Block Toeplitz Matrix Inversion using Levinson Polynomials

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ABSTRACT

In this paper, we propose detection methods for gradual scene changes such as .dissolve, pan, and zoom. The proposal method to detect a dissolve region uses scene features based on spatial statistics of the image. The spatial statistics to define shot boundaries are derived from squared means within each local area. We also propose a method of the camera motion detection using four representative motion vectors in the background. Representative motion vectors are derived from macroblock motion vectors which are directly extracted from MPEG streams. To reduce the implementation time, we use DC sequences rather than fully decoded MPEG video. In addition, to detect the gradual scene change region precisely, we use all types of the MPEG frames(I, P, B frame). Simulation results show that the proposed detection methods perform better than existing methods.

I. Introduction

In this paper we consider the problem of inverting positive definite hermitian block Toeplitz matrices. These matrices occur in a wide variety of scenarios such as time-series analysis, multichannel maximum entropy spectrum estimation, and multiuser detection have been developed for their inversion [1-9]. Our objective in this paper is to derive the desired inverse formulas in terms of the associated matrix Levinson polynomial coefficients in an elementary manner. The Levinson polynomials can be iteratively computed from the given data without involving any inversion, and this makes the whole solution very attractive from a computational viewpoint. Similar formulas have been originally obtained by Gohberg and Semencul for a general invertible Toeplitz matrix in a purely algebraic format [5,6], and in the present approach the positive definite case is examined from a spectral analysis viewpoint thereby exhibiting the interrelationship between positivity of Toeplitz matrices, the strictly bounded nature of the associated reflection coefficient matrices, and the minimum phase[1] character of the matrix Levinson polynomials.

II. Positive definite Block Hermitian Toplitz Matrix Inverses

Block hermitian Toeplitz matrices such as[2]

$$\mathbf{T}_n \equiv \begin{bmatrix} \mathbf{r}_0 & \mathbf{r}_1 & \cdots & \mathbf{r}_n \\ \mathbf{r}_1^* & \mathbf{r}_0 & \cdots & \mathbf{r}_{n-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{r}_n^* & \mathbf{r}_{n-1}^* & \cdots & \mathbf{r}_0 \end{bmatrix} = \mathbf{T}_n^* > 0, \quad n = 0 \to \infty \tag{1}$$

occur in multichannel situations where several inputs interact simultaneously to generate several outputs. Here, the matrices, $\mathbf{r}_k$, $k=0\to n$ are of size $m\times m$ and they can be interpreted as the first $(n+1)$ autocorrelation matrices of a jointly wide sense stationary stochastic vector $\mathbf{x}(nT)\equiv[x_1(nT), x_2(nT), \cdots, x_m(nT),]^T$ with power spectral density matrix

$$\mathrm{S}(\theta) = \sum_{k=-\infty}^{\infty} \mathbf{r}_k e^{jk\theta} \geq 0 \tag{2}$$

The nonnegativity property of the power spectral density matrix $\mathrm{S}(\theta)$ in (2) is equivalent to the nonnegativity of every block Toeplitz matrix $\mathbf{T}_k$ as in (1) for $k=0\to\infty$ [10]. Further, the positivity of $\mathbf{T}_k$'s follows from the finite

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entropy condition [11].

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \ln det\, \mathbf{S}(\theta)d\theta > -\infty \tag{3}$$

In this context, the objective is to obtain the inverse of $\mathbf{T}_n$ in a fast and efficient manner.

Toward this, consider four matrix polynomials $\mathbf{A}_n(z)$, $\mathbf{B}_n(z)$, $\mathbf{C}_n(z)$ and $\mathbf{D}_n(z)$ that satisfy the recursions

$$\mathbf{E}_k^* \mathbf{A}_k(z) = \mathbf{A}_{k-1}(z) - z\, \mathbf{S}_k\, \tilde{\mathbf{C}}_{k-1}(z) \tag{4}$$

$$\mathbf{E}_k^* \mathbf{B}_k(z) = \mathbf{B}_{k-1}(z) + z\, \mathbf{S}_k\, \tilde{\mathbf{D}}_{k-1}(z) \tag{5}$$

$$\mathbf{C}_k(z)\, \mathbf{F}_k = \mathbf{C}_{k-1}(z) - z\, \tilde{\mathbf{A}}_{k-1}(z)\, \mathbf{S}_k \tag{6}$$

and

$$\mathbf{D}_k(z)\, \mathbf{F}_k = \mathbf{D}_{k-1}(z) - z\, \tilde{\mathbf{B}}_{k-1}(z)\, \mathbf{S}_k \tag{7}$$

starting with

$$\begin{aligned} \mathbf{A}_0(z) &= \mathbf{C}_0(z) = \mathbf{r}_0^{-1/2} \\ \mathbf{B}_0(z) &= \mathbf{D}_0(z) = \mathbf{r}_0^{1/2}/2 \end{aligned} \tag{8}$$

Here $\mathbf{S}_k$, $k=1\to n$ are a sequence of free matrix parameters that are strictly bounded by unity, i.e.,

$$\mathbf{I} - \mathbf{S}_k \mathbf{S}_k^* > 0, \quad k=1\to n \tag{9}$$

and further, the matrices $\mathbf{E}_k$ and $\mathbf{F}_k$ in (4)-(7) satisfy the matrix factorizations

$$\begin{aligned} \mathbf{E}_k^* \mathbf{E}_k &= \mathbf{I} - \mathbf{S}_k \mathbf{S}_k^* \\ \mathbf{F}_k^* \mathbf{F}_k &= \mathbf{I} - \mathbf{S}_k^* \mathbf{S}_k \end{aligned} \tag{10}$$

For uniqueness, these matrix factors $\mathbf{E}_k$ and $\mathbf{F}_k$ may be chosen to be lower triangular with positive diagonal elements. Further,

$$\begin{aligned} \tilde{\mathbf{A}}_k(z) &\equiv z^k\, \mathbf{A}_{k*}(z) \\ &\equiv z^k\, \mathbf{A}_k^*(1/z^*), \quad k\geq 1 \end{aligned} \tag{11}$$

represents the matrix polynomial reciprocal to $\mathbf{A}_k(z)$. The bounded character of $\mathbf{S}_k$'s together with (8), guarantee the above polynomials to be minimum phase. A direct induction argument using (4)-(10) also shows that the polynomials defined above are interrelated through the nested relations [9]

$$\mathbf{A}_n(z)\, \mathbf{B}_{n*}(z) + \mathbf{B}_n(z)\, \mathbf{A}_{n*}(z) = \mathbf{I} \tag{12}$$

$$\mathbf{C}_{n*}(z)\, \mathbf{D}_n(z) + \mathbf{D}_{n*}(z)\, \mathbf{C}_n(z) = \mathbf{I} \tag{13}$$

$$\mathbf{B}_n(z)\, \mathbf{C}_n(z) - \mathbf{A}_n(z)\, \mathbf{D}_n(z) = 0 \tag{14}$$

and

$$\mathbf{A}_{n*}(z)\, \mathbf{A}_n(z) - \mathbf{C}_n(z)\, \mathbf{C}_{n*}(z) = 0 \tag{15}$$

We can make use of the freedom present in the $\mathbf{S}_k$'s in (4)-(7) to relate these polynomials to the given $\mathbf{r}_k$, $k=0\to n$. Toward this, let

$$\mathbf{A}(z) = \mathbf{A}_0 + \mathbf{A}_1 z + \cdots + \mathbf{A}_n z^n \tag{16}$$

$$\mathbf{B}(z) = \mathbf{B}_0 + \mathbf{B}_1 z + \cdots + \mathbf{B}_n z^n \tag{17}$$

$$\mathbf{C}(z) = \mathbf{C}_0 + \mathbf{C}_1 z + \cdots + \mathbf{C}_n z^n \tag{18}$$

$$\mathbf{D}(z) = \mathbf{D}_0 + \mathbf{D}_1 z + \cdots + \mathbf{D}_n z^n \tag{19}$$

and suppose $\mathbf{S}_k$, $k=1\to n$ are chosen so as to satisfy

$$\begin{aligned} 2\,\mathbf{A}_n^{-1}(z)\, \mathbf{B}_n(z) &= 2\,\mathbf{D}_n(z)\, \mathbf{C}_n^{-1}(z) \\ &= \mathbf{r}_0 + 2\sum_{k=1}^{n} \mathbf{r}_k e^{jk\theta} + O(z^{n+1}) \end{aligned} \tag{20}$$

This gives

[1] A matrix function is said to be analytic in $|z|<1$ if all it entries are analytic in $|z|<1$. If, in addition, its determinant is also nonsingular in $|z|<1$, then it is said to be minimum-phase

[2] In this paper, lower case regular letters denote scalars, lower case bold type letters denote vectors, and upper case bold type letters denote matrices. Thus a, $\boldsymbol{a}$ and $\mathbf{A}$ denote scalar, vector and matrix in that order. Further $\mathbf{A}^T$ represents the transpose of $\mathbf{A}$, $\mathbf{A}^*$ denotes the complex conjugate transposeof $\mathbf{A}$, and $det\,\mathbf{A}$ is the determinant of $\mathbf{A}$.

$$\begin{aligned} &\mathbf{A}_n^{-1}(z)\ \mathbf{B}_n(z)+(\mathbf{A}_n^{-1}(z)\ \mathbf{B}_n(z))_* \\ &= \mathbf{D}_n(z)\ \mathbf{C}_n^{-1}(z)+(\mathbf{D}_n(z)\ \mathbf{C}_n^{-1}(z))_* \\ &= \mathbf{r}_0+\sum_{k=1}^{n}(\mathbf{r}_k z^k+\mathbf{r}_k^* z^{-k})+O(z^{\pm(n+1)}) \end{aligned}$$

and using (12)-(13), the above equation simplifies to

$$\begin{aligned} &\Big(\mathbf{r}_0+\sum_{k=1}^{n}(\mathbf{r}_k z^k+\mathbf{r}_k^* z^{-k}) \\ &+O(z^{\pm(n+1)})\Big)\ \tilde{\mathbf{A}}_n(z)=z^n\ \mathbf{A}_n^{-1}(z) \end{aligned} \qquad (21)$$

$$\begin{aligned} &\tilde{\mathbf{C}}_n(z)\Big(\mathbf{r}_0+\sum_{k=1}^{n}(\mathbf{r}_k z^k+\mathbf{r}_k^* z^{-k}) \\ &+O(z^{\pm(n+1)})\Big)=z^n\ \mathbf{C}_n^{-1}(z) \end{aligned} \qquad (22)$$

Comparing coefficients of z^k, $k=0\rightarrow n$ on both sides of (21)-(22), we obtain

$$\begin{aligned} &[\ \mathbf{A}_0,\ \mathbf{A}_1,\ \cdots,\ \mathbf{A}_n]\ \mathbf{T}_n \\ &=[\ \mathbf{A}_0^{*-1},\ 0,\ \cdots,\ 0] \end{aligned} \qquad (23)$$

and

$$\begin{aligned} &[\ \mathbf{C}_n^*,\ \mathbf{C}_{n-1}^*,\ \cdots,\ \mathbf{C}_0^*]\ \mathbf{T}_n \\ &=[\ 0,\ 0,\ \cdots,\ \mathbf{C}_0^{-1}] \end{aligned} \qquad (24)$$

Direct substitution of (4), (6) into (23)-(24) shows that $\mathbf{A}_n(z)$ and $\mathbf{C}_n(z)$ satisfy the recursions in (4)-(10), provided $\mathbf{S}_k$'s are chosen to be [9]

$$\begin{aligned} \mathbf{S}_k&=\Big\{\mathbf{A}_{k-1}(z)\Big(\sum_{i=1}^{k}\mathbf{r}_i z^i\Big)\Big\}\ \mathbf{C}_{k-1}(0) \\ &= \mathbf{A}_{k-1}(0)\Big\{\Big(\sum_{i=1}^{k}\mathbf{r}_i z^i\Big)\ \mathbf{C}_{k-1}(z)\Big\} \end{aligned} \qquad (25)$$

The boundedness of $\mathbf{S}_k$'s follows from the positivity of the $\mathbf{T}_k$'s, and hence they represent matrix reflection coefficient. With $\mathbf{S}_k$'s so defined, (4)-(7) represent the standard forward and backward matrix Levinson polynomials of the first and second kind respectively.

To make further progress, returning back to (20), we get

$$\begin{aligned} &2\ \mathbf{B}_n^*(z) \\ &=\Big(\mathbf{r}_0+2\sum_{k=1}^{n}\mathbf{r}_k^* z^{*k}+O(z^{*n+1})\Big)\ \mathbf{A}_n^*(z) \end{aligned}$$

$$\begin{aligned} &2\ \mathbf{D}_n^*(z) \\ &= \mathbf{C}_n^*(z)\Big(\mathbf{r}_0+2\sum_{k=1}^{n}\mathbf{r}_k^* z^{*k}+O(z^{*n+1})\Big) \end{aligned}$$

and comparing coefficients of like powers on both sides and rearranging them, we obtain

$$\mathbf{R}\,\mathbf{M}_0=2\,\mathbf{M}_1 \text{ and } \mathbf{N}_0\,\mathbf{R}=2\,\mathbf{N}_1 \qquad (26)$$

where

$$\mathbf{M}_0\cong\begin{bmatrix} \mathbf{A}_0^* & 0 & \cdots & 0 \\ \mathbf{A}_1^* & \mathbf{A}_0^* & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_n^* & \mathbf{A}_{n-1}^* & \cdots & \mathbf{A}_0^* \end{bmatrix} \quad \mathbf{M}_1\cong\begin{bmatrix} \mathbf{B}_0^* & 0 & \cdots & 0 \\ \mathbf{B}_1^* & \mathbf{B}_0^* & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_n^* & \mathbf{B}_{n-1}^* & \cdots & \mathbf{B}_0^* \end{bmatrix} \qquad (27)$$

$$\mathbf{N}_0\cong\begin{bmatrix} \mathbf{C}_0^* & 0 & \cdots & 0 \\ \mathbf{C}_1^* & \mathbf{C}_0^* & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_n^* & \mathbf{C}_{n-1}^* & \cdots & \mathbf{C}_0^* \end{bmatrix} \quad \mathbf{N}_1\cong\begin{bmatrix} \mathbf{D}_0^* & 0 & \cdots & 0 \\ \mathbf{D}_1^* & \mathbf{D}_0^* & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{D}_n^* & \mathbf{D}_{n-1}^* & \cdots & \mathbf{D}_0^* \end{bmatrix} \qquad (28)$$

and

$$\mathbf{R}\cong\begin{bmatrix} \mathbf{r}_0 & 0 & \cdots & 0 \\ 2\mathbf{r}_0^* & 2\mathbf{r}_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 2\mathbf{r}_n^* & 2\mathbf{r}_{n-1}^* & \cdots & \mathbf{r}_0 \end{bmatrix}$$

so that

$$\begin{aligned} \mathbf{T}_n&=\frac{\mathbf{R}+\mathbf{R}^*}{2} \\ &= \mathbf{M}_1\ \mathbf{M}_0^{-1}+\mathbf{M}_0^{*-1}\ \mathbf{M}_1^* \end{aligned} \qquad (29)$$

as well as

$$\mathbf{T}_n=\mathbf{N}_1^*\ \mathbf{N}_0^{-1*}+\mathbf{N}_0^{-1}\ \mathbf{N}_1 \qquad (30)$$

similarly, from (12), we obtain

$$\begin{aligned} &\begin{bmatrix} \mathbf{M}_0^* & \mathbf{M}_n \\ 0 & \mathbf{M}_0^* \end{bmatrix}\begin{bmatrix} \mathbf{M}_2^* & \mathbf{M}_1 \\ 0 & \mathbf{M}_2^* \end{bmatrix} \\ &+\begin{bmatrix} \mathbf{M}_1^* & \mathbf{M}_2 \\ 0 & \mathbf{M}_1^* \end{bmatrix}\begin{bmatrix} \mathbf{M}_n^* & \mathbf{M}_0 \\ 0 & \mathbf{M}_n^* \end{bmatrix} \\ &=\begin{bmatrix} 0 & \mathbf{I}_{n+1} \\ 0 & 0 \end{bmatrix} \end{aligned} \qquad (31)$$

where

$$\mathbf{M}_n \cong \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ \mathbf{A}_n & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \mathbf{A}_1 & \mathbf{A}_2 & \cdots & \mathbf{A}_n & 0 \end{bmatrix} \quad \mathbf{M}_2 \cong \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ \mathbf{B}_n & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \mathbf{B}_1 & \mathbf{B}_2 & \cdots & \mathbf{B}_n & 0 \end{bmatrix} \qquad (32)$$

Expanding and comparing terms, we get

$$\begin{aligned} &\mathbf{M}_0^* \mathbf{M}_1 + \mathbf{M}_n \mathbf{M}_2^* + \mathbf{M}_1^* \mathbf{M}_0 + \mathbf{M}_2 \mathbf{M}_n^* = \mathbf{I}_{n+1} \\ &\mathbf{M}_0^* \mathbf{M}_2^* + \mathbf{M}_1^* \mathbf{M}_n^* = 0 \end{aligned} \qquad (33)$$

so that

$$\mathbf{M}_2 = -\mathbf{M}_n \mathbf{M}_1 \mathbf{M}_0^{-1} \qquad (34)$$

Substituting (34) into (33) gives

$$\mathbf{M}_0^* \mathbf{M}_1 + \mathbf{M}_1^* \mathbf{M}_0 - \mathbf{M}_n (\mathbf{M}_0^{-1} \mathbf{M}_1^* + \mathbf{M}_1 \mathbf{M}_0^{-1}) \mathbf{M}_n^* = \mathbf{I}_{n+1}$$

and hence

$$\mathbf{M}_0^* (\mathbf{M}_1 \mathbf{M}_0^{-1} + \mathbf{M}_0^{*-1} \mathbf{M}_1^*) \mathbf{M}_0 - \mathbf{M}_n (\mathbf{M}_1 \mathbf{M}_0^{-1} + \mathbf{M}_0^{*-1} \mathbf{M}_1^*) \mathbf{M}_n^* = \mathbf{I}_{n+1}. \qquad (35)$$

Using (29), the above expression simplifies to

$$\mathbf{M}_0^* \mathbf{T}_n \mathbf{M}_0 - \mathbf{M}_n \mathbf{T}_n \mathbf{M}_n^* = \mathbf{I}_{n+1} \qquad (36)$$

Similarly, from (13), we obtain

$$\begin{bmatrix} \mathbf{N}_n^* & \mathbf{N}_0 \\ 0 & \mathbf{N}_n^* \end{bmatrix} \begin{bmatrix} \mathbf{N}_1^* & \mathbf{N}_2 \\ 0 & \mathbf{N}_1^* \end{bmatrix} + \begin{bmatrix} \mathbf{N}_2^* & \mathbf{N}_1 \\ 0 & \mathbf{N}_2^* \end{bmatrix} \begin{bmatrix} \mathbf{N}_0^* & \mathbf{N}_n \\ 0 & \mathbf{N}_0^* \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{I}_{n+1} \\ 0 & 0 \end{bmatrix} \qquad (37)$$

where

$$\mathbf{N}_n \cong \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ \mathbf{C}_n & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{C}_1 & \mathbf{C}_2 & \cdots & \mathbf{C}_n & 0 \end{bmatrix} \quad \mathbf{N}_2 \cong \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ \mathbf{D}_n & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{D}_1 & \mathbf{D}_2 & \cdots & \mathbf{D}_n & 0 \end{bmatrix} \qquad (38)$$

Expanding (37), we obtain

$$\begin{aligned} &\mathbf{N}_n^* \mathbf{N}_2 + \mathbf{N}_0 \mathbf{N}_1^* + \mathbf{N}_2^* \mathbf{N}_n + \mathbf{N}_1 \mathbf{N}_0^* = \mathbf{I}_{n+1} \\ &\mathbf{N}_n^* \mathbf{N}_1^* + \mathbf{N}_2^* \mathbf{N}_0^* = 0 \end{aligned} \qquad (39)$$

so that

$$\mathbf{N}_2 = -\mathbf{N}_0^{-1} \mathbf{N}_1 \mathbf{N}_n \qquad (40)$$

A direct substitution of (40) into (39) gives

$$\mathbf{N}_0 \mathbf{N}_1^* + \mathbf{N}_1 \mathbf{N}_0^* - \mathbf{N}_n^* (\mathbf{N}_0^{-1} \mathbf{N}_1 + \mathbf{N}_1^* \mathbf{N}_0^{*-1}) \mathbf{N}_n = \mathbf{I}_{n+1} \qquad (41)$$

As before, using (30), this gives the compact form

$$\mathbf{N}_0 \mathbf{T}_n \mathbf{N}_0^* - \mathbf{N}_n^* \mathbf{T}_n \mathbf{N}_n = \mathbf{I}_{n+1} \qquad (42)$$

Finally, from (14), we get

$$\begin{bmatrix} \mathbf{M}_1^* & \mathbf{M}_2 \\ 0 & \mathbf{M}_1^* \end{bmatrix} \begin{bmatrix} \mathbf{N}_0^* & \mathbf{N}_n \\ 0 & \mathbf{N}_0^* \end{bmatrix} + \begin{bmatrix} \mathbf{M}_0^* & \mathbf{M}_n \\ 0 & \mathbf{M}_0^* \end{bmatrix} \begin{bmatrix} \mathbf{N}_1^* & \mathbf{N}_2 \\ 0 & \mathbf{N}_1^* \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (43)$$

and this gives

$$\mathbf{M}_1^* \mathbf{N}_n + \mathbf{M}_2 \mathbf{N}_0^* - \mathbf{M}_0^* \mathbf{N}_2 - \mathbf{M}_n \mathbf{N}_1^* = 0 \qquad (44)$$

And

$$\mathbf{M}_1 \mathbf{M}_0^{-1} = \mathbf{N}_0^{-1} \mathbf{N}_1 \text{ or } \mathbf{M}_0^{*-1} \mathbf{M}_1^* = \mathbf{N}_1^* \mathbf{N}_0^{*-1} \qquad (45)$$

To proceed further, Substituting (34) and (40) into (44) gives

$$\mathbf{M}_1^* \mathbf{N}_n - \mathbf{M}_n \mathbf{M}_1 \mathbf{M}_0^{-1} \mathbf{N}_0^* + \mathbf{M}_0^* \mathbf{N}_0^{-1} \mathbf{N}_1 \mathbf{N}_n - \mathbf{M}_n \mathbf{N}_1^* = 0$$

or,

$$(\mathbf{M}_1^* - \mathbf{M}_0^* \mathbf{N}_0^{-1} \mathbf{N}_1) \mathbf{N}_n - \mathbf{M}_n (\mathbf{M}_1 \mathbf{M}_0^{-1} \mathbf{N}_0^* + \mathbf{N}_1^*) = 0$$

or,

$$\mathbf{M}_0^* (\mathbf{M}_0^{*-1} \mathbf{M}_1^* + \mathbf{N}_0^{-1} \mathbf{N}_1) \mathbf{N}_n - \mathbf{M}_n (\mathbf{M}_1 \mathbf{M}_0^{-1} + \mathbf{N}_1^* \mathbf{N}_0^{*-1}) \mathbf{N}_0^* = 0 \qquad (46)$$

Moreover, by making use of (45), the above equation can be rewritten as

$$\begin{aligned} &\mathbf{M}_0^*(\mathbf{M}_0^{*-1}\mathbf{M}_1^* + \mathbf{M}_1\mathbf{M}_0^{-1})\mathbf{N}_n - \\ &\mathbf{M}_n(\mathbf{M}_1\mathbf{M}_0^{-1} + \mathbf{M}_0^{*-1}\mathbf{M}_1^*)\mathbf{N}_0^* \\ &= 0 \end{aligned} \tag{47}$$

and together with (29), this gives

$$\mathbf{M}_0^*\mathbf{T}_n\mathbf{N}_n - \mathbf{M}_n\mathbf{T}_n\mathbf{N}_0^* = 0 \tag{48}$$

As a result,

$$\mathbf{M}_n\mathbf{T}_n = \mathbf{M}_0^*\mathbf{T}_n\mathbf{N}_n\mathbf{N}_0^{*-1} \tag{49}$$

and

$$\mathbf{T}_n\mathbf{N}_n = \mathbf{M}_0^{*-1}\mathbf{M}_n\mathbf{T}_n\mathbf{N}_0^* \tag{50}$$

Finally, substituting (49) into (36), we obtain

$$\mathbf{M}_0^*\mathbf{T}_n(\mathbf{M}_0 - \mathbf{N}_n\mathbf{N}_0^{-1}\mathbf{M}_n^*) = \mathbf{I}_{n+1}$$

Thus

$$\begin{aligned} &\mathbf{T}_n(\mathbf{M}_0 - \mathbf{N}_n\mathbf{N}_0^{*-1}\mathbf{M}_n^*) = \mathbf{M}_0^{*-1} \text{ or} \\ &\mathbf{T}_n^{-1} = \mathbf{M}_0\mathbf{M}_0^* - \mathbf{N}_n\mathbf{N}_0^{*-1} \end{aligned} \tag{51}$$

Alternatively, substituting (50) into (42), we get

$$(\mathbf{N}_0 - \mathbf{N}_n^*\mathbf{M}_0^{*-1}\mathbf{M}_n)\mathbf{T}_n\mathbf{N}_0^* = \mathbf{I}_{n+1}$$

Hence

$$\begin{aligned} &(\mathbf{N}_0 - \mathbf{N}_n^*\mathbf{M}_0^{*-1}\mathbf{M}_n)\mathbf{T}_n = \mathbf{N}_0^{*-1} \text{ or} \\ &\mathbf{T}_n^{-1} = \mathbf{N}_0^*\mathbf{N}_0 - \mathbf{N}_n\mathbf{N}_0^{*-1}\mathbf{M}_n^*\mathbf{M}_0^* \end{aligned} \tag{52}$$

Finally, the remaining equation (15) can be used to further simplify (51)-(52). Rewriting (15), we get

$$\begin{aligned} &\begin{bmatrix} \mathbf{M}_n^* & \mathbf{M}_0 \\ 0 & \mathbf{M}_n^* \end{bmatrix} \begin{bmatrix} \mathbf{M}_0^* & \mathbf{M}_n \\ 0 & \mathbf{M}_0^* \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{N}_0^* & \mathbf{N}_n \\ 0 & \mathbf{N}_0^* \end{bmatrix} \begin{bmatrix} \mathbf{N}_n^* & \mathbf{N}_0 \\ 0 & \mathbf{N}_n^* \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned} \tag{53}$$

and this gives

$$\mathbf{M}_n^*\mathbf{M}_0^* - \mathbf{N}_0^*\mathbf{N}_n^* = 0.$$

Thus

$$\begin{aligned} &\mathbf{N}_n^* = \mathbf{N}_0^{*-1}\mathbf{M}_n^*\mathbf{M}_0^* \text{ or} \\ &\mathbf{M}_n^* = \mathbf{N}_0^*\mathbf{N}_n^*\mathbf{M}_0^{*-1} \end{aligned} \tag{54}$$

and using this in (51)-(52), we obtain

$$\begin{aligned} \mathbf{T}_n^{-1} &= \mathbf{M}_0\mathbf{M}_0^* - \mathbf{N}_n\mathbf{N}_n^* \\ &= \mathbf{N}_0^*\mathbf{N}_0 - \mathbf{M}_n^*\mathbf{M}_n \end{aligned} \tag{55}$$

the desired formula for the inverse of a positive definite Hermitian block Toeplitz matrix. To summarize, the inverse of the positive hermitian block Toeplitz matrix generated from, $\mathbf{r}_0, \mathbf{r}_1, \cdots, \mathbf{r}_n$, involves only the coefficients of the forward and backward matrix Levinson polynomials of the first kind, that can be computed recursively from (4),(6),(8),(10) and (25). In the single channel case, from (4)-(10) since, $\mathbf{C}_n(z) = \mathbf{A}_n(z)$, in particular, we have $\mathbf{N}_0 = \mathbf{M}_0$, $\mathbf{N}_n = \mathbf{M}_n$ and (55) reduces to one representation for $\mathbf{T}_n^{-1}$. Finally, if the given block Toeplitz matrix $\mathbf{T}_n$ is nonsingular, but not positive, the $\mathbf{S}_k$'s in (25) will not be bounded by unity (see(9)), and hence $\mathbf{E}_k$'s and $\mathbf{F}_k$'s are undefined in (10). Moreover, the matrix polynomials in (4)-(7) will not be minimum phase; nevertheless it is possible to derive formulas similar to that in (55) for the inversion of general block Toeplitz matrices [5,6].

Ⅲ. Conclusions

Inversion formulas for positive definite hermitian block Toplitz matrices are expressed here in terms of the coefficients of the associated forward and backward matrix Levinson Polynomials of the first kind. When the block Toeplitz matrix is positive definite, these matrix polynomials are minimum phase, and further they can be recursively computed from the given block matrix entries without involving inversions of any kind.

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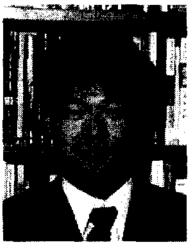

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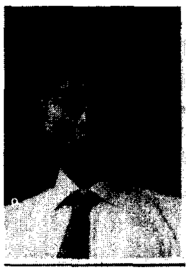

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