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Research on mathematics teachers' knowledge

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Introduction

The question "What does it mean to know?" has been raised in all areas of study. In the past it was assumed that knowledge in any discipline was simply an accumulation of bits of information arranged in some sequential order. Today this assumption has been challenged. The alternative perspective involves a notion of authentic knowledge. A new view of learning, in cognitive science, is an outgrowth of the revolution in psychology that has become dominant during the past decade (Romberg,1992). *In this view, acquisition of knowledge implies* changes in schemata, not just the aggregation of information. Then, we can think about knowledge as a process of development considering that knowledge is continually changing and developing, that knowledge is not static (Fennema & Franke.1992).

In mathematics, some research studies have addressed the question "What does it mean to know mathematics?" and how one comes to know mathematics. Lampert(1986) has investigated how mathematics might be taught and learned in classrooms. She reported that students can "do" mathematics and think mathematically. Even though the ideal mathematics teacher is described as being one who lets students "do" mathematics, the mathematics students are taught to "do" must be put into a framework that is understandable by the learners. The teacher must help students to see the relationship between their knowledge and the new knowledge that they are to learn, so that the teacher's knowledge is important to promote a good environment in classroom for students' mathematics learning.

The Professional Standards for Teaching Mathematics (NCTM, 1990)presents the guidelines for a "good mathematics teaching" based on the *Curriculum and Evaluation Standards for School* Mathematics (NCTM, 1989). However the decisions that are made in implementing these guidelines rest upon teachers' confidence in the appropriateness of doing so. Teachers' confidence in changing their practice and the way teachers interpret and implement curricula seem to be influenced by their knowledge and beliefs about mathematics, mathematics teaching and learning(Clark&Peterson,1986).

Moreover, "some inconsistencies between teachers' professed beliefs and practices may also be manifestations of espoused teaching ideals that

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cannot be realized because the teachers don't possess the skills and knowledge necessary to implement them"(Thomson,1992).

If knowledge is important, what does it mean to discuss mathematics teachers' knowledge?

Thinking about the content knowledge, there are some studies stating that there is little support for a direct relationship between mathematics teachers' knowledge and student learning, but we have to consider how these studies measured mathematics teachers' content knowledge. The National Longitudinal Study of Mathematical Abilities(1972) and Eisenberg(1977) defined teachers' knowledge as the number of university-level mathematics courses successfully completed. No attempt was made to measure what the teachers knew about mathematics of to ascertain accurately the mathematics covered in the various courses completed. The General Accounting Office (1984) used some form of a standardized test such as the National Teachers Examination to identify teachers' knowledge, and, once again, no attempt was made to measure the complexity of teachers' knowledge or the relationship between the formal mathematics that teachers knew and what they taught. Perhaps the inadequate measures of knowledge and relatively limited research methodology concealed any relationship that existed between mathematics teachers' knowledge and student learning.

In this case there was a crisis because theses results were so discouraging to continue this kind of research. Begle(1979) reported that "attempts to improve mathematics education would not profit from further studies of teachers". And, this crisis produced a paradigm shift in research on teaching, making an analogy with views of Kuhn(1970).

As a consequence of the shift of paradigm for research on teaching, recently, instead of using correlational techniques to measure the relationship between some measure of teachers' knowledge and their students' learning, scholars have been looking at teaching itself, with the interpretive tradition and have concentrated on providing rich descriptions of teacher's actions in their classrooms.

Teachers' knowledge has been studied and some frameworks and cognitive models have been proposed by researchers that indicate how the various components of knowledge might be organized.

Peterson(1988) argued that to be effective, teachers of mathematics need three kinds of knowledge: How students think in specific content areas, how to facilitate growth in students' thinking, and self-awareness of their own cognitive processes. She didn't ignore the content knowledge necessary to teach, but argued that this knowledge must be held in relation to the three categories she has identified.

Leinhardt(1986) and colleagues have the goal describing the mental structures of skilled teachers in depth. Their work is based on the belief that teaching is "a complex cognitive skill amenable to analysis in a manner similar to other skills described by cognitive psychology" (Leinhardt & Greeno, 1986). They defined two fundamental and related systems of knowledge: Subject matter (content knowledge) and lesson structure (practical knowledge). The structuring of a lesson takes priority and is both supported and constrained by the teacher's content knowledge.

Elbaz(1983) suggested that the practical, personal conception of teacher knowledge posits knowledge as dynamic, context driven, and related across past, present, and future. The structure of teachers' knowledge includes three dimensions: rules of practice, practical principles, and images. The rules and principles embody the instructional knowledge, while the images direct the decision-making process.

Another framework that can be applied to the understanding of teachers' knowledge is that of situated knowledge. When discussing situated knowledge and its acquisition, several have pointed out the contrast of knowledge gained by students in schools and out of school(e. g., Brown et al., 1989). This research have theorized that in-school knowledge is acquired by well-defined problems. Out-of-school knowledge is acquired by working in a social situation to decide on the causes of events, to solve ill-defined problems. The idea that knowledge acquired either in or out of school is not independent of the situation in which it is acquired (Brown et al., 1989). All knowledge is situated and is partly a result of the activity, context, and culture in which it is developed.

Research on mathematics teachers' knowledge

The models of Shulman, Peterson, and Leinhardt and colleagues have been applied to study mathematics teachers' knowledge. There is some evidence that elementary teachers don't have the content knowledge related to different mathematical topics in order to teach mathematics conceptually (Brow et al., 1990). For example, Ball(1988) reported on 19 pre-service teachers' abilities to develop a representation of 1 3/4 : 1/2. Ball developed a framework for exploring teachers' content knowledge in mathematics. Knowledge of mathematics is closely related to Shulman's dimension of substantive knowledge, and it includes both procedural and propositional knowledge. Knowledge about Mathematics is related to Shulman's dimension of syntactic knowledge, and it includes an understanding of the nature of knowledge in the discipline. She analyzed questionnaires and interviews to describe the mathematical understanding they brought to teacher education as well as their ideas about teaching, learning, students, and teachers' roles. In general, more students were able to give correct answers than were able to explain the reasons that their answers were correct. Ball reported that pre-service teachers themselves recognized the need for broader, deeper understanding of mathematics in order to teach conceptually.

It is interesting to note that Ginther et al. (1987) found that mathematics courses alone don't address the apparent mathematical deficiencies that seem to characterize elementary teachers. Although twice as many elementary teachers who were trained during the period 1983-1985 had 4 or more years of high school mathematics than did their counterparts from the 1967-1969 era and that the percentage of teachers taking three of more college mathematics courses increased from 4% to 23% from these same eras, their understanding of mathematics decreased.

Feiman-Nemser et al.(1986) concluded that, without guidance, pre-service teachers find it difficult to make the transition to pedagogical thinking. The researchers identified the transition to pedagogical thinking as a major component of learning to teach.

Research studies of Schram et al. (1988) provided evidence regarding the complexity of the process of changing prospective teachers' beliefs and knowledge about mathematics, mathematics teaching and learning. After the intervention, the way the pre-service teachers thought about mathematics for themselves was different from the way they thought it for young children. The participants consisted of 24 pre-service teachers. Data were collected during their two-year teacher preparation program and their first year of teaching and consisted of field notes, videotapes of class sessions for mathematics and methods courses, and questionnaires. The researchers examined changes in undergraduate education major's knowledge about mathematics, mathematics teaching and learning as they progressed through a sequence of three innovative mathematics courses. The courses emphasized conceptual development, group work, and problem-solving activities. The phenomenon of assimilation without accommodation was documented by Scharm and Wilcox(1988) in a case study: The student appeared to take in the new experiences and conceptual ideas by modifying them to fit into her original conceptions. The researchers concluded that a single, 10-week course, is insufficient to persuade teachers to resist the contextual constraints impeding conceptual approaches to teaching in elementary classrooms.

Some studies indicate that for teacher's knowledge of content to positively influence classroom instruction the knowledge must be organized in a particular way. For example, Steinberg et al. (1985) investigated the impact of interrelated knowledge on teaching. The participants were 20 student teachers in the 1984-85 academic year. The teachers whose mathematical knowledge appeared to be connected and conceptual were also more conceptual in their teaching and engaged students in more problem-solving activities, while those without this type of knowledge were more rule-based. The limitations in subject matter knowledge and pedagogical content knowledge were associated with a difficulty in making the transition to pedagogical thinking, an inability to connect topics during classroom instruction, and a focus on procedural rather than on conceptual understanding.

Also, the expertise studies demonstrated the same results. Leinhardt(1986) investigated how subject matter knowledge is translated into curricular events in classrooms, videotaping lessons of fractions. They obtained teachers comments on these lessons and mapped complex relationships between teachers' subject matter knowledge and their strategies and routines for engaging students in the content. Their subjects were eight four-grade mathematics teachers, four novices (student teachers) and four experts (teachers whose students had showed unusual and consistent growth scores over a five-year period). The expert teachers appeared to know not only the procedural rules of solving problems but understood the interrelationships of the procedures; they exhibited a more refined hierarchical structure to their knowledge.

Then, an important factor in a positive relationship between content knowledge and classroom instruction seems to be the mental organization of the knowledge. As stated by Brophy(1991), "where teachers knowledge is more explicit, better connected, and more integrated, they will tend to teach the subject more dynamically, represent it in more varied ways, and encourage and respond fully to student comments and questions. Where their knowledge is limited, they will tend to depend on the text for content, emphasize interactive discourse in favor of seat work assignments, and in general, portray the subject as a collection of static factual knowledge".

The view of children as constructors of their own mathematical knowledge has been discussed and accepted but it has not been accompanied by a similar view of research on teachers' knowledge. The study of D'Ambrosio and Campos(1992) demonstrated that experiences with prospective teachers in a constructivist view may be fruitful experiences. The researchers gathered evidence of change in pre-service teachers' conceptions of children's knowledge of fractions. They focused on the conflicts generated by the gaps between the pre-service teachers' research findings and their representations of the children's knowledge of fractions. The researchers based the study on the assumption that decision-making is a central skill of teaching. Five pre-service teachers were provided of several readings of research studies on children's representations of fractions to establish the theoretical background, and developed themselves research study for getting the knowledge of children's representations of fractions. The subjects were involved in the project in two roles: First, as subjects of the report; second, as members of a research group addressing children's understanding of fractions.

Concerning the in-service programs, the CGI project(Carpenter et al., 1989) provides evidence that teachers can attend to individual students when they have appropriate and well organized knowledge. The study involved 40 first-grade teachers, twenty of whom were randomly assigned to a treatment group. The treatment teachers attended a one-month summer workshop on research regarding the learning of addition and subtraction concepts, but they were not trained in specific techniques for altering their teaching or the curriculum. The researchers compared what experimental teachers and control teachers knew about the thinking of children in their classrooms by asking children to solve problems and then asking the teachers to predict how the children would solve those problems. Results indicated that students in the experimental classes performed more favorably on measures of problem solving and also on recall of number facts. The group of experimental teachers spent more time on number facts problems. Experimental teachers focused more often on the process students used to solve problems, and control teachers focused more frequently on the answer to the problems. In addition, experimental teacher allowed students a wider variety of strategies to solve problems. It seems that the studies of the CGI project illustrate that knowledge derived from research on learner's thinking can be used by teachers in a way that has impact on educational outcomes. Maybe this knowledge was so helpful because it was specific in regard to children's problem solving in a particular domain that was already part of the teacher's curriculum.

The study of Cobb et al.(1988) underscored the importance of teachers seeing their current practice as problematic as some sort of prerequisite mental state necessary for beneficial collaboration between teachers and researchers or staff developers in in-service programs. Insightful analyses and detailed accounts of how teachers internalized new ideas and developed new instructional practices contributed to the understanding of the cognitive processes involved in teachers changing their conceptions and practices. At the beginning of the project a second-grade class was used in a year-long experiment. The researchers worked closely with an experienced but traditional teacher to develop instructional activities that allowed students to construct their knowledge. Some activities had been developed in the year preceding, but many more were constructed and modified during the experimental year partly as a response to differences in the new students' mathematical conceptions. The study was considered successfully by both the researchers and the district personnel, as a consequence, an increased pool of teachers was able to implement the instructional procedures in a third year of the project. Eighteen teachers participated in the first round of the in-service program; 10 participated in an assessment of the project. Teacher and student data from the 10 classrooms were compared to data from the 8 non-project classrooms. Project teachers' pedagogical content beliefs were more compatible with constructivism than were those of their of their non-project colleagues. Project students were superior in conceptual understanding at the end of the school year and attributed less importance to conforming to the solution methods of others, being competitive.

Findings from both the CGI project and the study of Cobb et al. indicate that in-service programs can affect teachers' pedagogical content knowledge and beliefs, and that these changes in knowledge and beliefs are associated with changes in classroom practices and student achievement. But, we have to remember that teachers of the experimental group in the study of Cobb et al. were volunteers, and in the CGI project experimental teachers and students didn't outperform control teachers and students on several measures of knowledge and beliefs. The pattern of mixed finding was also supported by investigations in other content domains. The project was a multi-year, multi-phase program of research in which Good et al. (1979) developed and tested a model for whole-class instruction in mathematics. The research focused on effective mathematics teaching practices, instead of on teacher knowledge or thought process, but the researchers speculated that one reason for the low level of implementation might be that the manual that comprised the in-service program might not have been extensive enough to enable the growth in content knowledge or pedagogical content knowledge necessary to alter significantly teachers' presentation of new skills and concepts. The researchers concluded that the process of helping teachers improve their content knowledge and pedagogical content knowledge requires careful attention over an extended period of time.

The study of Lappan et al. (1988) also demonstrated that the presentation of detailed teaching units in a 2-week summer workshop to middle school mathematics teachers was sufficient to teach the teachers the project's units but not to enable transfer to other parts of the curriculum. They were committed to help teachers to adopt a more conceptual approach to teaching mathematics. They reported that transfer of knowledge and integration require a substantial, long-term, staff-development programs of at least 2 years, including intellectual and emotional support in addition to any provision of materials.

Generally, the studies point out the need to study knowledge of subject matter as it interacts with other aspects of teacher knowledge. The results of the studies are intriguing and lend credence to the belief that content knowledge of teachers is positively related to the structure of their classrooms, but we need more research addressing this theme.

Conclusions

The questions dealing with research on teachers' knowledge are complex. As beliefs and knowledge are related to each other, it seems more helpful to focus research studies on teachers' conceptions, encompassing both beliefs and any aspect of the teachers' knowledge.

To study mathematics teachers' knowledge the models of Shulman, Peterson, and Leinhardt and colleagues have been applied, but we still have to try the model of Elbaz and the model of teachers knowledge as situated. Also, research has been done on the elementary and middle school; researchers need to look at the secondary and post-secondary levels. And, we might think about the validity of mixing either teachers from different levels, or pre-service and in-service teachers in these studies.

Knowledge should be structured about how teachers can be helped to increase and develop their understanding of mathematics. Prospective teachers' beliefs and knowledge seem to be shaped by their own experiences as students of mathematics; the task of modifying long-held, deeply rooted conceptions of mathematics and its teaching and learning in a short period of time remains a major problem in mathematics teacher education. Generally, studies on change in teachers' beliefs and knowledge involving pre-service and in-service teachers have not provided the detailed analyses necessary to shed light on the question of why it is so difficult for teachers to accommodate their schemas and internalize new ideas. How we use information about teachers' beliefs about mathematics and its teaching and learning to help teachers reflect on their own beliefs and practice is an issue that deserves attention.

Measures need to be developed that can account for teachers' differing orientations. Researchers must also be aware of their own orientations, as they may influence the measures they choose and the results that they gain.

Valuable detailed knowledge can be gained by studying how students learn mathematics, but we need more studies about the relevance of such knowledge for teachers.

Finally, the broad acceptance of children as constructors of their own mathematical knowledge has not been accompanied by a similar view in research on teachers' knowledge. It should be studied about the relationship between mathematics teachers' knowledge and students' learning, and about the extent to which teachers and students' conceptions interact during instruction.

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수학 교사의 지식 연구

이 중 권 · 김 용 기

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