ENVIRONMENTAL SCIENCES 1999 Vol. 3, No. 2, 81~89
Bulletin of the Korean Environmental Sciences Society

Estimating Correlation Dimensions of Land-Sea Breeze Phenomenon

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(Manuscript received on 9 September 1999)

This study estimates the correlation dimensions of the land-sea breeze phenomenon, that has a clear diurnal cycle, in order to gain a more detailed understanding of this phenomenon.

The data adopted include north-south wind velocity component(v) and temperature(T) time series that were observed at Kimhae Airport and Inje University over a period of 5 days, from the 4th to the 8th of August, 1994. The embedding phase space of the time series were reconstructed from 2 to 14 dimensions, and the correlation dimensions of the attractors were then estimated.

The results show that the land-sea breeze phenomenon exhibits a deterministic chaos with non-integer correlation dimension values between 2 and 3. Accordingly, 3 is the minimum number of independent variables required to model the dynamics of the landsea breeze phenomenon in the Kimhae area. Since the saturated embedding dimension, when the correlation dimension remains unchanged, is larger for the wind velocity v-component than for temperature, this indicates that wind velocity is susceptible to topology.

Key words : attractor, correlation dimension, land-sea breeze phenomenon, deterministic chaos

1. Introduction

There are many kinds of flow that exist in nature, and the analyses of these flows have important significance for meteorology, oceanography, engineering, and so on. In meteorology, research activities related to the accurate analysis and understanding of the dynamics of atmospheric flows are extremely important for improving prediction models. Since atmospheric flows are originated from the turbulent behavior of fluids and the flows exhibit a strange attractor when they are spanned in phase spaces, they are recognized as very complicated and statistically nonperiodic flows(D. Rulle and F. Takens[1], 1971). However, if these flows can be considered as the result of deterministic nonlinear dynamic systems that depend on only a few degree of freedom, then it is possible to accurately predict their future state using their past. As a result, deterministic models can be used to predict the future of such flows, and the results used to characterize, monitor, or control them(E. Ott *et al*[2], 1994).

When a complex natural phenomenon, such as a fluid flow, is analyzed, it is important to establish whether it originates from a low-dimensional chaos. Methods for making this determination include measurements of the attractor dimension(B. B. Mandelbrot[3], 1977; Grassberger, P. and I. Procaccia(a)[4], (b)[5], 1983) and Lyapunov exponents(G. Benettin *et al*[6], 1976).

This study applied the technique developed by Fraedrich[7](1985), which can estimate the correlation dimensions of long-time atmospheric flows, to the land-sea breeze phenomenon. Since the land -sea breeze phenomenon has a distinct diurnal cycle, a short time series (about 5 days) was considered adequate for estimating the dimensions.

2. Basic concepts

Generally, Navier-Stokes equations are used as the governing equations for specific fluid flows. To simulate the motion, the fluid velocity is expanded into a Fourier series with regard to three spatial coordinates. It is assumed that the Fourier series converges and the Fourier coefficients have a finite set, then the resulting set of ordinary differential equations,

$$dx_j/dt=f_j(x_1,\cdots,x_n),\ j=1,\cdots n, \tag{1}$$

defines the time development of n expansion coefficients $x_j(t)$, where t is a n time independent variable, and f_j is the function relation of the continuous first-order partial differentiable equations. In this system, the phase space containing the time evolution of the underlying physical process is spanned by the n-dimensional Euclidean phase space which is constructed by the coordinates of variables $x_1, \cdots, x_n$. The phase portraits of the time evolution of the system are shown by attractors which trap the trajectories in the n-dimensional phase space after the transients of the initial conditions disappear. As a result, topological information, such as the attractor dimensions, can be drawn from the system attractors.

To deduce such measurements from observed turbulent or chaotic flows, the embedding theory can be used, which is a reconstruction of the phase space portraits of an attractor through the addition of further independent coordinates of a single state variable until the successive derivatives have no more information(Packard *et al.*[8], 1980). Thereafter, phase portraits of the dynamic system can be constructed in the new m-dimensional phase space spanned by such additional coordinates. The phase space attractors of observed turbulent or chaotic flows can be regarded as m-dimensional embedded phase space attractors that are reconstructed by the single state variable $x(t)$ and its m-1 derivatives,

$$\vec{X}(t) = [x(t), x'(t), \cdots, x^{(m-1)}(t)] \tag{2}$$

Accordingly, instead of the continuous variable $x(t)$ and its derivatives, $x^{(m-1)}(t)$, a discrete time series $x(t)$, and its shifts by (m-1) time lags, (m-1) τ, can be utilized to identify the structures in the time evolution of a single state variable (Ruelle[9], 1981),

$$\vec{X}(t)=f x(t), x(t+\tau), \cdots, x[t+(m-1)\tau]. \tag{3}$$

Although the embedding theory is very useful, the embedding phase space can only be properly constructed when the time shift coordinates are linearly independent. Generally, this can be achieved by choosing the delay time, τ, as when the autocorrelation function first passes through zero.

In this study, the delay time, τ, was chosen from the time series of the observed data, and the embedding phase portraits were then reconstructed relative to this delay time τ.

The dimension of the embedding phase portrait is calculated by using the correlation dimension,

$$d = \ln C(l)/(\ln l/l_0), \tag{4}$$

which is the slope of the log-log plot where $C(l)$ is the correlation integral, l is a prescribed threshold distance, and l_0 is an arbitrary distance(Grassberger, P. and I. Procaccia (a)[4], (b)[5], 1983). Therefore, if the embedding dimensions of the phase portraits or time evolutions from one to m are added, then the distribution of $C(l)$ in successively higher dimensional phase spaces,

$$C_m(l) \sim l^{d(m)}, (d_\infty = d(m) = d(m+1) = \cdots), \tag{5}$$

will increase until it is saturated at certain values, at which point the correlation dimension $d(m)$ of the attractor can be determined. However, it is unavoidable that random noise will destroy part of an attractor, particularly, if the attractor is constructed by observed data. This problem can be solved by estimating the dimensions at a length scale, l, which is larger than the noise (Ben-Mizrachi *et al*[10], 1984).

3. Fractal dimensions of weather variables when land-sea breeze occurs

3.1 Data

The method for estimating the correlation dimensions was applied to a time series of weather variables, that is, the v-component of wind velocity and temperature observed at 10-minute intervals at Kimhae Airport and Inje University using an AWS(Automatic Weather System). The period of observation was from the 4th to the 8th of August, 1994, during the occurrence of land-sea breezes. Figure 1 shows the observation stations. In this figure, Inje University is located in an urban area with very complicated topography, while Kimhae Airport is in a flatland area in the suburbs. The evaluating flow chart proposed by Jeon[11](1993), which determined the sea-breeze days, was used to construct the time series used in this study.

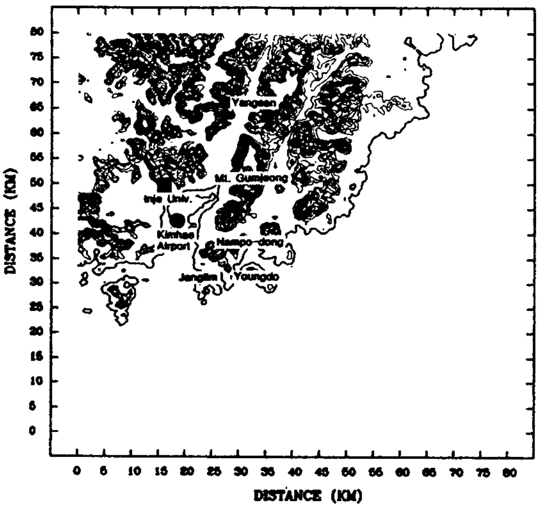


Fig. 1. Observation points used in this study. (● : Kimhae Airport, ■ : Inje University)

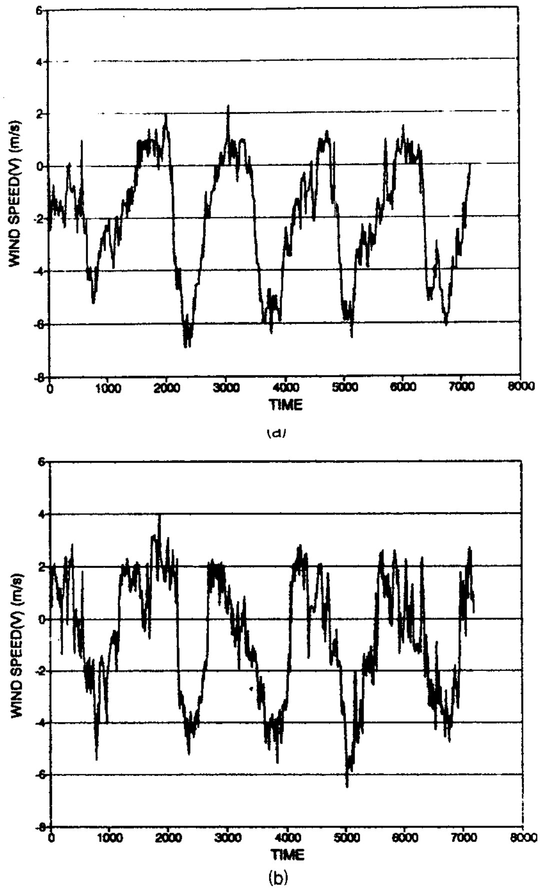


Fig. 2. Time series of wind velocity v-component when land-sea breeze occurred. (a) Kimhae Airport, (b) Inje University

3.2 Estimation of dimensions of observed data attractors

3.2.1 Dimension values of v-component of wind velocity

The time series of the wind velocity v-component observed at the two stations are depicted in Fig. 2. This figure shows that although the periodic wave was very dominant, small perturbations were also detected. A comparison of the two time series indicates that the topographic effect is more important at Inje University.

The time when the autocorrelation first passes through zero, is regarded as the delay time for extracting the random noise before the correlation dimension is estimated from the time series. The autocorrelation of the v-component is shown in Fig. 3. In this figure, the first zero autocorrelation appeared at Kimhae Airport at about five hours and forty minutes and at five thirty at Inje University. Accordingly, the embedding phase space was spanned from $m = 2$ to 14 in relation to these delay times. 2- and 3-dimensional perspectives of the phase space trajectories for all the observed data in relation to the wind velocity v-component are shown in Figs. 4 and 5, respectively. The space fillings in the phase spaces of these figures ensure that every embedding coordinate axis is independent and that the delay time is properly selected. When the embedding dimension m increases from 2 to 14, the correlation dimensions can be estimated from the $\log_2 C(l)$-$\log_2(l/l_0)$ plot(Fig. 6). Figures 7 and 8 show that the dimensionalities of every wind speed v-component obser

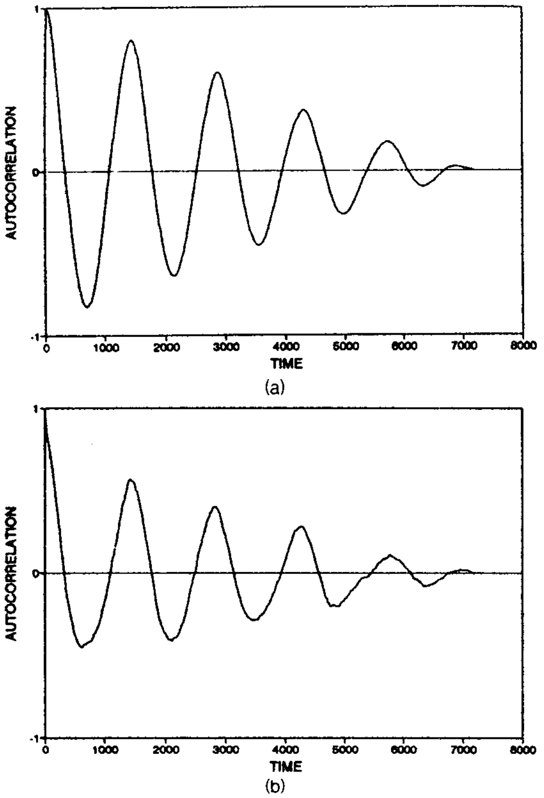


Fig. 3. Autocorrelation functions for wind velocity v-component in the two regions.

ved at the two stations converged to a noninteger value between 2 and 3. More precisely, the correlation dimension of the v-component was 2.57 ± 0.001 at Kimhae Airport and 2.84 ± 0.002 at Inje University, which implies that there are lower fractal dimensionalities with larger attractor dimensions at Inje University. It can also be seen that the saturation of the dimensionality was achieved at a higher dimension at Kimhae Airport(m= 12) than at Inje University(m = 8).

From the above results, several physical interpretations can be suggested about the land-sea breeze phenomenon at Kimhae Airport and Inje University. According to Fraerich's illustration, the fractal dimension value indicates the deterministic chaos of the land-sea breeze phenomenon along with its predicted sensitivity to the initial conditions. The integer part of the saturation dimension d_∞ defines the number of fundamental

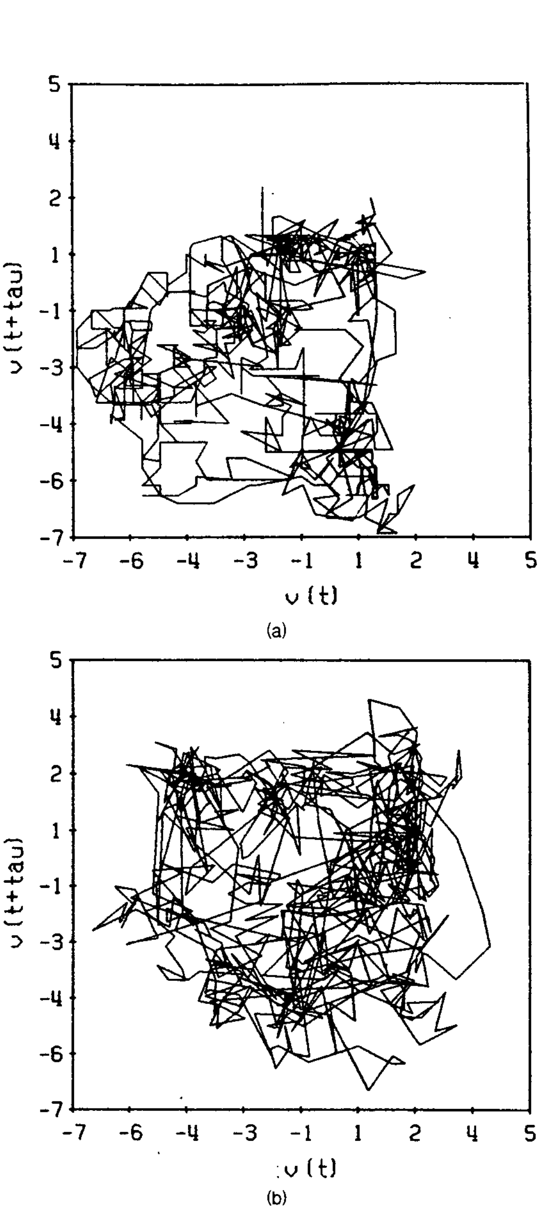


Fig. 4. Two-dimensional embedding phase portraits of wind velocity v-component at (a) Kimhae Airport (b) Inje University when uncorrelation occurred.

periods in the phenomenon, and the next integer above the noninteger dimension(d_∞ < 3) provides the minimum variables necessary to model the dynamic system. The saturated embedding dimension m < 13 from both stations was a adequate upper boundary for the number of variables required to model the dynamics of the attractor.

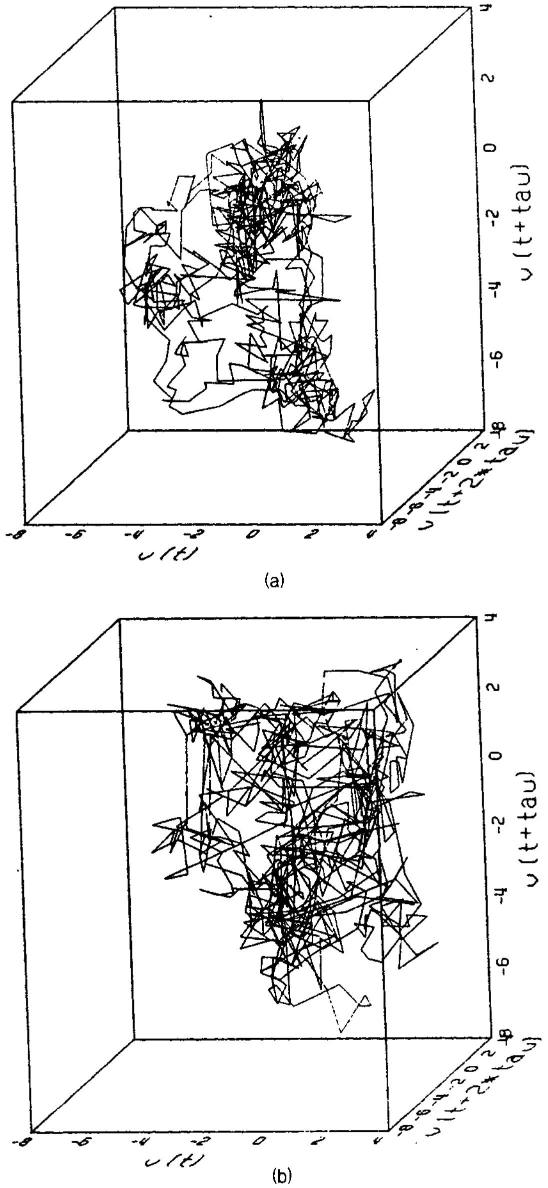


Fig. 5. Three-dimensional embedding phase portraits of wind velocity v-component at (a) Kimhae Airport (b) Inje University when uncorrelation occurred.

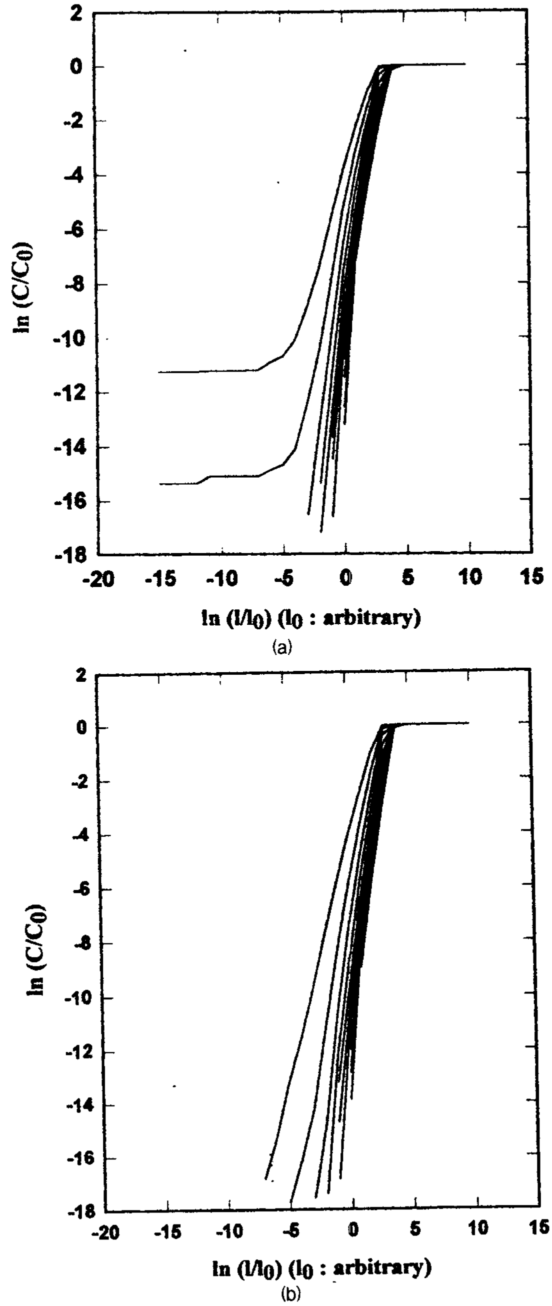


Fig. 6. Correlation integral of wind velocity v-component that evolved in 2 ~ 14 dimensional phase space at (a) Kimhae Airport, (b) Inje University when uncorrelation occurred.

3.2.2 Dimension values of temperature

The same methods applied to the wind velocity v-component were also applied to temperature, T. Figure 8 shows the temperature time series. As above, the diurnal difference of temperature was more affected by topology at Inje University than at Kimhae Airport. The autocorrelations of the time series are shown in Fig. 9. Since the first zero value times both appeared around five forty, this time was selected as the delay time, τ, for the embedding phase portraits from 2 to 14. 2-and 3-dimensional phase trajectories are shown in Figs. 10. and 11, respectively. As with the v-component,

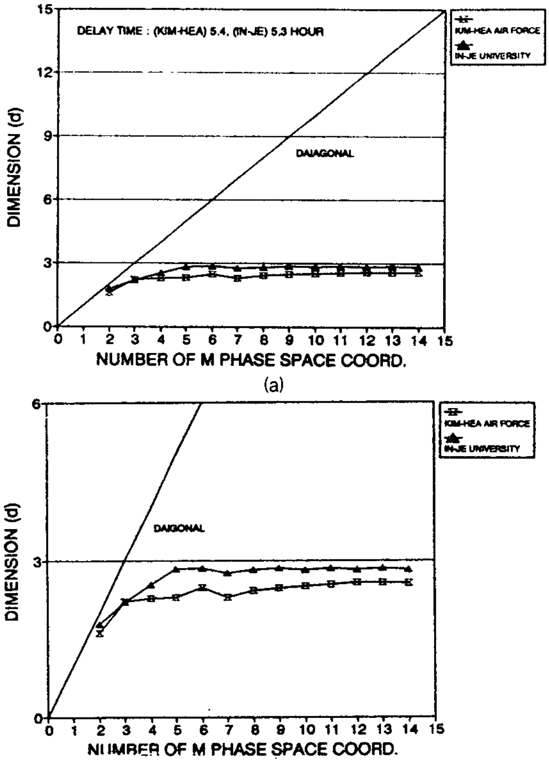
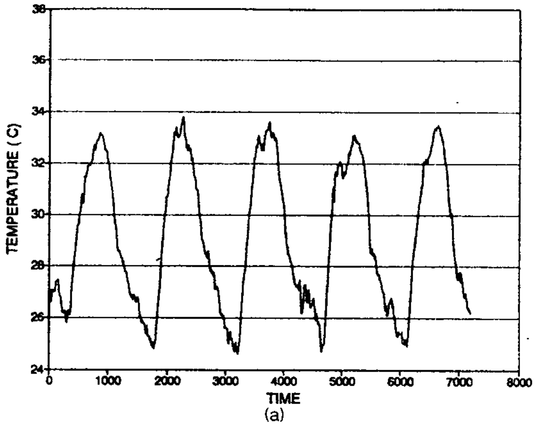


Fig. 7. (a) Correlation dimensions of wind velocity v-component at two stations as embedding evolved between 2 ~ 14 when uncorrelation occurred.
(b) Close-up of correlation dimension sections in (a)



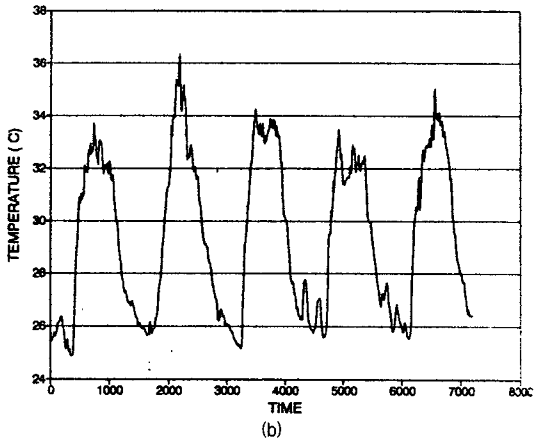


Fig. 8. Time series of temperature at (a) Kimhae Airport (b) Inje University when land-sea breeze occurred.

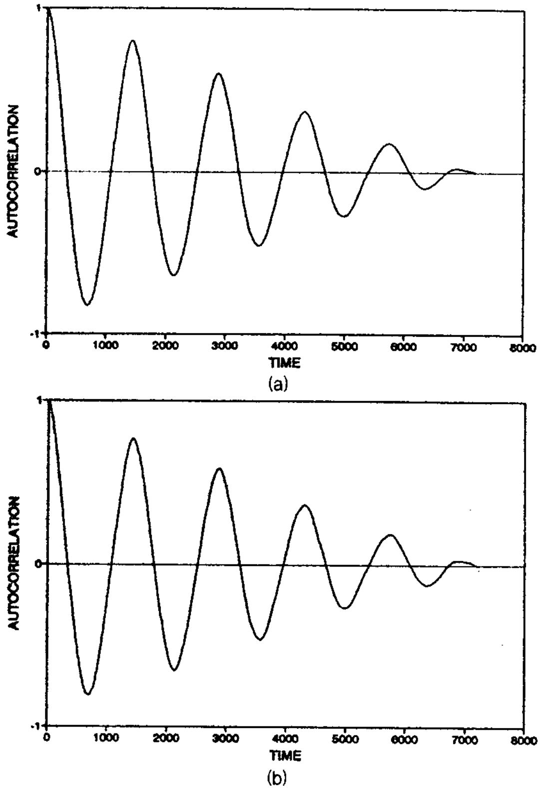


Fig. 9. Autocorrelation functions for temperature in the two regions.

phase space fillings also occurred at the delay time of five forty, therefore, the delay time was confirmed as appropriate for constructing the embedding phase space for the temperatures of the

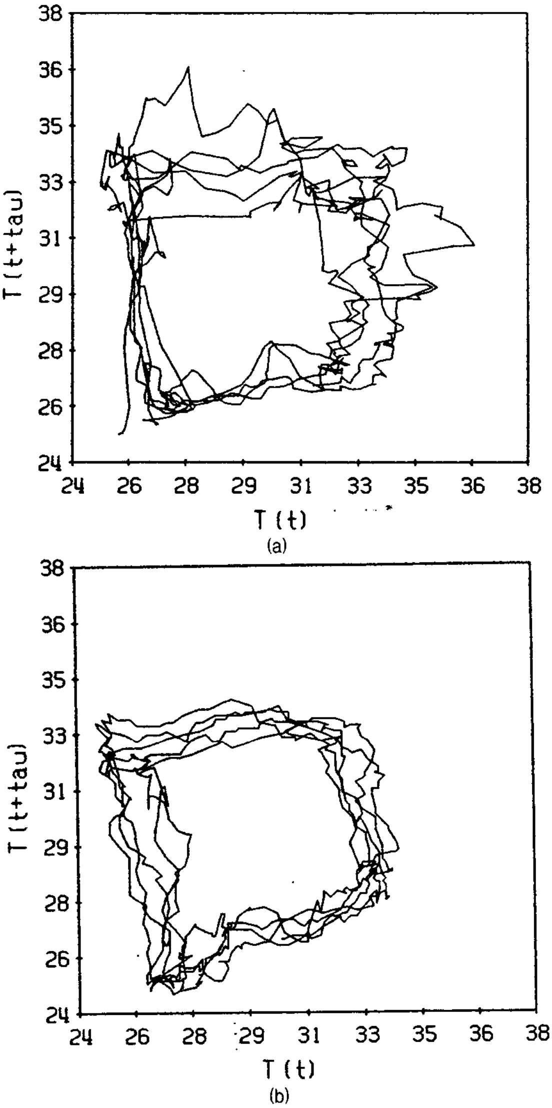


Fig. 10. Two-dimensional embedding phase portraits of temperature at (a) Kimhae Airport (b) Inje University when uncorrelation occurred.

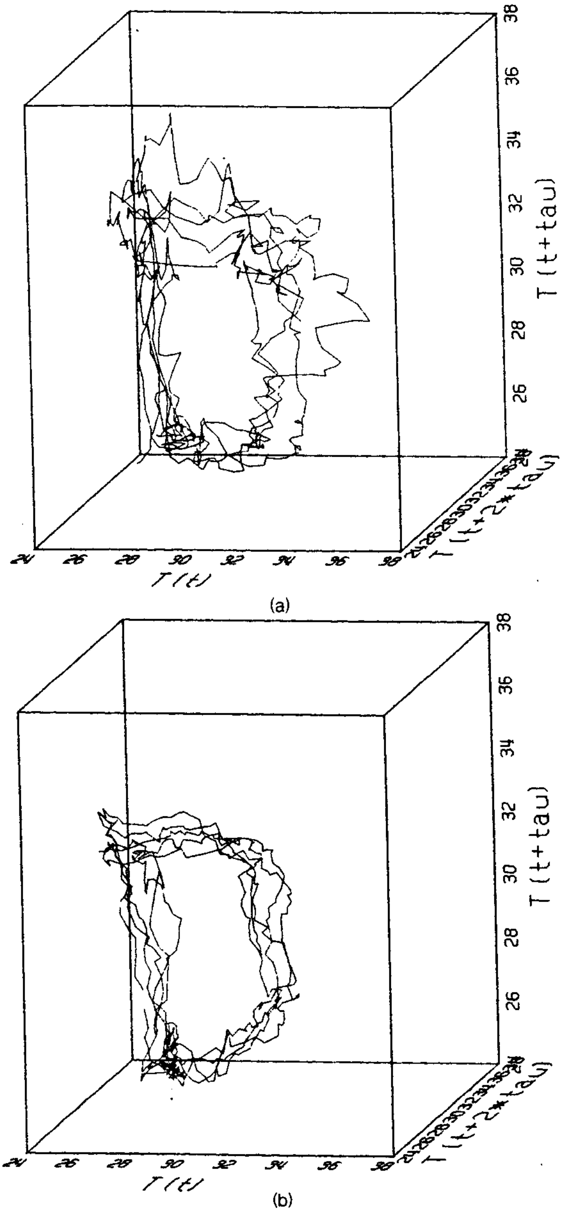
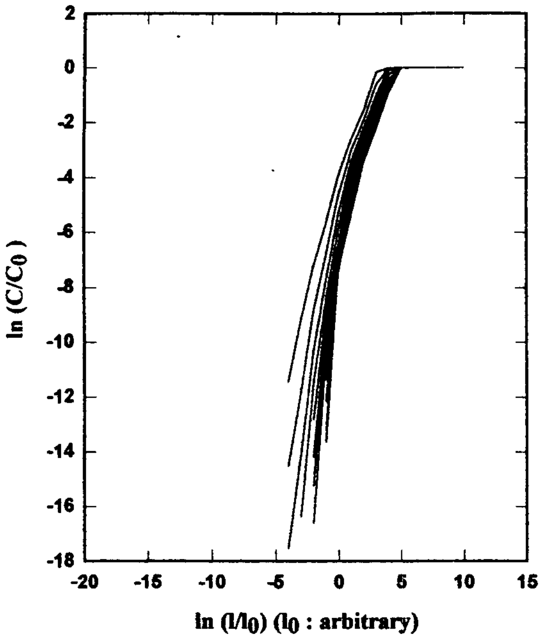
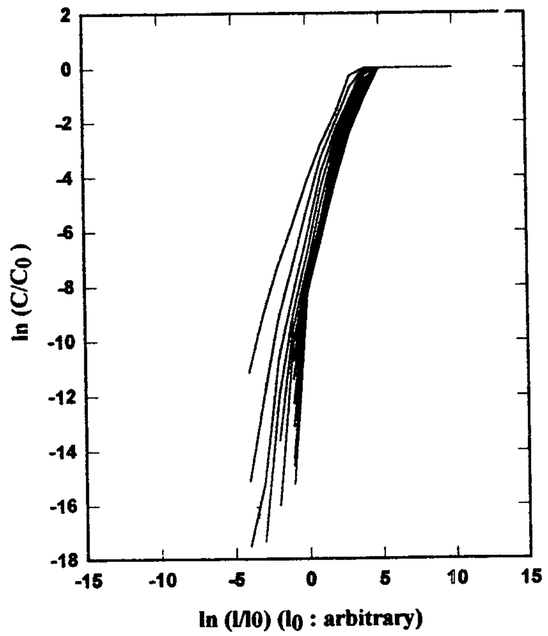


Fig. 11. Three-dimensional embedding phase portraits of temperature at (a) Kimhae Airport (b) Inje University when uncorrelation occurred.

two stations. The $\log_2 C(l)$-$\log_2(l/l_0)$ plots for the embedding phase portraits form 2 to 14 are presented in Fig. 12. As with the v-component, the correlation dimensions were estimated above the noise level dimensions were estimated above the noise level that produced a sharp slope on the plot. The resulting correlation dimension alities for the temperatures are shown in Fig. 13. From the above results, the correlation dimensions for temperature also exhibited noninteger values, that were fractal dimensions, between 2 and 3. More precisely, the estimated correlation dimension at Kimhae Airport was 2.16 ± 0.001 and 2.28 ± 0.001 at Inje University. Accordingly, the latter was slightly larger than the former and the saturated



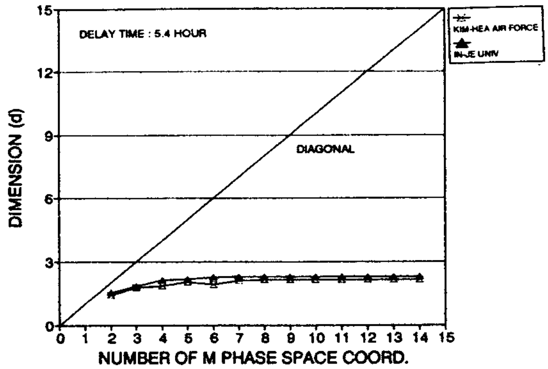
(a)



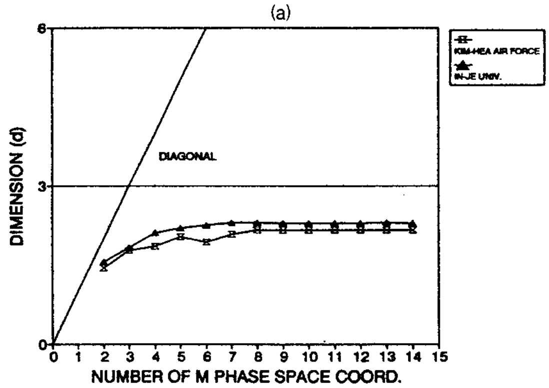
(b)

Fig. 12. Correlation integrals of temperature that evolved in 2 ~ 14 dimensional phase space at (a) Kimhae Airport, (b) Inje University when uncorrelation occurred.

embedding dimension m was below 10 for both stations.



(a)



(b)

Fig. 13. (a) Correlation dimensions of temperature at the two stations as embedding evolved between 2 ~ 14 when uncorrelation occurred.
(b) Close-up of correlation dimension sections in (a)

4. Conclusion

The correlation dimensions of two meteor ological variables, wind velocity v-components and temperatures, observed at two stations in Kimhae were estimated during the occurrence of the land -sea breeze phenomenon. The principal findings related to the correlation dimensions of these two meteorological variables were as follows:

(1) The correlation dimensions estimated from the time series of the observation data set exhibited low fractal dimensions, i.e., non-integer dimensions between 2 and 3. Due to the difference of the topology, the correlation dimensions were larger at Inje University than at Kimhae Airport. Based on these facts, it is suggested that 3 essential independent variables are needed to model the

land-sea breeze phenomenon at Kimhae and 12 variables are sufficient to model the dynamics of the attractor.

(2) The saturated embedding dimensions, when the correlation dimensions remain unchanged, were larger for the wind velocity v-component than for temperature. Accordingly, since wind velocity is more susceptible to topology, more variables are required to model the attractor for wind velocity than that for temperature.

(3) The land-sea breeze phenomenon can be shown by low-dimensional chaotic behavior yet can not be explained by a periodicity which has an integer dimension.

In further studies, a time series with a shorter time step and longer data length will be adopted to estimate correlation dimensions for a more accurate analysis of the land-sea breeze phenomenon in the Kimhae area. In particular, the connection between this work and previous studies may provide a more detailed description of the dynamic mechanism of the land-sea breeze phenomenon in this area. Although the application of this kind of method to atmospheric phenomena still have many problems, the proposed methods can still provide further insight and understanding of the physics involved in the land-sea breeze phenomenon.

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