협대역 무선채널에서 최적의 다이버시티 수신알고리즘 연구

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Optimal Decoding Algorithm with Diversity Reception for a Fading Channel

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요 약

본 논문에서는 무선 협대역 채널에서 다이버시티 수신 알고리즘을 제안하였다. 수신 알고리즘은 통계학분야에서 Maximum-Likelihood Sequence Estimation의 근사 추정치를 계산하는데 활용되는 Expectation-Maximization(EM) 알고리즘을 기본으로 유도하였다. 알고리즘의 특성은 파일럿 심볼을 이용하여 반복적으로 블록 디코딩을 수행하며 시뮬레이션 결과를 기존의 파일럿 심볼을 이용하는 방식(PSI)에 비교하여 매우 우수한 성능을 보였다. 수신 알고리즘의 성능은 컴퓨터 시뮬레이션을 이용하여 검증하였으며 다양한 차수의 다이버시티 수신단과 Trellis Coded Modulation (TCM)을 이용한 시스템에 알고리즘을 적용하였다.

ABSTRACT

In this paper, the problem of decoding transmitted data sequence with diversity reception in the presence of nonselective fading is studied. The expectation maximization (EM) algorithm is employed to derive an interactive algorithm. The algorithm performs block-by-block coherent decoding with the aid of pilot symbols. It is shown that the complexity of the algorithm grows linearly as a function of sequence length. The performance of the algorithm is shown to be better than that of the conventional pilot symbol aided (PSI) algorithm. Simulation results are presented to assess the performance of the algorithm and the results are compared with that of the conventional PSI algorithm.

I. INTRODUCTION

In this paper, an algorithm based on the EM algorithm for coherent decoding with diversity reception is proposed. The algorithm uses periodically inserted pilot symbols to obtain its initial fading estimates and iterates until estimates converge. It will be shown that the number of iteration is mostly two, which makes the algorithm practical. There are other pilot symbol aided algorithms [1, 2]. The PSI algorithm in [1] assumes the land mobile fading channel [3] and uses the pilot symbols only for extracting fading estimates. The proposed algorithm uses the pilot symbols as well as the tentative decoded data symbols for estimation of fading. It is shown that the algorithm effectively suppresses the error floor associated with the PSI algorithm in [1] under limited decoding delay and pilot symbol insertion rate.

The EM algorithm is a two-step iterative

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algorithm for obtaining maximum-likelihood (ML) sequence estimates [4, 5]. The algorithm may provide a practical ML sequence estimator when the optimal ML sequence estimator is not available or when its implementation is difficult. The optimal ML sequence estimator for the fading channel can easily be derived. However, its implementation requires evaluation of a likelihood function for all possible transmitted sequences.

The EM algorithm applied for sequence estimation in the presence of random parameters (random phase, fading) was published in [6], and part of this paper focused on uncoded performance without diversity reception was presented in [7]. In this paper, the results presented in [6.7] are generalized to include diversity reception and the coded performance is assessed with the use of trellis coded modulation (TCM).

Ⅱ. THE CHANNEL MODEL

A sequence of complex-valued symbols $\{s_k\}$ is transmitted over D frequency -nonselective Rayleigh fading channels with diversity reception. After demodulating and sampling the received signal, the baseband data for the k-th symbol interval through the l-th channel is

$$r_k(l) = a_k(l)\, s_k(l) + n_k(l), \quad \text{for } l = 1 \ldots D, \tag{1}$$

where $n_k(l)$ is a complex zero-mean additive white Gaussian noise (AWGN). The complex fading $a_k(l)$ is the piece-wise constant approximation to the l-th channel continuous fading process $a_t(l)$ over the k-th signaling interval. Here, we use the land mobile fading process [3] for describing the fading process $a_t(l)$ and its autocorrelation function can be written as

$$\frac{1}{2} E[\, a_t(l)\, a_{t+\tau}(l)^*] = J_0(2\pi B\tau), \tag{2}$$

where $J_0(\cdot)$ is the zero-order Bessel function, and B is the Doppler spread factor. The transmitted symbols are assumed to be zero mean and unit variance, i.e. $E(s_k)=0$, $E(|s_k|^2)=1$. The variance of the AWGN can be written as $\frac{D}{b\, SNR_b}$, where b is a number of information bits per symbol and SNR_b is the signal to noise ratio per bit. It is also assumed that transmitting channels are identically distributed and independent.

Ⅲ. DECODER BASED ON THE EM ALGORITHM

The standard format to begin an sequence estimation is to express the received signal in (1) for the l-th channel as a vector form

$$\mathrm{r}(l) = \mathrm{Sa}(l) + \mathrm{n}(l) \tag{3}$$

where the matrix S is an N-dimensional diagonal matrix with its (i,i)-th element equal to the transmitted symbol in the i-th symbol interval, and its off-diagonal elements are zero. The AWGN vector n(l) is described with its diagonal covariance matrix of $\frac{D}{b\, SNR_b} \times \mathrm{I}$, where I is N-dimensional unit matrix. The fading vector a(l) is assumed to have zero mean and covariance matrix Q.

Assuming phase shift keying (PSK) signaling, the optimal ML sequence estimate is

$$\begin{aligned} \hat{\mathrm{s}} &= \arg\max_{\mathrm{s}}\, p(\{\mathrm{r}(1), \ldots, \mathrm{r}(D)\} | \mathrm{s}) \\ &= \arg\min_{\mathrm{s}} \sum_{l=1}^{D} \mathrm{r}(l)^{+}\ \mathrm{S}(\tilde{Q})^{-1}\ \mathrm{S}^{*}\mathrm{r}(l) \end{aligned} \tag{4}$$

where $\mathrm{r}(l)^{+}$ is conjugate transpose of $\mathrm{r}(l)$ and $\tilde{Q} = Q + \frac{D}{b\, SNR_b}\mathrm{I}$. In the above equation, the transmitted symbol sequence is expressed in vector form as s and in diagonal matrix form as S. The maximization step of the above decoder requires evaluation of its likelihood function for every possible sequence, which is not practical.

The EM algorithm has been studied in the area of statistics [4,5] and it offers an alternative ML sequence estimation strategy when the optimal estimator cannot be used. The formulation of the

EM algorithm for the proposed algorithm is briefly explained in the following. Define a *complete* data X such that it is many-to-one mapped to the available data $R=\{r(1),\ldots,r(D)\}$. The available data R is referred as the *incomplete* data, and the X is defined as the *incomplete* data R along with the fading $A=\{a(1),\ldots,a(D)\}$. The EM algorithm uses the conditional log-likelihood function of the *complete* data X in a two-step iterate procedure. The i-th step of the procedure can be written as:

1. Expectation-step : Evaluate

$$L(s|s^i) \equiv E[\log p(X|s)|R,\ s^i] \tag{5}$$

2. Maximization-step : Solve for

$$s^{i+1} = \arg\max_s L(s|s^i), \tag{6}$$

where s^i is the *i*-th sequence estimate and $X=\{R,A\}$.

Assuming that transmitting sequences and the fading are statistically independent, the likelihood function $L(s|s^i)$ for the Expectation -step can be written as

$$L(s|s^i) = \sum_{l=1}^{D}\sum_k \left\{ Re[\, r_k(l)^* s_k m_1(k,l)^i] - \frac{1}{2}|s_k|^2 m_2(k,l)^i \right\} \tag{7}$$

where $m_1(k,l)^i$ and $m_2(k,l)^i$ are the first and the second order conditional moments of the fading process. The required moments are expressed as

$$m_2(k,l)^i = E[\,|a_k(l)|^2 | r(l),\ s^i]$$
$$= [Q - Q\,(S)^{i*} H^{-1} S^i Q]_{k,k} + |m_1(k,l)^i|^2$$
$$m_1(k,l)^i = E[\,a_k(l)| r(l),\ s^i]$$
$$= [Q\,(S^i)^* H^{-1} r(l)]_k, \tag{8}$$

where

$$H = \left[S^i Q\,(S^i)^* + \frac{D}{b\,SNR_b} I \right]. \tag{9}$$

For the special case of PSK signaling, the Expectation-step is reduced to evaluating

$$L(s|s^i) = \sum_{l=1}^{D}\sum_k Re[\, r_k(l)^* s_k m_1(k,l)^i]. \tag{10}$$

The likelihood function for the Expectation-step is the sum of the likelihood functions for each channels used for diversity reception. Also, note that the Maximization-step of the algorithm can be implemented as symbol -by-symbol maximization. This implies linear growth of the algorithm complexity as a function of sequence length for a fixed number of iteration.

Ⅳ. IMPLEMENTATION OF THE EM ALGORITHM

The implementation of the EM algorithm requires a scheme for obtaining an initial fading estimates. The initial estimates is obtained using a interpolator along with pilot symbols. Assume that the length of the interpolation windows is

$$N_T = M(L-1)+1, \tag{11}$$

where L is the number of pilot symbols and M is the pilot symbol insertion period in the window. As an example, Figure 1 shows an interpolation window with $M=5, L=3$, and $N_T=11$ where the transmitted pilot symbols are in the shaded slots.

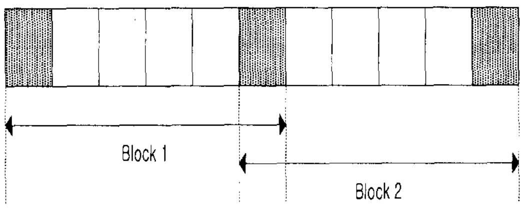

Fig. 1 Interpolation window with M=5, L=3, NT=11 and two Sub-Blocks

The Wiener filter is used for interpolation, and the interpolated fading estimate for the *k*-th symbol interval of the *l*-th channel is

$$\hat{a}_k(l) = \sum_{i=1}^{L} h_{i,k}\, r_{1+M(i-1)}(l)\; s^*_{1+M(i-1)}$$

$$, k=1, \ldots, N_T \tag{12}$$

where $h_{i,k}$ is filter coefficient for i-th channel interpolator.

Note that for PSK signaling assumed, the Wiener-Hopf equation is

$$E\{ a_{k,error}(l)[r_{1+M(i-1)}(l)^* s_{1+M(i-1)}]\}=0$$

$$\text{for } i=1, \ldots, L \tag{13}$$

$$where,\ a_{k,error}= a_k(l) - \hat{a}_k(l)$$

The above equation is solved for $h_{i,k}=[\mathrm{h}_k]_i$ for use in equation (12) to yield

$$\mathrm{h}_k= \left[Q_s+\frac{D}{b\, SNR_b}\mathrm{I}\right]^{-1} \mathrm{q}_k, \tag{14}$$

where

$$[Q_s]_{i,j}=\frac{1}{2}E(a_{1+M(i-1)}(l)\, a_{1+M(j-1)}(l)^*)$$

and

$$[\mathrm{q}_k]_i=\frac{1}{2}E(a_k(l)\, a_{1+M(i-1)}(l)^*).$$

The initial fading estimates for EM algorithm is provided by the interpolator using the pilot symbols.

The optimal implementation of the EM algorithm for a finite length interpolator window can be summarized as follows:

1. Given a received data R and pilot symbols, use equation (12) to obtain the initial fading estimates.
2. Given the fading estimates, use equation (7) for symbol-by-symbol decoding.
3. Given a decoded sequence, evaluate the fading estimates using equation (8).
4. Goto step 2 and iterate until estimates converge.

Note that the algorithm uses the decoded data sequence with the pilot symbols for fading estimates. The decoding part is implemented as symbol-by-symbol maximizati -on which implies a linear growth of the algorithm complexity as a function of the sequence length for a fixed number of iterations.

The optimal implementation of the algorithm requires inversion of a matrix with dimension of N_T for evaluation of the coefficients for the linear estimator in equation (8). This may be too complicated when the length of the interpolation window is large. Thus, a scheme for suboptimal implementation of the EM algorithm is proposed. Given an interpolation window with length of N_T, the window can be divided into a number of sub-block of length N. After interpolation for obtaining the initial fading estimates for the entire window, the EM algorithm with sequence length of N is applied. As an example, figure 1 shows an interpolation window of length $N_T=11$ with the known symbols inserted in the shaded slots. The window can be divided into two sub-blocks with length 6. The interpolator of order L=3 is used to interpolate the window to provide the initial fading estimate. Then the EM algorithm with N=6 can be applied for decoding each sub-block.

V. PERFORMANCE

The performance of the interpolator is a function of the pilot symbol transmission period M and the number of pilot symbols L in the interpolation window. The interpolation process is equivalent to reconstructing a fading process using its sampled values. Thus, the pilot symbol transmission rate $\frac{1}{M}$ has to be larger then the Nyquist rate $2BT$ (neglecting the AWGN). The transmission rate can be expressed as

$$\frac{1}{M}=2\left(1+\frac{\alpha}{100}\right)BT, \qquad \alpha\geq 0 \tag{15}$$

where α represents extra redundancy percentage for the pilot symbol insertion. The average decoding delay for a given values of M and L is

$$\text{Decoding Delay} = \frac{M(L-1)T}{2}. \tag{16}$$

Note that for actual transmission, the value of M need to be minimized for reducing the redundancy rate. Given a value of M, the decoding delay is minimized by choosing a small value of L. The transmission redundancy is taken into account by normalizing the SNR by factor of $\frac{M-1}{M}$ for all our simulated results in the following.

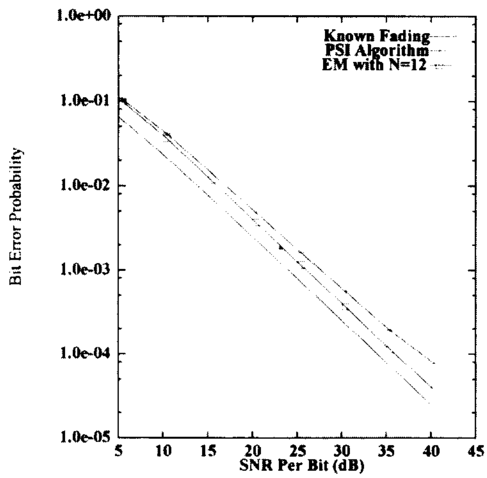

Fig. 2 Bit Error Probability for Uncoded 4-PSK(Land Mobile Fading with BT=0.03, M=11, L=9, α =51%)

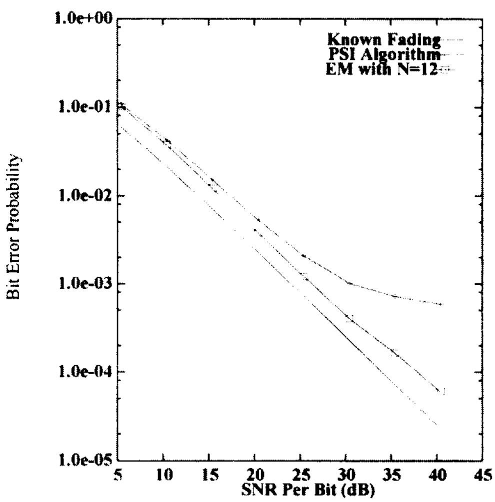

Fig. 3 Bit Error Probability for Uncoded 4-PSK(Land Mobile Fading with BT=0.03, M=11, L=5, α =51%)

The performance the EM algorithm is compared with that of the PSI algorithm as shown in Figures 2 and 3. The suboptimal EM algorithm with N equal to 12 and the PSI algorithm are simulated. The extra redundancy rate α of 51% is used with the insertion number L of 9 and 5 respectively. It is shown that for a small value of L (smaller decoding delay), the performance of the PSI algorithm shows an error floor. The performance of the EM algorithm shows only a small degradation.

The average number of iterations for the EM algorithm simulated in Figure 2 and 3 is shown in Table 1. The result show that mostly two iterations are needed, which suggest that the proposed algorithm converges fast.

Table 1. Average Number of Iterations (Uncoded 4-PSK with N=12)

SNR(dB)	L=9	L=5
10	2.2062	2.1975
20	2.0337	2.0400
30	2.0077	2.0095
40	2.0077	2.0065

Now the parameters for the interpolation window are fixed as $M=5, L=5$ and the EM algorithm is simulated for different values of BT as shown in Figures 4,5. The α values for each Figure are 100% and 43% respectively. The EM algorithm with sequence length N equal to 11 and

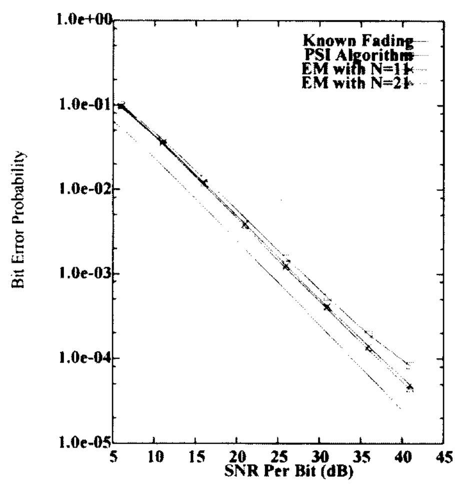

Fig. 4 Bit Error Probability for Uncoded 4-PSK for Various of N(Land Mobile Fading with BT=0.05, M=5, L=5, α =100%)

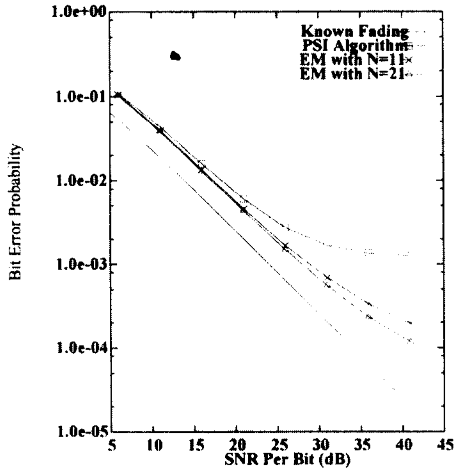

Fig. 5 Bit Error Probability for Uncoded 4-PSK for Various of N(Land Mobile Fading with BT=0.08, M=5, L=5, α =43%)

21 is simulated and its performance is compared to that of the PSI algorithm as well as to that with known fading. The results show that the PSI algorithm degrades rapidly for small α values. However, the EM algorithm with N=11 and N=21 shows marginal performance degradation. For small value of α (sampling of channel close to Nyquist rate), the performance difference between suboptimal PSI algorithm and the optimal EM algorithm gets large.

Double and triple diversity channels are simulated in Figures 6, 7 for various values of

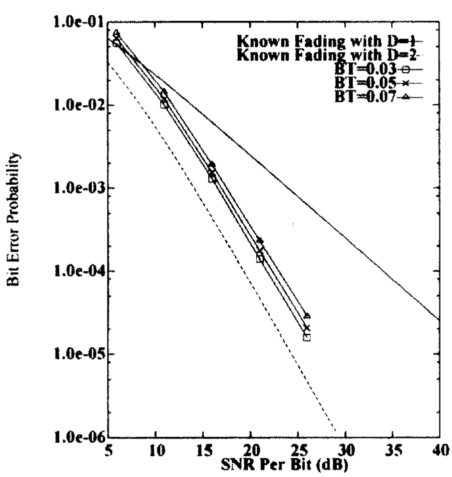

Fig. 6 Bit Error Probability for Uncoded 4-PSK(EM algorithm with N=11, Double Diversity Land Mobile Fading with D=2, M=5, L=5)

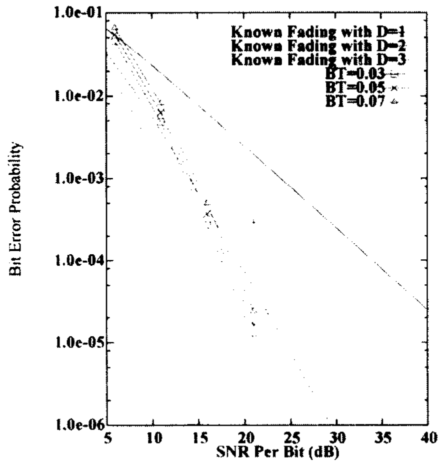

Fig. 7 Bit Error Probability for Uncoded 4-PSK(EM algorithm with N=11, Triple Diversity Land Mobile Fading with D=3, M=5, L=5)

BT. A fixed interpolation window ($M=5, L=5$) is used with suboptimal implementation of the EM algorithm with $N=11$. The results show that the performance degradation compared to that with known fading is 2.5 to 4 dB. The result show that the EM algorithm with moderate complexity can be used for varying BT values, and the performance shows only marginal degradation compared to that of known fading.

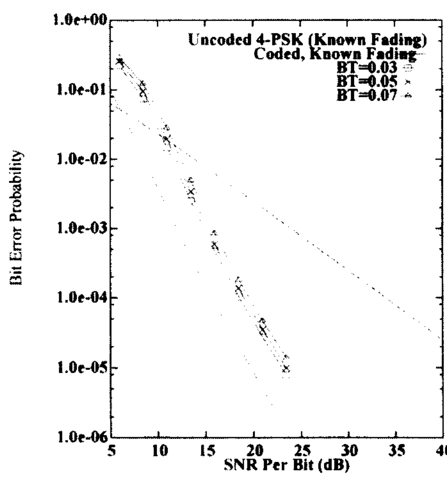

Fig. 8 Bit Error Probability for 8-State, Rate 2/3 Coded 8-PSK(EM algorithm with N=11, Land Mobile Fading with M=5, L=5)

A coded system using an 8-state, rate 2/3 code[8] is simulated with $M=5, L=5$ for the interpolation window, and its performance is shown in Figure 8. The EM algorithm with $N=11$ is used for providing the fading estimates and a perfect interleaver is assumed. The Viterbi algorithm [9] is used for soft decision decoding. The result are similar to that of the diversity channels.

Ⅵ. CONCLUSION

The EM algorithm is shown to provide a practical way to obtain ML sequence estimate in the fading channel. An optimal as well as a suboptimal algorithm is proposed with computer simulation results. It is shown that the algorithm converges mostly within two iterations and its suboptimal implementation with limited complexity performs as well as that of the optimal implementation. It is shown that the performance of the algorithm is not as sensitive as the PSI algorithm to the parameters of th interpolation window, i.e., M, L. This allows designing a decoder with small decoding delay and redundancy for a desired performance. Diversity channel and trellis coded channel are simulated for various values of BT. It is shown that the performance of the algorithm shows minimal loss compared to that the perfect decoder.

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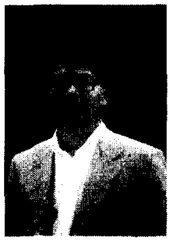

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