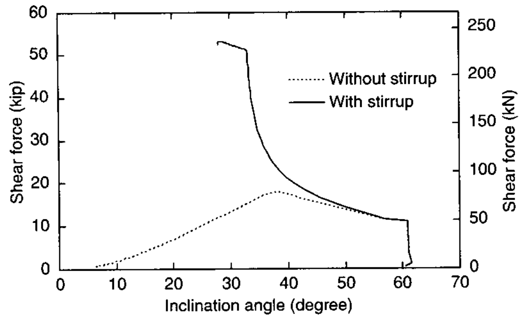


Fig.6 Shear force-inclination angle curves

As shown in Fig.6, the inclination angles of the compressive stress in the web concrete are about 60° before diagonal cracking. After cracking, inclination angles decrease radically and reach about 40° and 30° at their peak shear shear strengths, without stirrups and with stirrups, respectively. The inclination angle at stirrup yielding is about 33°, which is considerably lower than 45° assumed in the ACI code. This fact implies that the ACI code is conservative in evaluating the contribution of shear reinforcement to shear strength compared to the MCFT.

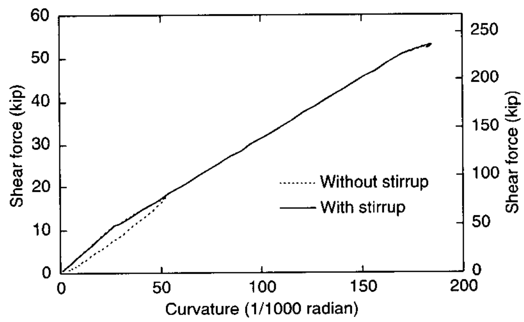


Fig.7 Shear force-curvature relationship

Fig.7 shows a clear linear relationship between the shear force and the curvature in the beams which fail in shear, unlike in the beams which fail in flexure. From this fact, it can be stated that shear failure is very brittle unlike flexural failure. Also, the beam without stirrups has little deformability.

4.2 Concrete Strength

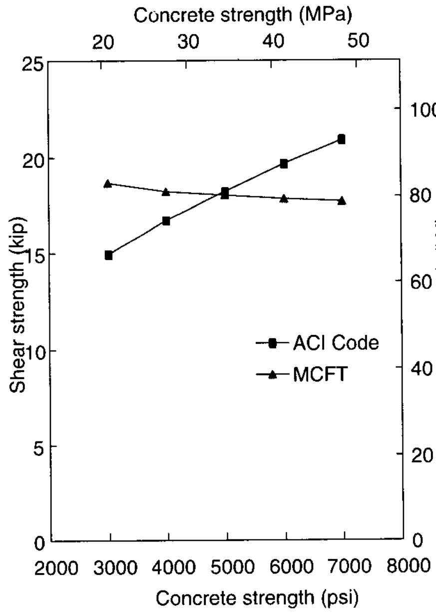


Fig.8 Influence of concrete strength on shear strength of beam without stirrups.

To evaluate the effect of concrete strength on shear strength, shear resisting capacities are predicted using the MCFT and the ACI code for different concrete strength.

As can be seen in Fig.8, the predicted shear strength by the ACI code increases curvi-linearly with large curvature (almost linearly) as the concrete compressive strength increases in the beam without stirrups. On the contrary, the shear strength predicted by the MCFT slightly decreases linearly with an increase of concrete strength. The decrease in shear strength by the MCFT is attributed to the fact that the principal stress in the web concrete increases faster than the cracking stress(f_{cr}) as the concrete strength increases. The shear strengths predicted by the MCFT are about 25% higher and 15% lower than those by the ACI code in the concrete of 3000psi(21MPa) and 7000psi(48MPa), respectively.

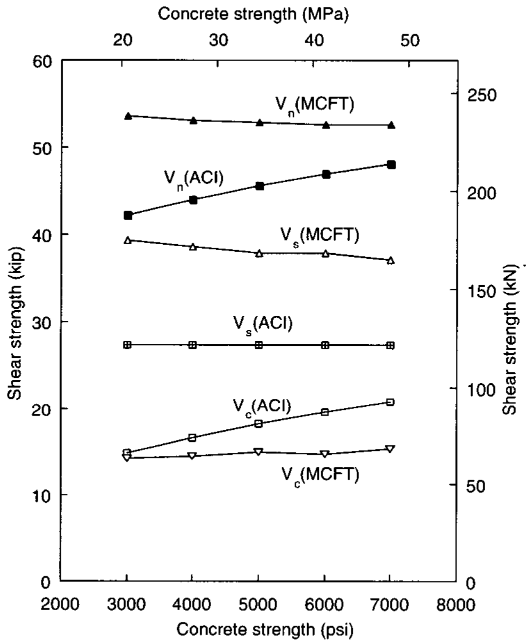


Fig.9 Effect of concrete strength on shear strength

The comparison of the shear strengths for different concrete strength in the beam with stirrups is shown in Fig.9. The concrete contribution to the shear strength predicted using the MCFT is little affected by the concrete compressive strength. The concrete contribution by the ACI code is about 5% and 35% higher than those by the MCFT in the concrete of 3000psi(21MPa) and 7000psi(48MPa).

While shear reinforcement contibution predicted by the MCFT slightly decreases linearly with an increase of concrete strength. The stirrup contribution to the shear strength by the MCFT is considerably higher(about 45% and 35% higher) than those by the ACI code in the the concrete of 3000psi(21MPa) and 7000 psi(48MPa), respectively. The reason for this higher prediction is that the inclined crack angle in the beam with stirrups is much smaller than 45° assumed in the ACI code. Consequently, the nominal shear strengths by the MCFT are about 25% and 10% higher than those by the ACI code in the concrete of 3000psi (21MPa) and 7000psi(48MPa), respectively.

4.3 Longitudinal Reinforcement Ratio

To investigate the effect of longitudinal reinforcement ratio, shear strengths were predicted using the MCFT and the ACI code for different longitudinal reinforcement ratio.

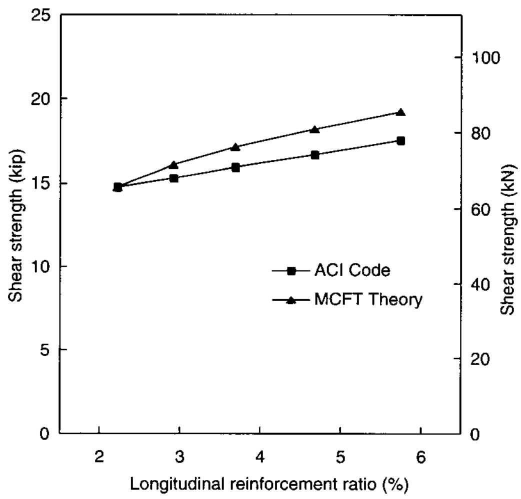


Fig.10 Effect of longitudinal reinforcement ratio on shear strength without stirrups.

As shown in Fig.10, the predicted shear strengths of the beam without shear reinforcement increases almost linearly with an increase of longitudinal reinforcement ratio in the two methods. The shear strength by the MCFT is about 10% higher in the beam with the maximum reinforcement ratio.

Unlike in the beam without stirrups, the concrete contribution in the beam with stirrups by the MCFT is smaller than that by the ACI code as shown in Fig.11. The stirrup contribution to shear strength increased curvi-lineraly as the reinforcement ratio increases. As a result, the nominal shear strengths by the MCFT are about 20% smaller and 20% higher in the beams with 30% and 106% of the maximum reinforcement, respectively. In normally reinforced beam with about 60% of the maximum reinforcement, the MCFT gives about 15% higher prediction of shear strength in this test beam.

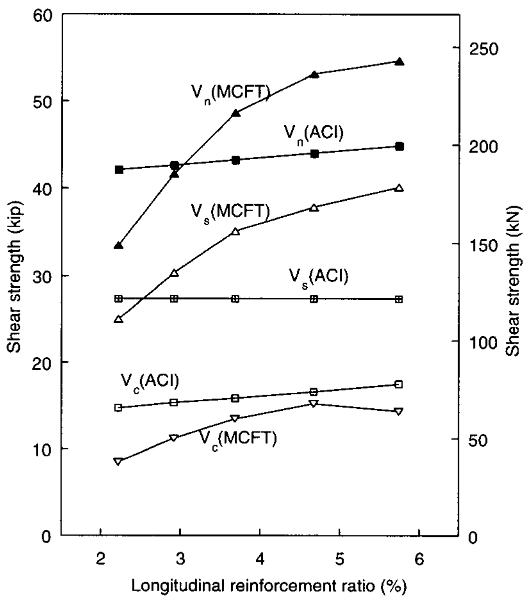


Fig.11 Effect of longitudinal reinforcement ratio on shear strength with stirrups.

4.4 Web Reinforcement Ratio

Beams with different shear reinforcement ratio were analyzed to investigate the effect of web reinforcement ratio on shear strength. As can be seen in Fig.12, the ACI code gives constant concrete contribution and linearly proportional stirrup contribution to web reinforcement ratio. On the other hand, the MCFT predicts lower contribution of the concrete and higher contribution of the shear reinforcement. Consequently, the MCFT gives higher nominal shear strength in the beam with lower shear reinforcement ratio. From the analysis results shown in Fig.12, it is assumed that the shear resistance capacity predicted by the MCFT may be lower than that by the ACI code if sufficient stirrups were provided enough to resist applied shear force up to flexural failure load. This is because all beams analyzed were designed to fail in shear.

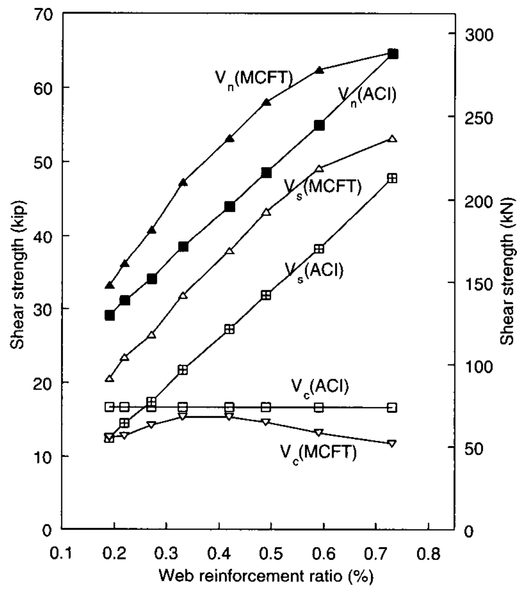


Fig.12 Effect of web reinforcement ratio on shear strength.

4.5 Shear Span to Depth Ratio

Figs.13 and 14 show the parametric analysis results for various span to depth ratios in beams without and with stirrups. As shown in Fig.13, the ACI code and the MCFT predict slightly decreasing shear strengths with an increase of shear span to depth ratio in the beams without stirrups. The difference between the predictions by the MCFT and by the ACI code is less than 10%.

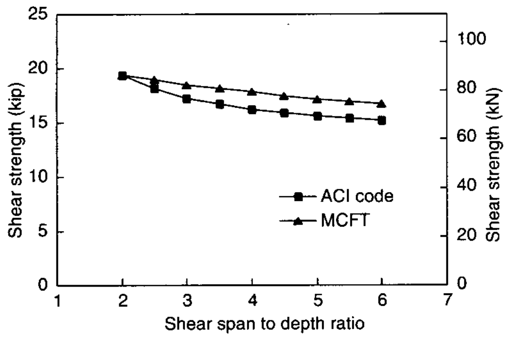


Fig.13 Effect of shear span to depth ratio on shear strength

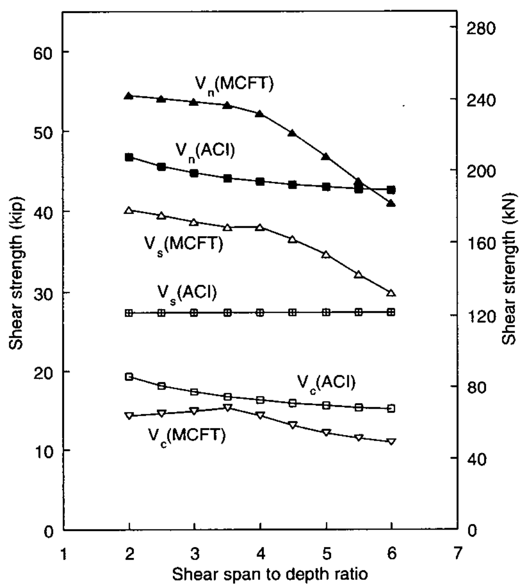


Fig.14 Effect of shear span to depth ratio on shear strength with stirrups.

In the beam with stirrups, the shear reinforcement contribution curve by the MCFT is roughly bi-linear with a change in its slope at the shear span to depth ratio of 4.0. After the change in slope, the shear strength provided by stirrups radically decreases as the shear span to depth ratio increases. As a result, the nominal shear strength curve is similar to stirrup contribituion curve in shape.

5. CONCLUSION

The compression field theory enables not only the prediction of the shear strength but also the prediction of the response of reinforced concrete beams subjected to shear, moment, and axial force. The most important aspect of this approach is that it is more general and rational than the ACI code. However, the procedure is complex and time consuming for designers to use the theory.

On the contrary, the ACI code has the most importent advantage of simplicity. However, it lacks generality and rationality in its shear equations based on empirical test results.

From this analytical study with a selected test beam, the following conclusions can be drawn.

1) In the normal strength concrete beam without stirrups, the predicted shear strength by the MCFT is a little higher than the one by the ACI code.
2) In the beam with stirrups, the concrete contribution to shear strength predicted by the MCFT is a little lower than the one by the ACI code.
3) The predicted shear reinforcement contribution by the MCFT is considerably higher than the one by

the ACI code. This is because the inclination angle of compressive stress in web concrete is much smaller than 45° assumed in the ACI code.

4) The nominal shear strength predicted by the MCFT depends on the contributions of concrete and stirrups. With lower web reinforcement ratio, the MCFT gives higher prediction of nominal shear strength than the ACI code. However, with higher web reinforcement ratio, the MCFT predicts lower shear resisting capacity than the ACI code.
5) The predicted shear strength by the MCFT is little affected by concrete compressive strength, while the concrete contribution by the ACI code is proportional to square root of concrete strength.
6) The shear strength predicted by the MCFT increases curvi-linearly with an increase of longitudinal reinforcement ratio, while the concrete contribution to shear strength by the ACI code increases linearly.
7) The shear strength by the MCFT increases curvi-linearly as the web reinforcement ratio increases, while the shear strength by the ACI code increases linearly.
8) The shear strength decreases with an increase of shear span to depth ratio. In the beams with stirrups, the shear strength predicted by the MCFT radically decreases with a larger shear span to depth ratio than 4.0.

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