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On fuzzy pairwise irresolute mappings

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ABSTRACT

In this paper, we further investigate some proterties of fuzzy pairwise irresolute mappings on fuzzy bitopological spaces.

1. Introduction

Kandil [6] introduced and studied a fuzzy bitopological space as a natural generalization of a fuzzy topological space. Sampath Kumar [10] defined and investigated a (T_i, T_j)-fuzzy semiopen ((T_i, T_j)-fuzzy semiclosed) set, a (T_i, T_j)-semi-interior ((T_i, T_j)-semi-closure) and a fuzzy pairwise semicontinuous mapping on fuzzy bitopological spaces. Recently, Park *et al.*[9] defined a fuzzy pairwise irresolute mapping on fuzzy bitopological spaces, and showed us that every fuzzy pairwise irresolute mapping is a fuzzy pairwise semicontinuous mapping, but the converse is not true in general.

In this paper, we further investigate some properties of fuzzy pairwise irresolute mappings on fuzzy bitopological spaces. We also show that a fuzzy pairwise continuous mapping and a fuzzy pairwise irresolute mapping do not have any specific relations.

2. Preliminaries

A system (X, T_1, T_2) consisting of a set X with two fuzzy topologies T_1 and T_2 on X is called a **fuzzy bitopological space** [*fbts* for shorts] [6]. Throughout this paper, the indices i, j take values in $\{1, 2\}$ with $i \neq j$. For a fuzzy set μ in a *fbts* (X, T_1, T_2), T_i-*fo* set μ and T_i-*fc* set μ mean T_i-fuzzy open set μ and T_i-fuzzy closed set μ respectively. Also, T_i-Intμ and T_i-Clμ mean the interior and closure of μ for the fuzzy topologies T_i and T_j respectively.

Defimition 2.1 [10] Let μ be a fuzzy set of a *fbts* X. Then μ is called;

(i) a (T_i, T_j)**-fuzzy semiopen** [(T_i, T_j)-*fso*] set of X if there exists a T_i-*fo* set ν of X such that $\nu \leq \mu \leq T_j$-Clν,

(ii) a (T_i, T_j)**-fuzzy semiclosed** [(T_i, T_j)-*fsc*] set of X if there exists a T_i-*fc* set ν of X such that T_j-Int$\nu \leq \mu \leq \nu$.

Lemma 2.2 [10] Let μ be a fuzzy set of a *fbts* X. Then the following statements are equivalent:

(i) μ is a (T_i, T_j)-*fsc* set.
(ii) μ^c is a (T_i, T_j)-*fso* set.
(iii) T_j-Int$(T_i$-Cl$\mu) \leq \mu$.
(iv) T_j-Cl$(T_i$-Int$(\mu^c)) \geq \mu^c$.

Theorem 2.3 [10] (i) Any union of (T_i, T_j)-*fso* sets is a (T_i, T_j)-*fso* set.

(ii) Any intersection of (T_i, T_j)-*fsc* sets is a (T_i, T_j)-*fsc* set.

It is clear that every T_i-*fo* (respectively T_i-*fc*) set is a (T_i, T_j)-*fso* (respectively (T_i, T_j)-*fsc*) set, but the converse need not be true. The intersection (respectively union) of any two (T_i, T_j)-*fso* (respectively (T_i, T_j)-*fsc*) sets needs not be a (T_i, T_j)-*fso* (respectively (T_i, T_j)-*fsc*) set. Even the intersection (respectively union) of a (T_i, T_j)-*fso* (respectively (T_i, T_j)-*fsc*) set with a T_i-*fo* (respectively T_i-*fc*) set may fail to be a (T_i, T_j)-*fso* (respectively (T_i, T_j)-*fsc*) set [10].

Theorem 2.4 [10] Let μ and ν be two fuzzy sets of a *fbts* X.

(i) If μ is a (T_i, T_j)-*fso* set and T_i-Int$\mu \leq \nu \leq T_j$-Clμ, then ν is a (T_i, T_j)-*fso* set.

(ii) If μ is a (T_i, T_j)-*fsc* set and T_j-Int$\mu \leq \nu \leq T_i$-Clμ, then ν is a (T_i, T_j)-*fsc* set.

Theorem 2.5 [10] Let μ be a fuzzy set of a *fbts* X. Then μ is a (T_i, T_j)-*fso* set if and only if there exists a (T_i, T_j)-*fso* set ν_{x_ω} such that $x_\omega \in \nu_{x_\omega} \leq \mu$ for every fuzzy point x_ω in μ.

Definition 2.6 [10] Let μ be a fuzzy set of a *fbts* X.

(i) The **(T_i, T_j)-semi-interior** of μ [(T_i, T_j)-sIntμ] is defined by

(T_i, T_j)-sIntμ = sup$\{\nu \mid \nu \leq \mu,\ \nu$ is a (T_i, T_j)-*fso* set$\}$.

(ii) The **(T_i, T_j)-semi-closure** of μ [(T_i, T_j)-sClμ)] is defined by

(T_i, T_j)-sClμ = inf$\{\nu \mid \nu \geq \mu,\ \nu$ is a (T_i, T_j)-*fsc* set$\}$.

From Lemma 2.2 and Theorem 2.3, we have the following obvious facts.

(T_i, T_j)-sInt μ is the greatest (T_i, T_j)-*fso* set which is contained in μ and (T_i, T_j)-sClμ is the lowest (T_i, T_j)-*fsc* set which contains μ, and we have,

T_i-Int $\mu \leq (T_i, T_j)$-sInt $\mu \leq \mu \leq (T_i, T_j)$-sCl$\mu \leq T_i$-Cl$\mu$.

If $\mu \leq \nu$, then

(T_i, T_j)-sInt $\mu \leq (T_i, T_j)$-sInt ν and (T_i, T_j)-sCl$\mu \leq (T_i, T_j)$-sClν.

In addition to those facts, μ is a (T_i, T_j)-*fso* set if and only if $\mu = (T_i, T_j)$-sInt μ, and μ is a (T_i, T_j)-*fsc* set if and only if $\mu = (T_i, T_j)$-sClμ.

3. Fuzzy pairwise irresolute mapping

Definition 3.1 [1] Let $f: (X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ be a mapping. Then f is called a **fuzzy pairwise continuous** [*fpc*] **mapping** if the induced mappings $f: (X, T_k) \to (Y, T_k^*)$ $(k = 1, 2)$ are fuzzy continuous mappings.

Definition 3.2 [10] Let $f: (X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ be a mapping. Then f is called a **fuzzy pairwise semicontinuous** [*fpsc*] **mapping** if $f^{-1}(\nu)$ is a (T_i, T_j)-*fso* set of X for each T_i^*-*fo* set ν of Y.

Definition 3.3 [9] Let $f: (X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ be a mapping. Then f is called a **fuzzy pairwise irresolute** [*fpi*] **mapping** if $f^{-1}(\nu)$ is a (T_i, T_j)-*fso* set of X for each (T_i^*, T_j^*)-*fso* set ν of Y.

From the above definitions, we show that every *fpc* is *fpsc*. But the converse is not true in general [10]. Also, every *fpi* is *fpsc* but the converse is not true in general [9].

Example 3.4. Let μ_1, μ_2 and μ_3 be fuzzy sets of $X = \{a, b, c\}$ and let ν_1 and ν_2 be a fuzzy set of $Y = \{x, y, z\}$, defined as follows;

$\mu_1(a) = 1,\ \mu_1(b) = 0.7,\ \mu_1(c) = 0.8,$

$\mu_2(a) = 0,\ \mu_2(b) = 0.2,\ \mu_2(c) = 0.2,$

$\mu_3(a) = 0,\ \mu_3(b) = 0.3,\ \mu_3(c) = 0.3,$

$\nu_1(x) = 0,\ \nu_1(y) = 0.2,\ \nu_1(z) = 0.5,$

$\nu_2(x) = 0,\ \nu_2(y) = 0.3,\ \nu_2(z) = 0.5.$

Consider fuzzy topologies $T_1 = \{0_X, \mu_2, 1_X\}$, $T_2 = \{0_X, \mu_1, \mu_2, \mu_3, 1_X\}$, $T_1^* = \{0_Y, \nu_1, 1_Y\}$ and $T_2^* = \{0_Y, \nu_2, 1_Y\}$. Define $f: (X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ by $f(a) = x, f(b) = y, f(c) = y$. Then f is a *fpsc* (in fact, *fpc*). On the other hand, ν_2 is a (T_1^*, T_2^*)-*fso* set, but $f^{-1}(\nu_2)$ is not a (T_1, T_2)-*fso* set. Thus f is not a *fpi*. □

Exmple 3.5. Let μ_1, μ_2 and μ_3 be fuzzy sets of $X = \{a, b, c\}$, defined as follows;

$\mu_1(a) = 1,\ \mu_1(b) = 0.7,\ \mu_1(c) = 0.8,$

$\mu_2(a) = 0.9,\ \mu_2(b) = 0.6,\ \mu_2(c) = 0.7,$

$\mu_3(a) = 0.1,\ \mu_3(b) = 0.4,\ \mu_3(c) = 0.5.$

Consider fuzzy topologies $T_1 = \{0_X, \mu_3, 1_X\}$, $T_2 = \{0_X, \mu_2, 1_X\}$, $T_1^* = \{0_X, \mu_1, \mu_2, 1_X\}$ and $T_2^* = \{0_X, \mu_2, 1_X\}$. Then identity mapping $i_X: (X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ is a *fpi*, but i_X is not a *fpc*. □

Example 3.4 and 3.5 show that *fpc* and *fpi* do not have any specific relations.

Theorem 3.6. Let $f: (X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ be a mapping. Then the following statements are equivalent:

(i) f is *fpi*.

(ii) For each fuzzy point x_ω in X and each (T_i^*, T_j^*)-*fso* set ν of Y containing $f(x_\omega)$, there exists a (T_i, T_j)-*fso* set μ of X containing x_ω such that $f(\mu) \leq \nu$.

(iii) The inverse image of each (T_i^*, T_j^*)-*fsc* set of Y is a (T_i, T_j)-*fsc* set of X.

(iv) $f^{-1}(\lambda) \geq T_j$-Int$(T_i$-Cl$(f^{-1}(\lambda)))$ for each (T_i^*, T_j^*)-*fsc* set λ of Y.

(v) $f^{-1}(\nu) \leq T_j$-Cl$(T_i$-Int$(f^{-1}(\nu)))$ for each (T_i^*, T_j^*)-*fso* set ν of Y.

Proof. (i) implies (ii): Since f is *fpi* and ν is a (T_i^*, T_j^*)-*fso* set of Y containing $f(x_\omega)$, $f^{-1}(\nu)$ is a (T_i, T_j)-*fso* set of X. Let $\mu = f^{-1}(\nu)$. Then μ is a (T_i, T_j)-*fso* set of X containing x_ω and $f(\mu) \leq \nu$.

(ii) implies (i): Let ν be a (T_i^*, T_j^*)-*fso* set of Y. Then $f^{-1}(\nu)$ is a fuzzy set of X. If $x_\omega \in f^{-1}(\nu)$, then $f(x_\omega) \in \nu$. Hence there exists a (T_i, T_j)-*fso* set μ_{x_ω} of X such that $x_\omega \in \mu_{x_\omega}$ and $f(\mu_{x_\omega}) \leq \nu$. That is, $x_\omega \in \mu_{x_\omega} \leq f^{-1}(\nu)$. Thus we have,

$f^{-1}(\nu) = \vee\{x_\omega \mid x_\omega \in f^{-1}(\nu)\} \leq \vee\{\mu_{x_\omega} \mid x_\omega \in f^{-1}(\nu)\} \leq f^{-1}(\nu)$.

Consequently $f^{-1}(\nu) = \vee \{\mu_{x_\omega} | x_\omega \in f^{-1}(\nu)\}$. Therefore, $f^{-1}(\nu)$ is a (T_i, T_j)-*fso* set.

(i) implies (iii): Let ν be a (T_i^*, T_j^*)-*fsc* set of Y. Then ν^c is a (T_i^*, T_j^*)-*fso* set of Y. Hence $f^{-1}(\nu^c)$ is a (T_i, T_j)-*fso* set of X. But $f^{-1}(\nu^c) = (f^{-1}(\nu))^c$. Therefore, $f^{-1}(\nu)$ is a (T_i, T_j)-*fsc* set of X.

(iii) implies (iv): Let λ be a (T_i^*, T_j^*)-*fsc* set of Y. Then $f^{-1}(\lambda)$ is a (T_i, T_j)-*fsc* set of X. Hence by Lemma 2.2,
$f^{-1}(\lambda) \geq T_j\text{-Int}(T_i\text{-Cl}(f^{-1}(\lambda)))$.

(iv) implies (v): Obvious.

(v) implies (i): Let ν be a (T_i^*, T_j^*)-*fso* set of Y. Then $f^{-1}(\nu) \leq T_j\text{-Cl}(T_i\text{-Int}(f^{-1}(\nu)))$. Hence by Lemma 2.2, $f^{-1}(\nu)$ is a (T_i, T_j)-*fso* set of X. Therefore, f is *fpi*. □

Theorem 3.7 [9] A mapping $f: (X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ is *fpi* if and only if $f((T_i, T_j)\text{-sCl}\mu) \leq (T_i^*, T_j^*)\text{-sCl}(f(\mu))$ for each fuzzy set μ of X.

Proof. Let μ be any fuzzy set of X. Then $(T_i^*, T_j^*)\text{-sCl}(f(\mu))$ is a (T_i^*, T_j^*)-*fsc* set of Y. Thus $f^{-1}((T_i^*, T_j^*)\text{-sCl}(f(\mu)))$ is a (T_i, T_j)-*fsc* set of X. Now, $f(\mu) \leq (T_i^*, T_j^*)\text{-sCl}(f(\mu))$. Furthermore, $\mu \leq f^{-1}(f(\mu)) \leq f^{-1}((T_i^*, T_i^*)\text{-sCl}(f(\mu)))$. Therefore,

$$(T_i, T_j)\text{-sC}\mu \leq (T_i, T_j)\text{-sCl}(f^{-1}((T_i^*, T_j^*)\text{-sCl}(f(\mu)))) = f^{-1}((T_i^*, T_j^*)\text{-sCl}(f(\mu))).$$

This implies that

$$f((T_i, T_j)\text{-sCl}\mu) \leq f(f^{-1}((T_i^*, T_j^*)\text{-sCl}(f(\mu)))) \leq (T_i^*, T_j^*)\text{-sCl}(f(\mu)).$$

Conversely, let ν be a (T_i^*, T_j^*)-*fsc* set of Y and $\mu = f^{-1}(\nu)$. Then

$$f((T_i, T_j)\text{-sCl}\mu) \leq (T_i^*, T_j^*)\text{-sCl}(f(\mu)) = (T_i^*, T_j^*)\text{-sCl}(f(f^{-1}(\nu))) \leq (T_i^*, T_j^*)\text{-sCl}\nu = \nu.$$

Thus $f^{-1}(f((T_i, T_j)\text{-sCl}\mu)) \leq f^{-1}(\nu) = \mu$ and
$(T_i, T_j)\text{-sCl}\mu \leq f^{-1}(f((T_i, T_j)\text{-sCl}\mu)) \leq \mu$.

Hence $(T_i, T_j)\text{-sCl}\mu = \mu = f^{-1}(\nu)$. Therefore, μ is a (T_i, T_j)-*fsc* set of X. And consequently f is *fpi*. □

Theorem 3.8. A mapping $f: (X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ is *fpi* if and only if $(T_i, T_j)\text{-sCl}(f^{-1}(\nu)) \leq f^{-1}((T_i^*, T_j^*)\text{-sCl}\nu)$ for each fuzzy set ν of Y.

Proof. Let ν be any fuzzy set of Y. Then $f^{-1}(\nu)$ is a fuzzy set of X. Hence by Theorem 3.6,
$f((T_i, T_j)\text{-sCl}(f^{-1}(\nu))) \leq (T_i^*, T_j^*)\text{-sCl}(f(f^{-1}(\nu)))$.

Therefore,

$$(T_i, T_j)\text{-sCl}(f^{-1}(\nu)) \leq f^{-1}(f((T_i, T_j)\text{-sCl}(f^{-1}(\nu)))) \leq f^{-1}((T_i^*, T_j^*)\text{-sCl}(f(f^{-1}(\nu)))) \leq f^{-1}((T_i^*, T_j^*)\text{-sCl}\nu).$$

Conversely, let μ be any fuzzy set of X. Then $f(\mu)$ is a fuzzy set of Y, and
$(T_i, T_j)\text{-sCl}(f^{-1}(f(\mu))) \leq f^{-1}((T_i^*, T_j^*)\text{-sCl}(f(\mu)))$.

Hence

$$f((T_i, T_j)\text{-sCl}(f^{-1}(f(\mu)))) \leq f(f^{-1}((T_i^*, T_j^*)\text{-sCl}(f(\mu)))) \leq (T_i^*, T_j^*)\text{-sCl}(f(\mu)).$$

Therefore,

$$f((T_i, T_j)\text{-sCl}\mu) \leq f((T_i, T_j)\text{-sCl}(f^{-1}(f(\mu)))) \leq (T_i^*, T_j^*)\text{-sCl}(f(\mu)).$$

Consequently, by Theorem 3.7, f is *fpi*. □

Theorem 3.9. A mapping $f: (X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ is *fpi* if and only if $f^{-1}((T_i^*, T_j^*)\text{-sInt}\nu) \leq (T_i, T_j)\text{-sInt}(f^{-1}(\nu))$ for every fuzzy set ν of Y.

Proof. Let ν be any fuzzy set of Y. Then $(T_i^*, T_j^*)\text{-sInt}\nu$ is a (T_i^*, T_j^*)-*fso* set of Y. Clearly $f^{-1}((T_i^*, T_j^*)\text{-sInt}\nu)$ is a (T_i, T_j)-*fso* set of X and we have,

$$f^{-1}((T_i^*, T_j^*)\text{-sInt}\nu) = (T_i, T_j)\text{-sInt}(f^{-1}((T_i^*, T_j^*)\text{-sInt}\nu)) \leq (T_i, T_j)\text{-sInt}(f^{-1}(\nu)).$$

Conversely, let ν be a (T_i^*, T_j^*)-*fso* set of Y. Then $(T_i^*, T_j^*)\text{-sInt}\nu = \nu$ and
$f^{-1}(\nu) = f^{-1}((T_i^*, T_j^*)\text{-sInt}\nu) \leq (T_i, T_j)\text{-sInt}(f^{-1}(\nu))$.

Hence $f^{-1}(\nu) = (T_i, T_j)\text{-sInt}(f^{-1}(\nu))$. Therefore, $f^{-1}(\nu)$ is a (T_i, T_j)-*fso* set. And consequently f is *fpi*. □

Theorem 3.10. Let $f: (X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ be a bijective mapping. Then f is a *fpi* mapping if and only if for every fuzzy set μ of X $(T_i^*, T_j^*)\text{-sInt}(f(\mu)) \leq f((T_i, T_j)\text{-sInt }\mu)$.

proof. Let μ be any fuzzy set of X. Then $f(\mu)$ is a fuzzy set of Y. Then by Theorem 3.9,
$f^{-1}((T_i^*, T_j^*)\text{-sInt}(f(\mu))) \leq (T_i, T_j)\text{-sInt}(f^{-1}(f(\mu)))$.

Since f is bijective,

$$(T_i^*, T_j^*)\text{-sInt}(f(\mu)) = f(f^{-1}((T_i^*, T_j^*)\text{-sInt}(f(\mu)))) \leq f((T_i, T_j)\text{-sInt}(f^{1}(f(\mu)))) = f((T_i, T_j)\text{-sInt }\mu).$$

Conversely, let ν be a (T_i^*, T_j^*)-*fso* set of Y. Then
$(T_i^*, T_j^*)\text{-sInt}(f(f^{-1}(\nu))) \leq f((T_i, T_j)\text{-sInt}(f^{-1}(\nu)))$.
Since f is bijective, $(T_i^*, T_j^*)\text{-sInt } \nu \leq f((T_i, T_j)\text{-}$

sInt($f^{-1}(v)$).

This implies that

$$f^{-1}(T_i^*,\ T_j^*)\text{-sInt}v) \leq f^{-1}(f((T_i,\ T_j)\text{-sInt}(f^{-1}(v)))$$
$$= (T_i,\ T_j)\text{-sInt}(f^{-1}(v)).$$

Therefore, by Theorem 3.9, f is *fpi*. □

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