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The (k, t_p) Replacement Policy for the System subject to Two Types of Failure

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Abstract

In this paper, we consider a new preventive replacement policy for the system which deteriorates while it is in operation with an increasing failure rate. The system is subject to two types of failure. A type 1 failure is repairable while a type 2 failure is not repairable. In the new policy, a system is replaced at the age of t_p or at the instant the k^{th} type 1 failure occurs, whichever comes first. However, if a type 2 failure occurs before a preventive replacement is performed, a failure replacement should be made. We assume that a type 1 failure can be rectified with a minimal repair. We also assume that a replacement takes a non-negligible amount of time while a minimal repair takes a negligible amount of time. Under a cost structure which includes a preventive replacement cost, a failure replacement cost and a minimal repair cost, we develop a model to find the optimal (k, t_p) policy which ˙mizes the expected cost per unit time in the long run while satisfying a system ˙˙v constraint.

1. Introduction

A considerable amount of work has been done on the maintenance policies for the system that stochastically deteriorates with time[1,6]. In particular, when the system is repairable, a decision has to be made as to when to replace the system and when to repair it in order to operate the system most economically.

Nakagawa[3] considered a model for determining the optimal number of failures N^* before the scheduled replacement time T, in which a failed unit undergoes minimal repair between replacements regardless of the failure type. Nakagawa[2] also considered a replacement policy for the system which is exposed to two types of failure. In this study, it is assumed that a type 1 failure is repairable and can be rectified with a minimal repair while a type 2 failure is not repairable. Therefore, the system should be replaced whenever a type 2 failure occurs. For this system, he considered a replacement policy in which a system is replaced at a type 2 failure or at the k^{th} type 1 failure, whichever comes first.

In this paper, we combine these two results and consider a (k, t_p) policy for the system which is subject to two types of failure. In our (k, t_p) policy, a system is replaced at the age of t_p or at the instant the k^{th} type 1 failure occurs, whichever comes first. However, if a type 2 failure occurs before a preventive replacement is performed, a failure replacement should be made. The probability that each failure is classified as a type i failure is p_i ($p_1+p_2=1$) independent of time and other failures. We assume that a replacement takes a non-negligible amount of time while a minimal repair takes a negligible amount of time. We also assume that a minimal repair restores the system to the condition it was in immediately prior to the failure.

The objective of this study is to develop a model to find the optimal (k, t_p) policy which minimizes the expected cost per unit time in the long run while satisfing the constraint that the availability of the system should exceed a certain prespecified value.

2. Notation & Assumptions

Notation

C_m : cost of minimal repair

C_p : cost of preventive replacement

C_f : cost of failure replacement ($C_f > C_p > C_m$)

R_m : mean time required to perform minimal repair

R_p : mean time required to perform preventive replacement

R_f : mean time required to perform failure replacement ($R_f > R_p > R_m$)

t_p : planned preventive replacement age

k : maximum number of Type 1 failures for preventive replacement

$N_R(k, t)$: number of minimal repairs performed in the interval $(0, t)$

t : observed time from system replacement until next failure(Type 1 or Type 2) with failure rate $h(t)$

$Ni(t)$: number of Type i failures in period $(0, t)$, where t represents time to failure $(i = 1, 2)$

$f(t)$: probability density function for time to failure

$F(t)$: probability distribution function for time to failure

$G(t)$: probability distribution function for time to Type 2 failure

$B(t)$: probability distribution function for time to k^{th} Type 1 failure

$h(t)$: system failure rate at time t

$H(t)$: $\int_0^t h(x)dx$, cumulative hazard

p_i : probability of Type i failure ($p_1 + p_2 = 1$)

q_i : probability that case i occurs $(i = 1, 2, 3)$

T_k : time to k^{th} Type 1 failure

T_f : time to Type 2 failure

$UC(k, t_p)$: expected long run average cost under (k, t_p) policy

$M(k, t_p)$: mean length of operation time during a cycle under (k, t_p) policy

$A(k, t_p)$: availability of the system under (k, t_p) policy

Assumptions

1. The failure rate function $h(t)$ is increasing.
2. A failure is detected immediately.
3. A minimal repair takes a negligible amount of time and does not affect failure

rate of the system.

4. A replacement restores the system to *like new*.

3. The (k, t_p) Replacement Model

3.1 Replacement policy

We assume two types of failure are possible during system operation. Type 1 failure is rectified by minimal repair or preventive replacement (PR), and does not drastically affect system availability. Type 2 failure is irrepairable, meaning the system needs a failure replacement (FR). The probability of each type of failure is p_1 and p_2, respectively.

There can be three replacement scenarios :

Case 1 : preventive replacement at the k^{th} Type 1 failure

Case 2 : preventive replacement at age t_p

Case 3 : failure replacement immediately after Type 2 failure

Let q_i represent the probability of a case i occurrence. Type 1/Type 2 failures follow a non-homogeneous Poisson process with intensity functions $p_1h(t)/p_2h(t)$, respectively, both processes are independent, and the combined process creates a non-homogeneous Poisson process with intensity function $h(t)$. q_i can be computed as follows:

1) case 1

Case 1 happens if at least k failures occurs during $(0, t_p)$ and the first k failures of these are Type 1 failures. So,

$$q_1 = p_1^{\,k} \sum_{n=k}^{\infty} e^{-H(t_p)} \frac{H(t_p)^n}{n!} \qquad (1)$$

2) case 2

Case 2 happens if $min\{T_k, T_f, t_p\} = t_p$. That is to say, Type 2 failure does not occur during $(0, t_p)$ and the number of Type 1 failures during $(0, t_p)$ is less than k. So,

$$q_2 = e^{-p_2 H(t_p)} \sum_{n=0}^{k-1} e^{-p_1 H(t_p)} \frac{[p_1 H(t_p)]^n}{n!} \qquad (2)$$

3) case 3

Case 3 happens if $min\{T_k,\ T_f,\ t_p\} = T_f$.

This means that T_f is less than t_p and the number of Type 1 failures during $(0,\ T_f)$ is less than k. So,

$$q_3 = \int_0^{t_p} \sum_{n=0}^{k-1} e^{-p_1 H(t)} \frac{[p_1 H(t)]^n}{n!} dG(t) \tag{3}$$

or $q_3 = 1 - q_1 - q_2$

T_k and T_f have probability distribution function $B(t)$ and $G(t)$ respectively and $B(t)$, $b(t)$, $G(t)$, $g(t)$ can be computed as follows:

$$\begin{aligned} B(t) &= \Pr\{\ T_k \le t\ \} \\ &= \Pr\{\ \text{the number of Type 1 failures during } (0,t] \text{ is greater than or equal to } k\ \} \\ &= \Pr\{\ N(t) \ge k\ \} \\ &= 1 - \sum_{n=0}^{k-1} e^{-p_1 H(t)} \frac{[p_1 H(t)]^n}{n!} \end{aligned} \tag{4}$$

$$b(t) = \frac{dB(t)}{dt} = p_1 h(t)\, e^{-p_1 H(t)} \frac{[p_1 H(t)]^{k-1}}{(k-1)!} \tag{5}$$

$$\begin{aligned} G(t) &= \Pr\{\ T_f \le t\ \} \\ &= \Pr\{\ \text{Type 2 failure occurs at the age } t\ \} \\ &= 1 - e^{-p_2 H(t)} \end{aligned} \tag{6}$$

$$g(t) = \frac{dG(t)}{dt} = p_2 h(t)\, e^{-p_2 H(t)} \tag{7}$$

q_1, q_2, q_3 also can be obtained by

$$q_1 = \int_0^{t_p} (1 - G(t))\, dB(t) = \int_0^{t_p} e^{-p_2 H(t)} dB(t) \tag{8}$$

$$q_2 = \int_{tp}^{\infty} \int_{tp}^{\infty} dB(t)\, dG(t) = \{1 - B(t_p)\}\{1 - G(t_p)\} \tag{9}$$

$$\begin{aligned} q_3 &= \int_0^{t_p} (1 - B(t))\, dG(t) \\ &= \int_0^{t_p} \sum_{n=0}^{k-1} e^{-p_1 H(t)} \frac{[p_1 H(t)]^n}{n!} dG(t) \end{aligned} \tag{10}$$

3.2 Expected length of cycle, *E(L)*

The length of cycle(L) is completed each time a failure replacement or preventive replacement takes place and depends both on the time of failure replacement and preventive replacement.

$$L = \begin{cases} L1 : R_p + T_k & \text{with probability } q_1 \\ L2 : R_p + t_p & \text{with probability } q_2 \\ L3 : R_f + T_f & \text{with probability } q_3 \end{cases} \tag{11}$$

The expected length of cycle, $E[L]$, is

$$\begin{aligned} E[L] &= \int_0^{t_p} t(1-G(t))dB(t) + t_p*\{1-B(t_p)\}\{1-G(t_p)\} + \int_0^{t_p} t(1-B(t))dG(t) \\ &\quad + (q_1+q_2)*R_p + q_3*R_f \\ &= M(k,\ t_p) + (q_1+q_2)*R_p + q_3*R_f \end{aligned} \tag{12}$$

where $M(k,\ t_p)$ is the mean age of the system during a cycle

$$\begin{aligned} M(k,\ t_p) &= \int_0^{t_p} t(1-G(t))dB(t) + t_p*\{1-B(t_p)\}\{1-G(t_p)\} \\ &\quad + \int_0^{t_p} t(1-B(t))dG(t) \end{aligned} \tag{13}$$

3.3 Expected total cost, *E(C)*

A cycle is completed each time a replacement takes place, and costs(C) incurred in a cycle is given by the total costs of minimal repairs and the costs of either preventive replacement or failure replacement.

Let Ni represent the number of minimal repairs during a cycle given a case i occurrence $(\ i = 1,\ 2,\ 3\ )$.

To find the total costs of minimal repairs, we should know the expected number of minimal repairs for three separate cases, $E(Ni)$. The expected number of minimal repairs and expected total cost for each three case is given by

1) case 1

$$\begin{aligned} E(N1) &= E[N_R(k, t_p) \mid T_f < t, t_p < t, N_1(t) \ge k] \\ &= k-1 \end{aligned} \tag{14}$$

$$E(C1) = C_p + C_m * E(N1)$$

$$= C_p + C_m * (k-1) \tag{15}$$

2) case 2

$$E(N2) = E[N_R(k, t_p) \mid T_f > t_p, N_1(t_p) < k]$$

$$= \sum_{n=0}^{\infty} nP[N_1(t_p) = n \mid T_f > t_p, N_1(t_p) < k]$$

$$= \frac{\sum_{n=0}^{\infty} nP[N_1(t_p) = n,\ T_f > t_p, N_1(t_p) < k]}{P[T_f > t_p, N_1(t_p) < k]}$$

$$= \frac{P[T_f > t_p] \sum_{n=0}^{k-1} nP[N_1(t_p) = n]}{q_2}$$

$$= \frac{e^{-p_2 H(t_p)} \sum_{n=0}^{k-1} nP[N_1(t_p) = n]}{q_2}$$

$$= \frac{1}{q_2} e^{-p_2 H(t_p)} \sum_{n=0}^{k-1} n \ e^{-p_1 H(t_p)} \frac{[p_1 H(t_p)]^n}{n!} \tag{16}$$

$$E(C2) = C_p + C_m * E(N2)$$

$$= C_p + C_m * \frac{1}{q_2} e^{-p_2 H(t_p)} \sum_{n=0}^{k-1} n \ e^{-p_1 H(t_p)} \frac{[p_1 H(t_p)]^n}{n!} \tag{17}$$

3) case 3

$$E(N3) = E[N_R(k, t_p) \mid T_f < t_p, N_1(t_p) < k]$$

$$= \sum_{n=0}^{\infty} nP[N_1(T_f) = n \mid T_f < t_p, N_1(T_f) < k]$$

$$= \frac{\sum_{n=0}^{\infty} nP[N_1(T_f) = n,\ T_f < t_p, N_1(T_f) < k]}{P[T_f < t_p, N_1(T_f) < k]}$$

$$= \frac{\sum_{n=0}^{k-1} nP[N_1(T_f) = n,\ T_f < t_p]}{q_3}$$

$$= \frac{\int_0^{t_p} \sum_{n=0}^{k-1} nP[N_1(t) = n] dG(t)}{q_3}$$

$$= \frac{1}{q_3} \int_o^{t_p} \sum_{n=0}^{k-1} n \ e^{-p_1 H(t)} \frac{[p_1 H(t)]^n}{n!} dG(t) \tag{18}$$

$$E(C3) = C_f + C_m * E(N3)$$

$$= C_f + C_m * \frac{1}{q_3} \int_o^{t_p} \sum_{n=0}^{k-1} n \ e^{-p_1 H(t)} \frac{[p_1 H(t)]^n}{n!} dG(t) \tag{19}$$

Probabilities for these cases are q_1, q_2, q_3 respectively, and expected total cost $E(C)$ is computed as follows:

$$E(C) = q_1*E(C1) + q_2*E(C2) + q_3*E(C3) \qquad (20)$$

3.4 Expected long run average cost, $UC(k, t_p)$

In maintenance policy study, a cycle is completed when either a failure replacement or preventive replacement is made. The system probabilistically begins again, and each replacement continues a renewal.

Because the length of a cycle varies, it may be meaningless to compare alternatives based upon total cost for the cycle. Therefore it may be necessary to employ the concept of stochastical average cost, that is expected long run average cost, to the problem. By using the Renewal Reward Process [5], expected long run average cost $UC(k, t_p)$ is given by:

$$UC(k, t_p) = \frac{\text{Expected total cost incurred during a cycle}}{\text{Expected length of cycle}}$$

$$= \frac{E[C]}{E[L]} \qquad (21)$$

3.5 System availability, $A(k, t_p)$

Assuming only two states for a system, up or down, system availability $A(k,t)$ will describe the expected amount of time the system is up during a cycle divided by the expected time of a cycle,

$$A(k,t) = \frac{\text{Mean life during a cycle}}{\text{Expected length of a cycle}}$$

We assume downtime for minimal repairs as negligible, so system availability for age t_p, $A(k, t_p)$ is

$$A(k,,t_p) = \frac{M(k, t_p)}{E[L]} \qquad (22)$$

where $M(k, t_p)$ denotes the mean operational time during a cycle (see equation (13))

4. Weibull Example

Let us assume, as is often the case, the system has a Weibull distribution with shape parameter α and scale parameter λ. The probability distribution function $F(t)$ is given by

$$F(t) = 1 - e^{-(\lambda t)^{\alpha}} \qquad \alpha>1,\ \lambda>0,\ t\geq 0 \tag{23}$$

the probability density function f(t) by

$$f(t) = \alpha\ \lambda^{\alpha}\ t^{\alpha-1}\ e^{-(\lambda t)^{\alpha}} \qquad \alpha>1,\ \lambda>0,\ t\geq 0 \tag{24}$$

failure rate $h(t)$, a conditional probability density function, by

$$h(t) = \frac{f(t)}{1-F(t)} = \alpha\ \lambda^{\alpha}\ t^{\alpha-1} \qquad \alpha>1,\ \lambda>0,\ t\geq 0 \tag{25}$$

and the hazard function $H(t)$ by

$$H(t)= \int_0^t h(s)ds = (\lambda t)^{\alpha} \qquad \alpha>1,\ \lambda>0,\ t\geq 0 \tag{26}$$

If the shape parameter α is 1.0, then the failure rate $h(t)$ will be the constant λ, which corresponds to an Exponential distribution with function

$$F(t) = 1 - e^{-(\lambda t)} \qquad \alpha>1,\ \lambda>0,\ t\geq 0 \tag{27}$$

It means that, by virtue of its memoryless property, preventive replacement is unnecessary as the new system will be equally as good and bad as the old one.

There are two types of failure, (Type 1, Type 2) as previously shown, with probabilities of p_1 and p_2, respectively.

The failure rate of Type 1 failure is $p_1h(t)$ at age t,
and the failure rate of Type 2 failure is $p_2h(t)$ at age t.

From (25) and (26) above, the probability functions $B(t)$, $b(t)$, $G(t)$, $g(t)$ for $\alpha>1$, $\lambda>0$, $t\geq 0$, and the probabilities for cases q_1, q_2, q_3 are

$$B(t) = 1 - \sum_{n=0}^{k-1} e^{-p_1 H(t)} \frac{[p_1 H(t)]^n}{n!}$$

$$= 1 - e^{-p_1(\lambda t)^{\alpha}} \sum_{n=0}^{k-1} \frac{[p_1(\lambda t)^{\alpha}]^n}{n!} \tag{28}$$

$$b(t) = p_1 h(t)\, e^{-p_1 H(t)} \frac{[p_1 H(t)]^{k-1}}{(k-1)!}$$

$$= p_1 \alpha \lambda^{\alpha} t^{\alpha-1} e^{-p_1 (\lambda t)^{\alpha}} \frac{[p_1 (\lambda t)^{\alpha}]^{k-1}}{(k-1)!} \tag{29}$$

$$G(t) = 1 - e^{-p_2 H(t)}$$

$$= 1 - e^{-p_2 (\lambda t)^{\alpha}} \tag{30}$$

$$g(t) = p_2 h(t)\, e^{-p_2 H(t)}$$

$$= p_2 \alpha \lambda^{\alpha} t^{\alpha-1} e^{-p_2 (\lambda t)^{\alpha}} \tag{31}$$

$$q_1 = p_1^k \sum_{n=k}^{\infty} e^{-H(t_p)} \frac{H(t_p)^n}{n!}$$

$$= p_1^k e^{-(\lambda t_p)^{\alpha}} \sum_{n=k}^{\infty} \frac{(\lambda t_p)^{\alpha n}}{n!} \tag{32}$$

$$q_2 = e^{-p_2 H(t_p)} \sum_{n=0}^{k-1} e^{-p_1 H(t_p)} \frac{[p_1 H(t_p)]^n}{n!}$$

$$= e^{-(\lambda t_p)^{\alpha}} \sum_{n=0}^{k-1} \frac{[p_1 (\lambda t_p)^{\alpha}]^n}{n!} \tag{33}$$

$$q_3 = \int_0^{t_p} \sum_{n=0}^{k-1} e^{-p_1 H(t)} \frac{[p_1 H(t)]^n}{n!}\, dG(t)$$

$$= \int_0^{t_p} \sum_{n=0}^{k-1} \frac{[p_1 (\lambda t)^{\alpha}]^n}{n!}\, p_2 \alpha \lambda^{\alpha} t^{\alpha-1} e^{-(\lambda t)^{\alpha}} dt \tag{34}$$

$$E(C) = q_1 * E(C1) + q_2 * E(C2) + q_3 * E(C3)$$

$$= q_1 * [\, C_p + C_m * (k-1)\,]$$

$$+ q_2 * [\, C_p + C_m * \frac{1}{q_2} e^{-p_2 (\lambda t_p)^{\alpha}} \sum_{n=0}^{k-1} n\, e^{-p_1 (\lambda t_p)^{\alpha}} \frac{[p_1 (\lambda t_p)^{\alpha}]^n}{n!}\,]$$

$$+ q_3 * [\, C_f + C_m * \frac{1}{q_3} \int_0^{t_p} \sum_{n=0}^{k-1} n\, e^{-p_1 (\lambda t)^{\alpha}} \frac{[p_1 (\lambda t)^{\alpha}]^n}{n!} * p_2 h(t)\, e^{-p_2 (\lambda t)^{\alpha}} dt\,]$$

$$= q_1 * [\ C_p + C_m * (k-1)\ ]$$

$$+ q_2 * [\ C_p + C_m * \frac{1}{q_2} e^{-(\lambda t_p)^\alpha} \sum_{n=0}^{k-1} n \frac{[p_1 (\lambda t_p)^\alpha]^n}{n!}\ ]$$

$$+ q_3 * [\ C_f + C_m * \frac{1}{q_3} \int_0^{t_p} \sum_{n=0}^{k-1} n \frac{[p_1 (\lambda t)^\alpha]^n}{n!}$$

$$* p_2 \alpha \lambda^\alpha t^{\alpha-1} e^{-(\lambda t)^\alpha} dt \tag{35}$$

$$E[L] = t_p * \sum_{n=0}^{k-1} e^{-p_1 (\lambda t_p)^\alpha} \frac{[p_1 (\lambda t_p)^\alpha]^n}{n!} e^{-p_2 (\lambda t_p)^\alpha}$$

$$+ \int_0^{t_p} t \sum_{n=0}^{k-1} e^{-p_1 (\lambda t)^\alpha} \frac{[p_1 (\lambda t)^\alpha]^n}{n!} p_2 h(t) e^{-p_2 (\lambda t)^\alpha} dt$$

$$+ \int_0^{t_p} t e^{-p_2 (\lambda t)^\alpha} p_1 h(t) e^{-p_1 (\lambda t)^\alpha} \frac{[p_1 (\lambda t)^\alpha]^{k-1}}{(k-1)!} dt$$

$$+ (q_1+q_2)*R_p + q_3*R_f$$

$$= t_p * \sum_{n=0}^{k-1} \frac{[p_1 (\lambda t_p)^\alpha]^n}{n!} e^{-(\lambda t_p)^\alpha}$$

$$+ \int_0^{t_p} \sum_{n=0}^{k-1} \frac{[p_1 (\lambda t)^\alpha]^n}{n!} p_2 \alpha (\lambda t)^\alpha e^{-(\lambda t)^\alpha} dt$$

$$+ \int_0^{t_p} e^{-(\lambda t)^\alpha} p_1 \alpha (\lambda t)^\alpha \frac{[p_1 (\lambda t)^\alpha]^{k-1}}{(k-1)!} dt$$

$$+ (q_1+q_2)*R_p + q_3*R_f \tag{36}$$

$$M(k, t_p) = t_p * \sum_{n=0}^{k-1} \frac{[p_1 (\lambda t_p)^\alpha]^n}{n!} e^{-(\lambda t_p)^\alpha}$$

$$+ \int_0^{t_p} \sum_{n=0}^{k-1} \frac{[p_1 (\lambda t)^\alpha]^n}{n!} p_2 \alpha (\lambda t)^\alpha e^{-(\lambda t)^\alpha} dt$$

$$+ \int_0^{t_p} e^{-(\lambda t)^\alpha} p_l \alpha (\lambda t)^\alpha \frac{[p_1(\lambda t)^\alpha]^{k-1}}{(k-1)!} dt \qquad (37)$$

In summary, the objective function of our model is

$$\underset{k,\ t_p}{Min}\ UC(k,\ t_p) = \underset{k,\ t_p}{Min} \frac{E[C]}{E[L]} \qquad (38)$$

And the constraint of our model is given by

$$A(k,\ t_p) \geq \xi \rightarrow \frac{M(k,\ t_p)}{E[L]} \geq \xi \qquad (39)$$

5. The Optimization Procedure

Due to the complex form of the equations, we could not prove any properties(such as convexity or unimodality) for the cost function which could be useful in the search procedure. However, our computational experience suggests that for a fixed value of k, the cost function $UC(k, t_p)$ is unimodal with respect to t_p. Furthermore, if we let $t_p^*(k)$ denote the optimal value of t_p for a given k, our computational experience suggests that $UC(k, t_p^*(k))$ is also unimodal with respect to k. These properties observed from the numerical experiments can be exploited effectively in the search procedure to find the optimal policy. To illustrate these properties, we provide the following example.

In the example we assume that the time to failure follows a Weibull distribution with a shape parameter α and a scale parameter λ. For a Weibull distribution, a failure rate function and a cumulative hazard function are given by $h(t) = \alpha\lambda^\alpha t^{\alpha-1}$, $H(t) = (\lambda t)^\alpha$ respectively. We assume that $\alpha > 1$ because a preventive maintenance does not have to be performed if the failure rate function is non-increasing. In the example we set $\alpha = 3.0$, $\lambda = 1/1{,}350$. Other parameter values are given by $p_l = 0.8$,

p_2=0.2, R_f=32, R_p=16, C_f=37,500, C_p=25,000, C_m=1,000, ξ=0.98.

Figure 1 shows the behavior of the cost function, $UC(k, t_p)$, for a given value of k as k varies from 1 to 8. As shown in the figure, the cost function, $UC(k, t_p)$, is unimodal with respect to t_p for a fixed value of k. To show the behavior of the cost function, $UC(k, t_p^*(k))$, we provide the policy comparisons for each value of k in Table 1. In this table, the values of $A(k, t_p^*(k))$ as well as those of $UC(k, t_p^*(k))$ are provided. From the table, we see that $UC(k, t_p^*(k))$ is unimodal with respect to k and the optimal policy occurs when k is 5 and t_p is 2,255. When the optimal policy is used, the expected cost incurred per unit time is 18.682 and the availability of the system is 0.9863.

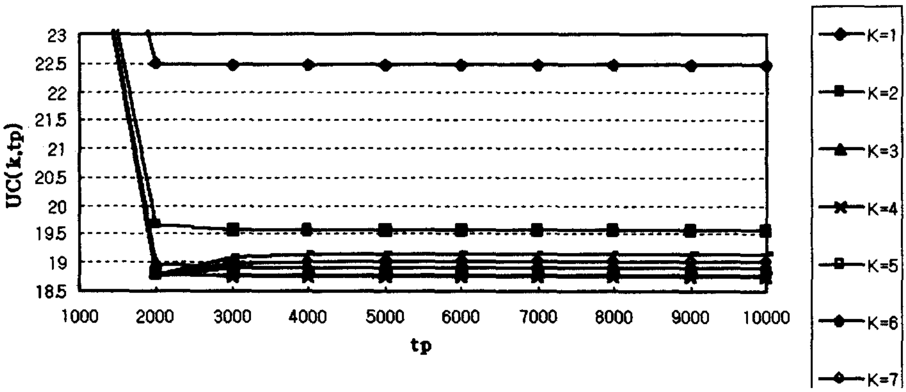

Figure 1. Behavior of the cost function $UC(k, t_p)$

Table 1. Values of $t_p^*(k)$, $UC(k, t_p^*(k))$ and $A(k, t_p^*(k))$ for each value of k

k	1	2	3	4	5	6	7	8
$t_p^*(k)$	2754	2499	2383	2308	**2255**	2219	2197	2186
$UC(k, t_p^*(k))$	22.454	19.562	18.881	18.707	**18.682**	18.691	18.701	18.712
$A(k, t_p^*(k))$	0.9843	0.9860	0.9862	0.9863	**0.9863**	0.9863	0.9863	0.9863

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[99년 3월 2일 접수, 99년 9월 13일 최종수정]