Bit Error Probability of Noncoherent M-ary Orthogonal Modulation over Generalized Fading Channels

Marvin K. Simon and Mohamed-Slim Alouini

Abstract: **Using a method recently reported in the literature for analyzing the bit error probability (BEP) performance of noncoherent M-ary orthogonal signals with square-law combining in the presence of independent and identically distributed Nakagami-m faded paths, we are able to reformulate this method so as to apply to a generalized fading channel in which the fading in each path need not be identically distributed nor even distributed according to the same family of distributions. The method leads to exact expressions for the BEP in the form of a finite-range integral whose integrand involves the moment generating function of the combined signal-to-noise ratio and which can therefore be readily evaluated numerically. The mathematical formalism is illustrated by applying the method to some selected numerical examples of interest showing the impact of the multipath intensity profile (MIP) as well as the fading correlation profile (FCP) on the BEP performance of M-ary orthogonal signals over Nakagami-m fading channels. These numerical results show that both MIP and FCP induce a non-negligible degradation in the BEP and have therefore to be taken into account for the accurate prediction of the performance of such systems.**

Index Terms: **M-ary noncoherent orthogonal modulation, M-ary frequency shift keying, Square-law combining, Postdetection equal gain combining, Nakagami fading.**

I. INTRODUCTION

The bit error probability (BEP) performance of noncoherent M-ary orthogonal modulation (or equivalently M-ary frequency shift keying (M-FSK)) operating over fading channels (both with and without diversity reception) has long been of interest. Hahn [1] and Lindsey [2] were the first to consider this problem for square-law combining (also called postdetection equal-gain combining) over Rayleigh and Rice fading channels, respectively. For the more general Nakagami-m channel [3], which is often a better fit to experimental data for urban mobile radio [4]–[6] as well indoor radio [7] propagation channels, analogous results were obtained by Crepeau [8] for the case of no diversity, and more recently by Weng and Leung [9] for the case of square-law diversity combining.

In addition, during the last decade, the problem has been reexamined in the context of its application to the performance analysis of the reverse link of direct-sequence code-division multiple access (DS-CDMA) systems over frequency-selective fading channels and with RAKE reception [10]–[13]. By modeling the multiple access (MA) interference among users as an equivalent additive Gaussian noise process as can be justified for many applications [14], these works evaluate performance in the same manner as would be done for the traditional additive white Gaussian noise (AWGN) channel using a total noise variance equal to the sum of that due to the ever-present thermal noise and that due to the multiple access interference. However, for mathematical tractability, most of the channel models considered in all these cited studies typically assume a uniform (constant) multipath intensity profile (MIP) (also called power delay profile (PDP)) along the paths and/or a particular fading correlation profile (FCP) across the diversity paths, namely a constant or an exponential correlation profile [13]. These idealizations are not always realistic since the average fading power may vary from one path to the other. For example, experimental measurements indicate that the radio channel is rather characterized by an exponentially decaying MIP for indoor office buildings [15] as well as urban [16], [17] and suburban areas [18]. Even in these idealized fading environments these studies still have to resort to approximation techniques to simplify the analysis. In particular, Jalloul and Holtzman [11] expand the conditional BEP using central differences to obtain an accurate approximation of the average BEP in terms of the two first moments of the sum of the multipath energies. On the other hand, Dallas and Pavlidou [12] approximate the mixed Rayleigh-lognormal probability density function (PDF) by an equivalent lognormal PDF then use a union bound to evaluate their final average BEP.

A unified approach to the analysis of noncoherent and differentially coherent communications over generalized fading channels[1] was recently presented by the authors [19], [20]. The approach was based on an alternative representation[2] of the gen-

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[1] A generalized fading channel is a multilink channel model in which the paths are not necessarily identically distributed nor even distributed according to the same family of distribution. For example, in the case of independent diversity paths, one option corresponds to all of them being chosen from the same probability distribution however the MIP is not restricted to be uniform. Alternately, each path may have its own probability distribution different from (or if desired the same as) the others. Finally, the diversity paths can be correlated with an arbitrary correlation matrix.

[2] By "alternative representation" we mean a finite-range integral form in which the integrand is an exponential function of the instantaneous combined signal-to-noise ratio (SNR).

eralized (mth order) Marcum Q-function as discussed in [21], which allowed obtaining expressions for average BEP in terms of a single integral with finite limits and an integrand involving the moment generating function (MGF) of the instantaneous combined SNR. One of the cases for which a direct application of the above-mentioned approach fails corresponds to noncoherent detection of M-ary ($M > 2$) orthogonal signals. In fact, even in the case of binary ($M = 2$) signaling, the average BEP result can only be obtained as a limiting case of the generic result in [19, Eq. (76) with $\beta \to 0$] which cannot be expressed in closed form. However, since the MGF of the instantaneous combined SNR of a large variety of fading conditions is straighforward to evaluate, as we will see next, it is still desirable to arrive at a generic expression for average BEP in the form of an integral (preferably with finite limits) having an integrand that depends on this MGF.

In this paper, we show that the original method used by Weng and Leung [9] to derive the BEP of noncoherent M-ary orthogonal modulation with square-law combining over independent identically distributed (i.i.d.) Nakagami-m fading channels in fact enables this to occur. More specifically, we reformulate in the next section this method so as to apply to a generalized fading channel with an arbitrary MGF and thereby arrive at *exact* results for average BEP in the above-mentioned desired form. To avoid unnecessary repetition, we shall draw heavily on the results in [22] only going into detail where necessary to distinguish between the specific Nakagami-m and generalized fading cases. We then show that the desired generic result can be further simplified for the special cases of Nakagami-m fading with arbitrary MIP and/or FCP. Finally, we present in the last section some numerical examples illustrating the impact of MIP and FCP on the BEP performance of noncoherent M-ary orthogonal modulation with square-law combining diversity reception.

II. EXACT ANALYSIS OF AVERAGE BIT ERROR PROBABILITY

Consider a noncoherent M-ary orthogonal system operating over an L-path frequency selective fading channel and using an L-fingers RAKE receiver with square-law combining. Our analysis assumes that the RAKE receiver is able to resolve perfectly the L multipaths and as such neglects the signal-dependent self-noise [23, p. 733]. Denote the outputs of the M combiners by U_i, ($i = 1, 2, \ldots, M$). Assume without loss in generality that the first combiner, i.e., U_1 corresponds to the information-bearing signal whereas the remaining $M-1$ combiner outputs contain noise only. Then, the average BEP for such a system can be expressed as [22, Eq. (8)]

$$P_b(E) = \frac{2^{(\log_2 M)-1}}{2^{\log_2 M}-1} P_s(E) = \frac{2^{(\log_2 M)-1}}{2^{\log_2 M}-1} \times \int_0^\infty \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathcal{M}_{U_1}(j\omega)\, e^{-j\omega u_1} g(u_1)\, d\omega\, du_1, \tag{1}$$

where $\mathcal{M}_{U_1}(j\omega)$ is the MGF of the first combiner output U_1 and

$$g(u_1) = \sum_{i=1}^{M-1} \binom{M-1}{i} (-1)^{i+1} e^{-iu_1/2} \sum_{k=0}^{i(L-1)} \beta_{ki}. \tag{2}$$

In (2) the coefficients $\{\beta_{ki}\}$ are obtained via the expansion

$$\left(\sum_{k=0}^{L-1} \frac{x^k}{k!} \right)^i = \sum_{k=0}^{i(L-1)} \beta_{ki}\, x^k, \tag{3}$$

and can be evaluated recursively by ([1, Eq. (23)] or equivalently [13, Eq. (32)])

$$\beta_{ki} = \sum_{n=k-L+1}^{k} \frac{\beta_{n(i-1)}}{(k-n)!} I_{[0,(i-1)(L-1)]}(i), \tag{4}$$

where $\beta_{00} = \beta_{0i} = 1$, $\beta_{k1} = 1/k!$, $\beta_{1i} = i$, and $I_{[a,b]}(i)$ is the indicator function defined by

$$I_{[a,b]}(i) = \begin{cases} 1 & a \le i \le b \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Since U_1 is of the form [22, Eq. (1)] (for more details see [23, Section 7.7])

$$U_1 = \sum_{l=1}^{L} \left| \alpha_l \sqrt{2E_s/N_0}\, e^{-j\phi_l} + n_{l,1} \right|^2, \tag{6}$$

where α_l and ϕ_l are the fading amplitude and phase associated with the lth diversity path, E_s/N_0 is the symbol SNR and $n_{l,1}$, ($l = 1, 2, \ldots, L$) are complex-valued i.i.d. zero mean unit variance random variables, then conditioned on the fading, U_1 is a noncentral chi-squared random variable with $2L$ degrees of freedom and MGF [24, Eq. (5A.8)]

$$\begin{aligned} \mathcal{M}_{U_1}\left(s \,|\, \{\alpha\}_{l=1}^{L}\right) &= \frac{1}{(1-2s)^L} \exp\left[\frac{2s}{1-2s} \frac{E_s}{N_0} \sum_{l=1}^{L} \alpha_l^2 \right] \\ &= \frac{1}{(1-2s)^L} \exp\left[\frac{2s}{1-2s} \sum_{l=1}^{L} \gamma_l \right], \end{aligned} \tag{7}$$

where $\gamma_l = \alpha_l^2 E_s/N_0$ denotes the instantaneous symbol SNR of the lth diversity branch. Averaging (7) over the fading yields the unconditional MGF of U_1, namely

$$\begin{aligned} \mathcal{M}_{U_1}(s) &= E\{\mathcal{M}_{U_1}(s|\{\alpha\}_{l=1}^{L})\} \\ &= \frac{1}{(1-2s)^L} \int_0^\infty \int_0^\infty \cdots \int_0^\infty \exp\left[\frac{2s}{1-2s} \sum_{l=1}^{L} \gamma_l \right] \\ &\quad \times p_{\gamma_1,\gamma_2,\ldots,\gamma_L}(\gamma_1, \gamma_2, \ldots, \gamma_L)\, d\gamma_1 d\gamma_2 \cdots d\gamma_L \\ &= \frac{1}{(1-2s)^L} \int_0^\infty \exp\left[\frac{2s}{1-2s} \gamma_l \right] p_{\gamma_l}(\gamma_l)\, d\gamma_l \\ &= \frac{1}{(1-2s)^L} \mathcal{M}_{\gamma_l}\left(\frac{2s}{1-2s} \right), \end{aligned} \tag{8}$$

where $\gamma_t = \sum_{l=1}^{L} \gamma_l$ is the total instantaneous symbol SNR at the combiner output and $\mathcal{M}_{\gamma_t}(s)$ is its MGF. Finally, substituting (8) in (1), reversing the order of integration and then integrating over u_1 (with the help of the integral in [25, Eq. (3.351.3)]) we obtain after some manipulation and a change of variables

$$P_b(E) = \left(\frac{2^{(\log_2 M)-1}}{2^{\log_2 M}-1}\right)\frac{1}{2\pi}\sum_{i=1}^{M-1}\binom{M-1}{i}(-1)^{i+1} \times \sum_{k=0}^{i(L-1)}\beta_{ki}\int_{-\infty}^{\infty}\frac{1}{(1-jv)^L}\frac{k!}{(i+jv)^{k+1}}\mathcal{M}_{\gamma_t}\left(\frac{jv}{1-jv}\right)dv. \tag{9}$$

To put (9) in the form of an integral with finite limits we now make the change of variables $v = \tan\theta$. Furthermore, recognizing that $P_b(E)$ is real, it is only necessary to take the real part of the right hand side of (9) since the imaginary part must equate to zero. Finally, after making this change of variables, performing some routine complex algebraic manipulations, and taking advantage of the fact that the resulting integrand is an even function of θ, we obtain the desired generic result, namely,

$$P_b(E) = \left(\frac{2^{(\log_2 M)-1}}{2^{\log_2 M}-1}\right)\frac{1}{\pi}\int_0^{\pi/2}\sum_{i=1}^{M-1}\binom{M-1}{i}(-1)^{i+1} \times \sum_{k=0}^{i(L-1)}\beta_{ki}\left[\mathcal{R}\left\{\mathcal{M}_{\gamma_t}\left(\frac{j\tan\theta}{1-j\tan\theta}\right)\right\}\cos\Phi_k - \mathcal{I}\left\{\mathcal{M}_{\gamma_t}\left(\frac{j\tan\theta}{1-j\tan\theta}\right)\right\}\sin\Phi_k\right] \times \frac{k!\,(\cos\theta)^{L-2}}{\left(i^2+\tan^2\theta\right)^{\frac{k+1}{2}}}\,d\theta, \tag{10}$$

where $\mathcal{R}\{\cdot\}$ and $\mathcal{I}\{\cdot\}$ denote the real and imaginary parts, respectively, and

$$\Phi_k \triangleq L\theta - (k+1)\arctan\left(\frac{\tan\theta}{i}\right). \tag{11}$$

For binary modulation, $M = 2$ and $\beta_{k1} = 1/k!$ and hence (10) simplifies further to

$$P_b(E) = \frac{1}{\pi}\int_0^{\pi/2}\sum_{k=0}^{(L-1)}\left[\mathcal{R}\left\{\mathcal{M}_{\gamma_t}\left(\frac{j\tan\theta}{1-j\tan\theta}\right)\right\} \times\cos((L-k-1)\theta) - \mathcal{I}\left\{\mathcal{M}_{\gamma_t}\left(\frac{j\tan\theta}{1-j\tan\theta}\right)\right\} \times\sin((L-k-1)\theta)\right]\frac{(\cos\theta)^{L-2}}{(1+\tan^2\theta)^{\frac{k+1}{2}}}d\theta, \tag{12}$$

which can be shown to agree numerically with the previously mentioned result obtained from substituting [19, Eq. (76)], and letting $\eta = 1$, $b = 1$, then taking the limit as $\beta \to 0$. Also recall that the conditional BEP of binary orthogonal signaling and differential phase-shift-keying (DPSK) [23, Eq. (4.4.13)] have the same functional dependence on the combined SNR γ_t, with the exception that γ_t is replaced by $2\gamma_t$ in the DPSK expression. However, since $\mathcal{M}_{\gamma_t}(s)$ is only a function of the normalized versions of its argument, i.e., $\{s\overline{\gamma}_l\}_{l=1}^L$, then $\mathcal{M}_{\gamma_t}(s) = E\left[e^{2\gamma_t s}\right] = E\left[e^{\gamma_t(2s)}\right]$ which from the foregoing is identical to $\mathcal{M}_{\gamma_t}(s)$ with $\{\overline{\gamma}_l\}_{l=1}^L$ replaced by $\{2\overline{\gamma}_l\}_{l=1}^L$. Thus we conclude that (12) applies also to the average BEP of DPSK with postdetection EGC when $\{\overline{\gamma}_l\}_{l=1}^L$ is replaced by $\{2\overline{\gamma}_l\}_{l=1}^L$.

A. *Independent But Not Necessarily Identically Distributed Fading*

Consider now the special case where the fading is independent from path to path but not necessarily identically distributed, i.e., each path is allowed to have arbitrary fading statistics. Because of the independence assumption, the MGF $\mathcal{M}_{\gamma_t}(s)$ partitions into the product

$$\mathcal{M}_{\gamma_t}(s) = \prod_{l=1}^{L}\mathcal{M}_{\gamma_l}(s;\overline{\gamma}_l), \tag{13}$$

where we explicitly include in the notation for the MGF of the instantaneous SNR per symbol of the lth path γ_l its dependence on the statistical average of γ_l denoted by $\overline{\gamma}_l$. As previously mentioned, $\mathcal{M}_{\gamma_l}(s;\overline{\gamma}_l)$ is readily available for a variety of different channel models and a summary of these MGF results is included here as Table I. In view of the above, the generic result for average BEP as given by (10) now becomes

$$P_b(E) = \left(\frac{2^{(\log_2 M)-1}}{2^{\log_2 M}-1}\right)\frac{1}{\pi}\sum_{i=1}^{M-1}\binom{M-1}{i}(-1)^{i+1} \times\sum_{k=0}^{i(L-1)}\beta_{ki}\int_0^{\pi/2}\left[\mathcal{R}\left\{\prod_{l=1}^{L}\mathcal{M}_{\gamma_l}\left(\frac{j\tan\theta}{1-j\tan\theta};\overline{\gamma}_l\right)\right\}\cos\Phi_k - \mathcal{I}\left\{\prod_{l=1}^{L}\mathcal{M}_{\gamma_l}\left(\frac{j\tan\theta}{1-j\tan\theta};\overline{\gamma}_l\right)\right\}\sin\Phi_k\right] \times\frac{k!\,(\cos\theta)^{L-2}}{\left(i^2+\tan^2\theta\right)^{\frac{k+1}{2}}}\,d\theta. \tag{14}$$

As a further specialization of (14), consider the case where all paths have Nakagami-m fading with the same fading parameter m as was done in [9] with, however, now an arbitrary MIP. Since for this type of fading the MGF of γ_l is given by (see Table I)

$$\mathcal{M}_{\gamma_l}(s;\overline{\gamma}_l) = (1 - s\overline{\gamma}_l/m)^{-m}, \tag{15}$$

then using this in (14) and simplifying the complex algebra we arrive at the result

$$P_b(E) = \left(\frac{2^{(\log_2 M)-1}}{2^{\log_2 M}-1}\right)\frac{1}{\pi}\int_0^{\pi/2}\sum_{i=1}^{M-1}\binom{M-1}{i}(-1)^{i+1} \times\sum_{k=0}^{i(L-1)}\beta_{ki}\frac{k!\,A\,\cos(\Phi_k+m\Theta)}{(i^2+\tan^2\theta)^{\frac{k+1}{2}}(\cos\theta)^{(2m-1)L+2}}d\theta, \tag{16}$$

Table 1. Moment generating function (MGF) of the SNR per symbol γ_l for some common multipath fading channels.

Type of Fading	Fading Parameter	$\mathcal{M}_{\gamma_l}(s;\overline{\gamma}_l)$
Rayleigh		$(1-s\overline{\gamma}_l)^{-1}$
Nakagami-q (Hoyt)	$0 \leq q_l \leq 1$	$\left(1-2s\overline{\gamma}_l+\frac{(2s\overline{\gamma}_l)^2 q_l^2}{(1+q_l^2)^2}\right)^{-1/2}$
Nakagami-n (Rice)	$0 \leq n_l$	$\frac{(1+n_l^2)}{(1+n_l^2)-s\overline{\gamma}_l}\exp\left(\frac{n_l^2 s\overline{\gamma}_l}{(1+n_l^2)-s\overline{\gamma}_l}\right)$
Nakagami-m	$\frac{1}{2} \leq m_l$	$\left(1-\frac{s\overline{\gamma}_l}{m_l}\right)^{-m_l}$

where

$$A \triangleq \prod_{l=1}^{L}\left[\left(1+\left(1+\tfrac{\overline{\gamma}_l}{m}\right)\tan^2\theta\right)^2+\left(\tfrac{\overline{\gamma}_l}{m}\tan\theta\right)^2\right]^{-m/2},$$
$$\Theta \triangleq \sum_{l=1}^{L}\arctan\left(\frac{\frac{\overline{\gamma}_l}{m}\tan\theta}{1+\left(1+\frac{\overline{\gamma}_l}{m}\right)\tan^2\theta}\right). \quad (17)$$

For i.i.d. Nakagami-m fading across the combined paths ($\overline{\gamma}_l = \overline{\gamma}$, $(l=1,2,\ldots,L)$), (16) agrees numerically with the symbol error probability formula given in [22, Eq. (12)] after conversion to BEP. For binary modulation, $M = 2$ and $\beta_{k1} = 1/k!$ and hence (16) simplifies further to

$$P_b(E) = \frac{1}{\pi}\int_0^{\pi/2}\sum_{k=0}^{L-1}\frac{A\cos\left[(L-k-1)\theta+m\Theta\right]}{(\cos\theta)^{(2m-1)L+1-k}}\,d\theta, \quad (18)$$

which agrees numerically with the result obtained from substituting [19, Eq. (50)] into [19, Eq. (76)], and letting $\eta = 1$, $b = 1$, then taking the limit as $\beta \rightarrow 0$. Also again because of the similarity of the conditional BEP for binary orthogonal signaling and DPSK (18) applies also for the average BEP of DPSK with $\overline{\gamma}_l$ replaced by $2\overline{\gamma}_l$. Hence for the i.i.d. case ($\overline{\gamma}_l = \overline{\gamma}$, $l = 1,2,\cdots,L$) (18) with the substitution of $\overline{\gamma}_l$ by $2\overline{\gamma}_l$ is equivalent to the closed-form expression given by Weng and Leung in [22, Eq. (11)].

B. Correlated Fading

Keeping in mind the application of our analysis to DS-CDMA systems [10]–[13] it is important to mention that there are many real life scenarios in which the combined paths have not only a decaying MIP but are also subject to correlated fading. In particular, for radio mobile macrocellular systems, Turin *et al.* [16] and Bajwa [26] observed correlation coefficients up to 0.6 between adjacent and second adjacent paths in the channel impulse response. More recently, based on a thorough statistical analysis of several macrocellular, microcellular, and indoor wideband channel impulse responses, Patenaude *et al.* [27], [28] reported correlation coefficients sometimes higher than 0.8 with no significant reduction in the correlation even for large path delay differences. Along these lines, many researchers investigated the effect of fading correlation on the performance of M-ary [13] and more particularly binary orthogonal signals [29], [30] with square-law combining. The performance analysis of such systems are typically restricted to the binary case (M=2) [29], [30] or dual diversity [30]. In addition, when the M-ary case with multiple diversity reception is considered, the channel is assumed to have a uniform MIP along the paths and a particular FCP, namely a constant or an exponential FCP [13].

In our case the generic expression for the average BEP of (10) can also be used to evaluate performance of M-ary signaling over L correlated fading channels with arbitrary MIP and FCP. For example in the particular case of Nakagami-m fading (for which Rayleigh is a special case) $\mathcal{M}_{\gamma_t}(s)$ in (10) can be expressed in closed-form using [20, Eq. (32)] as shown at the bottom of this page. In (19) ρ_{ij} is the correlation coefficient between the fading envelopes of paths i and j, and $\|[M]\|_{L\times L}$ denotes the determinant of the $L \times L$ matrix M. For $L = 2$, (19) further simplifies to

$$\mathcal{M}_{\gamma_t}(s) = \left(1-\frac{(\overline{\gamma}_1+\overline{\gamma}_2)}{m}s+\frac{(1-\rho)\overline{\gamma}_1\overline{\gamma}_2}{m^2}s^2\right)^{-m}. \quad (20)$$

III. NUMERICAL EXAMPLES

A. Impact of MIP

Fig. 1 is a plot of $P_b(E)$ for binary orthogonal FSK (obtained from (18)) versus the average SNR per bit of the first path $\overline{\gamma}_1$

$$\mathcal{M}_{\gamma_t}(s) = \prod_{l=1}^{L}\left(1-\frac{s\overline{\gamma}_l}{m}\right)^{-m}\left\|\begin{bmatrix} 1 & \sqrt{\rho_{12}}\left(1-\frac{m}{s\overline{\gamma}_2}\right)^{-1} & \cdots & \sqrt{\rho_{1L}}\left(1-\frac{m}{s\overline{\gamma}_L}\right)^{-1} \\ \sqrt{\rho_{12}}\left(1-\frac{m}{s\overline{\gamma}_1}\right)^{-1} & 1 & \cdots & \sqrt{\rho_{2L}}\left(1-\frac{m}{s\overline{\gamma}_L}\right)^{-1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \sqrt{\rho_{1L}}\left(1-\frac{m}{s\overline{\gamma}_1}\right)^{-1} & \sqrt{\rho_{2L}}\left(1-\frac{m}{s\overline{\gamma}_2}\right)^{-1} & \cdots & 1 \end{bmatrix}\right\|_{L\times L}^{-m}, \quad (19)$$

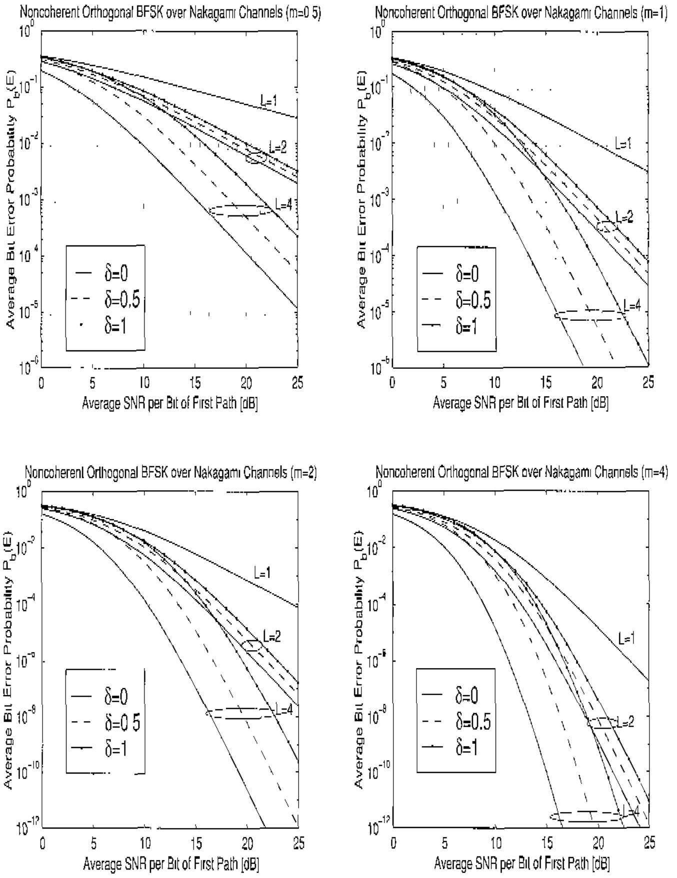

Fig. 1. Average bit error probability $P_b(E)$ of binary FSK with square-law combining versus the average SNR per bit of the first path $\overline{\gamma}_1$ over Nakagami-m channels with an exponentially decaying MIP.

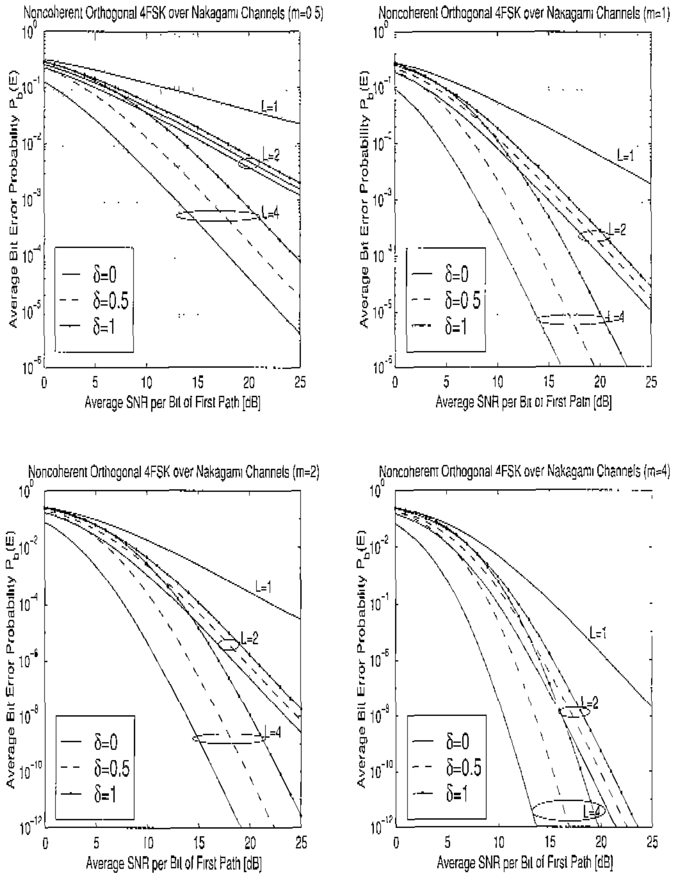

Fig. 2. Average bit error probability $P_b(E)$ of 4-ary FSK with square-law combining versus the average SNR per bit of the first path $\overline{\gamma}_1/2$ over Nakagami-m channels with an exponentially decaying MIP.

for different values of m and an exponentially decaying MIP ($\overline{\gamma}_l = \overline{\gamma}_1 \exp(-\delta(l-1))$, $(l = 1, 2, \cdots, L)$ and δ is the average fading power decay factor). In view of our previous remarks, in this figure as well as in the remaining figures concerned with binary orthogonal FSK, the average BEP of DPSK may be found by shifting the curves by 3 dB to the left. Analogous to Fig. 1, Figs. 2 and 3 are plots for the BEP $P_b(E)$ of 4-ary and 8-ary orthogonal FSK (obtained from (16) with $M = 4$ and $M = 8$) versus the average SNR per bit of the first path $\overline{\gamma}_1/\log_2 M$. From these figures there are two observations worth noting. First, in comparison with a system operating over a uniform MIP the BEP deterioration due to the exponentially decaying MIP can be quite important in all cases and this degradation is more accentuated for a large number of combined paths, as one may expect. However note that, in these figures, once the first path average SNR $\overline{\gamma}_1$ is fixed (as it is typically done in this type of illustrations (see for example [31])) the total average SNR is smaller for channels with an exponentially decaying MIP ($\delta > 0$) than for channels with a uniform MIP ($\delta = 0$) which explains in part the relatively important performance degradation due to the exponentially decaying MIP. Second, the use of M-ary ($M > 2$) instead of binary signaling still improves the BEP performance especially for channels subject to a low amount of fading (i.e., high fading parameter m).

B. Combined Effect of MIP and FCP

Fig. 4 is a plot of $P_b(E)$ (obtained from substituting (20) into (10)) for binary orthogonal FSK with dual diversity over correlated unbalanced ($\overline{\gamma}_2 = e^{-\delta}\overline{\gamma}_1$) Nakagami-$m$ fading channels. On the other hand Fig. 5 shows the average BEP performance of binary orthogonal FSK with square-law combining over a multilink channel with $L = 5$, an exponentially decaying MIP, and an exponential correlation profile (i.e., the ρ_{ij} in (19) are characterized by $\rho_{ij} = \rho^{|i-j|}$, $1 \leq i < j \leq L$). In both figures, for the parameters of interest, the BEP degradation induced by the MIP is higher than the degradation due to FCP, where the degradation here is with respect to a system operating over a uniform MIP with independent multipaths. Furthermore, comparing Figs 4 and 5 we conclude that this deterioration is more noticeable as the number of combined paths increases.

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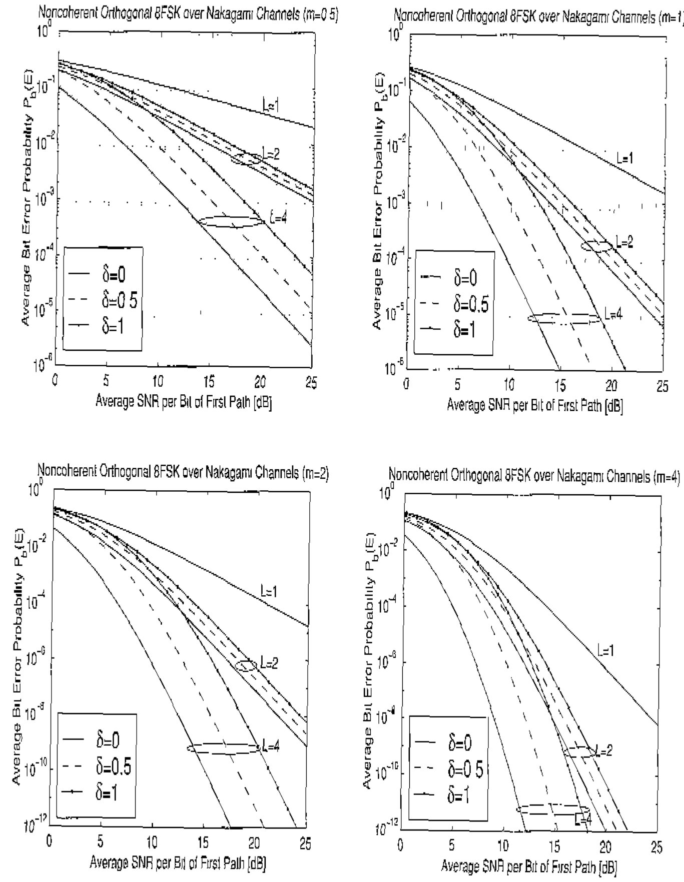

Fig. 3. Average bit error probability $P_b(E)$ of 8-ary FSK with square-law combining versus the average SNR per bit of the first path $\overline{\gamma}_1/3$ over Nakagami-m channels with an exponentially decaying MIP.

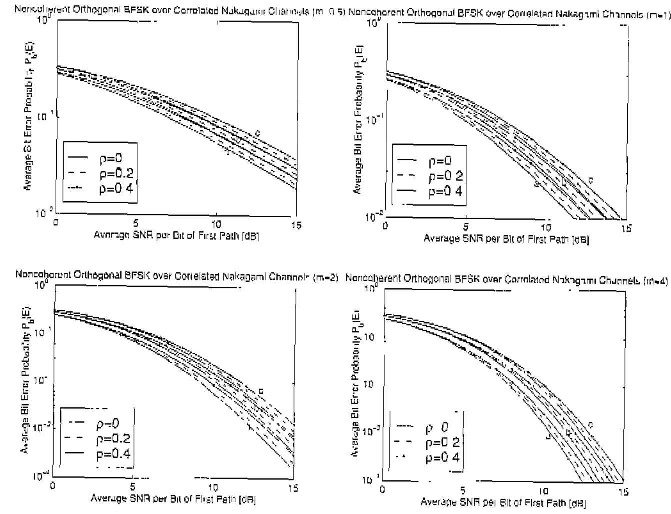

Fig. 4. Average bit error probability $P_b(E)$ of binary FSK with dual diversity ($L = 2$) square-law combining versus the average SNR per bit of the first path $\overline{\gamma}_1$ over unbalanced correlated Nakagami-m channels with an exponentially decaying MIP ((a) $\delta = 0$, (b) $\delta = 0.5$, and (c) $\delta = 1$).

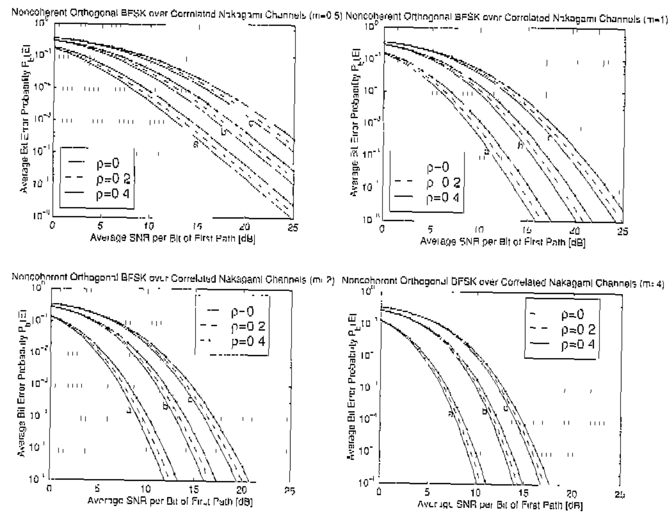

Fig. 5. Average bit error probability $P_b(E)$ of binary FSK with square-law combining ($L = 5$) versus the average SNR per bit of the first path $\overline{\gamma}_1$ over correlated Nakagami-m channels with an exponential FCP and an exponentially decaying MIP ((a) $\delta = 0$, (b) $\delta = 0.5$, and (c) $\delta = 1$).

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