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Two Priority Class Polling Systems with Batch Poisson Arrivals

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Abstract

In this paper we consider a polling system with two classes of stations: high priority and low priority. High priority stations are polled more frequently than low priority stations. We derive an exact and explicit formula for computing the mean waiting times for a message when the arrival processes are batch Poisson. In general, the formula requires to solve two sets of simultaneous equations. By specializing them to the case of two priority classes, we greatly reduce the number equations and provide a simple formula for the mean waiting times. We apply the results to a data communication processing system and show that the overall mean waiting time can be reduced by using priority polling.

1. Introduction

A polling system consists of a single server shared by multiple stations or queues. In this paper we consider a priority polling system with two priority classes. In this system each station is served in an order specified in a polling table, and high-priority stations are polled and served more frequently than low-priority stations. There are N_1 high-priority stations and $N_2 = J \times L$ low-priority stations. Each polling cycle consists of L phases, and every high-priority station is listed in each phase repeatedly but the low-priority stations are listed only once in one of the L phases. Thus, each phase has $N_1 + J$ entries and the polling table consists of $M = (N_1 + J) \times L$ entries. In one cycle, the high-priority stations are polled L times, but the low-priority stations are polled only once.

The priority polling policy can be employed in an AICPS(Advanced Information Communication Processing System) system. The AICPS system developed by

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ETRI(Electronics and Telecommunications Research Institute) provides a nation-wide platform for information communications. In the AICPS system, a HSSF(High Speed Switching Fabric), as a server, is connected to various kinds of delivery nodes(stations) such as PNAS(Packet Network Access System), WNAS(Web Network Access System), and ATM(Asynchronous Transmission Mode) network access system. In the AICPS, the ATM network access nodes usually have higher traffic load than the subscriber network access nodes. By making the HSSF poll the ATM access nodes more frequently than the subscriber access nodes, one may improve the overall system performance.

We consider stations with infinite capacity buffers. The message arrival process for each station is assumed to be a batch Poisson process. An example of batch Poisson process is an AICPS with a HSSF switching packets(messages). Here all packets arriving at each node are of fixed-length, but each node gets Poisson arrivals of variable-number packets. The batch Poisson processes are assumed to be independent with different rates across the stations. The number of messages in a batch is allowed to have an arbitrary distribution. The service times have arbitrary distributions and are independent too across the queues. The switch-over times, which start at the completion of serving one station and end at the polling instance to the very next station, have also arbitrary distributions and are independent across the queues. The service policy is exhaustive, meaning that once a station receives the token it transmits until its buffer is empty.

We assume that the system parameters for the low-priority stations depend only on their positions within the phases and not on which phase they belong to. For example, suppose there are three high-priority stations 1, 2 and 3, and four low-priority stations 4, 5, 6 and 7. Suppose there are two phases, and in the first one the station 1 is listed first and then followed by 2, 3, 4 and 5. In the second phase, the polling order is 1, 2, 3, 6, 7. In this case the above symmetry assumption means that the stations 4 and 6, and the stations 5 and 7 have the same batch arrival rates, batch size distributions, service time distributions, and switch-over time distributions. We allow, however, all high-priority stations have different parameters to each other.

The analysis of general cyclic polling systems can be found in Ferguson and Aminetzah (1985), and Takagi (1986) among others. Priority polling systems with general service order tables when the message arrival processes are independent Poisson have been investigated by Eisenberg (1972), and Baker and Rubin (1987). These two latter works have been extended to the case of batch Poisson arrival processes by Ryu *et al* (1998). In general, calculating the mean waiting times for a message, when the arrival processes are independent Poisson or independent batch Poisson, requires to solve a set of M simultaneous equations and a set of M^2 simultaneous equations. In this paper, we show that by specializing these sets of equations to the case of two priority classes and by utilizing the aforementioned symmetry property for the low-priority stations we are able to greatly simplify the equations to a single set of $(N_1+J)^2 \times L$ equations and provide a simple formula for the mean waiting times.

In the next section we briefly outline the results of Ryu *et al* (1998) which provide a formula for the mean waiting times for a priority polling system with a general service order table. In Section 3 we specialize these results to the case of two priority classes of stations, and apply the results to a data communication processing system showing that the overall mean waiting time can be reduced by using priority polling..

2. Priority Polling With a General Service Order Table

In a priority polling system with a service order table, each station is polled in turn according to the polling table. We call each entry in the polling table a *pseudostation*. We have $N=N_1+N_2$ stations with infinity capacity. Suppose there are $M(\geq N)$ pseudostations, i.e. M entries in the polling table, and let $I(k)$ denote the index of the underlying station which corresponds to the k-th pseudostation. For example, with a polling table $I=[1,2,1,3]$, there are three underlying stations indexed by 1, 2, 3, and four pseudostations with $I(1)=1$, $I(2)=2$, $I(3)=1$, $I(4)=3$.

The service policy is exhaustive, which means that the server continues to serve a station until it becomes empty. The messages arrive at each station in batch, and the batches arriving at the station i ($i=1,2,\ldots,N$) follows a Poisson process with arrival rate λ_i. These batch Poisson processes are assumed to be independent across the stations. Let m_i be the number of messages in a batch arriving at the station i, and S_i be the service time of a message in the station i. We assume that m_i's and S_i's are imdependent across the stations and are allowed to have arbitrary distributions.

Let D_k, $k=1,2,\ldots,M$, be the time spent for switch-over to the k-th pseudostation from the previous pseudostation. Define T_k, called *station time*, be D_k plus the time spent for serving messages in the k-th pseudostation. Note that each polling cycle has different station time. In equilibrium state, however, all station times for a pseudostation have the same distribution. Thus, we only consider one representative of them which we denote by T_k. Denote by J_k the time during which messages are accumulated for the k-th pseudostation. If $I(k)=i$, then J_k is the time elapses from the last departure to the next poll for the station

i. In the case of $I=[1,2,1,3]$, J_1 equals T_4+D_1 and J_2 equals $T_3+T_4+T_1+D_2$. We define $C_k=J_k+T_k-D_k$. If $I(k)=i$, then C_k is the time between two consecutive departures from the station i.

According to Ryu *et al* (1998), the mean of the waiting time for a message in the k-th pseudostation, denoted by W_k^*, is given by

$$\begin{aligned} \mathrm{E}(W_k^*) &= \frac{\mathrm{var}(J_k)}{2\mathrm{E}(J_k)} + \frac{\mathrm{E}(J_k)}{2} + \frac{\rho_{I(k)}}{2(1-\rho_{I(k)})} \cdot \frac{\mathrm{E}(S^2_{I(k)})}{\mathrm{E}(S_{I(k)})} \\ &\quad + \frac{\mathrm{E}(m^2_{I(k)}) - \mathrm{E}(m_{I(k)})}{2\lambda_{I(k)}(1-\rho_{I(k)})\{\mathrm{E}(m_{I(k)})\}^2} \end{aligned}$$

where $\rho_i=\lambda_i\mathrm{E}(m_i)\mathrm{E}(S_i)$, $i=1,2,\ldots,N$. Now let W_i be the waiting time for a message in the station i. To give a formula for computing the mean of W_i, let us introduce some more notations. Denote by C one full cycle time of the server. Note that it follows that $C=\sum_{k=1}^{M} T_k=\sum_{k|I(k)=i} C_k$ where $\sum_{k|I(k)=i}$ denotes summation over all pseudostation indices k such that $I(k)=i$. The mean waiting time for the station i is now given by

$$\mathrm{E}(W_i) = \frac{1-\rho}{\mathrm{E}(C)(1-\rho_i)} \cdot \sum_{k|I(k)=i} \mathrm{E}(J_k)\mathrm{E}(W_k^*)$$

where $\rho=\sum_{i=1}^{N}\rho_i$.

We see that the formula requires the first two moments of J_k. To give the formula for computing these moments, let $h_{k\ell}$ equal 1 if messages arriving at the k-th pseudostation during the station time T_ℓ are not served until the next visit to the k-th pseudostation, and equal 0 otherwise. For example, if $I=[1,2,1,3]$, then

$$H \equiv \begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{pmatrix}.$$

The basic relationships between the first moments of J_k, D_k, T_k and C_k are

$$\mathrm{E}(J_k) = \sum_{\ell \neq k}^{M} h_{k\ell}\mathrm{E}(T_\ell) + \mathrm{E}(D_k) \tag{2.1}$$

$$\mathrm{E}(C_k) = \sum_{\ell \neq k}^{M} h_{k\ell}\mathrm{E}(T_\ell) + \mathrm{E}(T_k) \tag{2.2}$$

$$\mathrm{E}(T_k) = \mathrm{E}(D_k) + \rho_{I(k)}\mathrm{E}(C_k) \tag{2.3}$$

$$\mathrm{E}(T_k) = \mathrm{E}(D_k) + \{\rho_{I(k)}/(1-\rho_{I(k)})\}\mathrm{E}(J_k). \tag{2.4}$$

From the above four identities we obtain a set of M simulataneous equations for computing the first moment of J_k :

$$\mathrm{E}(J_k) = \sum_{\ell \neq k}^{M} h_{k\ell}\left(\mathrm{E}(D_\ell) + \frac{\rho_{I(\ell)}}{1-\rho_{I(\ell)}}\mathrm{E}(J_\ell)\right) + \mathrm{E}(D_k). \tag{2.5}$$

The variance of J_k depends on the second moments of T_ℓ's. Note that the distribution of the cross product $T_k T_\ell$ depends not only on the pseudostation indices k and ℓ but also on which one is visited first. Therefore, we denote the covariance of T_k and T_ℓ by $r_{k\ell}$ when the ℓ-th pseudostation is visited before the k-th pseudostation. Then by Ryu *et al* (1998) we have

$$\mathrm{var}(J_k) = \frac{1-\rho_{I(k)}}{\rho_{I(k)}} \sum_{\ell \neq k}^{M} r_{k\ell} h_{k\ell} + \mathrm{var}(D_k). \tag{2.6}$$

Ryu *et al* (1998) give a system of M^2 equations for calculating $r_{k\ell}$'s $(1 \le k, \ell \le M)$ as follows :

(i) $\ell < k$

$$r_{k\ell} = \frac{\rho_{I(k)}}{1-\rho_{I(k)}}\left(\sum_{n=k+1}^{M} r_{\ell n} h_{kn} + \sum_{n=1}^{\ell-1} r_{\ell n} h_{kn} + \sum_{n=\ell}^{k-1} r_{n\ell} h_{kn}\right) \tag{2.7}$$

(ii) $\ell > k$

$$r_{k\ell} = \frac{\rho_{I(k)}}{1-\rho_{I(k)}}\left(\sum_{n=\ell}^{M} r_{n\ell} h_{kn} + \sum_{n=1}^{k-1} r_{n\ell} h_{kn} + \sum_{n=k+1}^{\ell-1} r_{\ell n} h_{kn}\right) \tag{2.8}$$

(iii) $\ell = k$

$$\begin{aligned} r_{kk} = {} & \frac{\mathrm{var}(D_k)}{(1-\rho_{I(k)})^2} + \frac{\lambda_{I(k)}\{\mathrm{E}(S_{I(k)})\}^2\{\mathrm{E}(m^2_{I(k)}) - \mathrm{E}(m_{I(k)})\}\mathrm{E}(J_k)}{(1-\rho_{I(k)})^2} \\ & + \lambda_{I(k)}\mathrm{E}(J_k)\{\mathrm{E}S^2_{I(k)}\mathrm{E}m_{I(k)} + \lambda_{I(k)}(\mathrm{E}S_{I(k)})^3(\mathrm{E}m_{I(k)})(\mathrm{E}m^2_{I(k)} - \mathrm{E}m_{I(k)})\} \\ & \div (1-\rho_{I(k)})^3 + \frac{\rho_{I(k)}}{1-\rho_{I(k)}} \sum_{n \neq k}^{M} r_{kn} h_{kn} \end{aligned} \tag{2.9}$$

3. Two Priority Classes

In this section we specialize the results of Section 2 to the case of two priority classes. Suppose that there are N_1 high-priority stations and $N_2 = J \times L$ low-priority stations. In the polling table there are L phases and each phase lists all the high-priority stations and J low-priority stations. Thus, while every high-priority station appears repeatedly in every phase, a low-priority station is listed only once in one of the L phases. Here the basic assumption is that all the system parameters, such as batch arrival rates and distributions of batch sizes, service times, switch-over times etc, for the low-priority stations depend only on their positions within in the phases and not on which phase they belong to. We will see that this assumption enables us to greatly reduce the number of equations to solve for computing the first two moments of J_k's. In fact, there is no need to solve the system of M equations given in (2.5), and the system of M^2 equations given in (2.7), (2.8) and (2.9) is reduced to a system of $(N_1+J)\times M$ equations.

Suppose that in each phase the N_1 high-priority stations are listed in the same order and before the low-priority stations. We note the following useful facts about $h_{k\ell}$'s :

(i) $1\le j\le N_1,\ 0\le n\le L-1$

$$h_{j+n(N_1+J),\,\ell}=\begin{cases}1 & \text{if } [j+1+(n-1)(N_1+J)]_M\le \ell \le [j-1+n(N_1+J)]_M \\ 0 & \text{otherwise}\end{cases} \tag{3.1}$$

(ii) $1\le j\le J,\ 0\le n\le L-1$

$$h_{N_1+j+n(N_1+J),\,\ell}=\begin{cases}1 & \text{if } \ell \ne N_1+j+n(N_1+J) \\ 0 & \text{otherwise}\end{cases} \tag{3.2}$$

where $[\,\cdot\,]_M$ denotes mod M.

3.1 First moments of J_k's

Here we consider the formula for calculating the means of J_k's.

<u>High-priority stations</u> ($J_{j+n(N_1+J)}$'s with $1\le j\le N_1,\ 0\le n\le L-1$)

From the equations (2.2) and (3.1) we have

$$\begin{aligned} \mathrm{E}(C_{j+n(N_1+J)}) &= \sum_{l=0,\,l\ne n}^{L-1}\sum_{i=1}^{N_1+J} h_{j+n(N_1+J),\,i+l(N_1+J)}\mathrm{E}(T_{i+l(N_1+J)}) \\ &\quad + \sum_{i=1,\,i\ne j}^{N_1+J} h_{j+n(N_1+J),\,i+n(N_1+J)}\mathrm{E}(T_{i+n(N_1+J)}) \\ &\quad + \mathrm{E}(T_{j+n(N_1+J)}) \\ &= \sum_{i=j+1}^{N_1+J}\mathrm{E}(T_{i+[n-1]_L(N_1+J)}) + \sum_{i=1}^{j-1}\mathrm{E}(T_{i+n(N_1+J)}) \\ &\quad + \mathrm{E}(T_{j+n(N_1+J)}). \end{aligned} \tag{3.3}$$

By the symmetry property of the low-priority stations it follows that

$$\mathrm{E}(T_{i+n(N_1+J)}) = \mathrm{E}(T_i), \quad 1 \le i \le N_1 + J, \quad 0 \le n \le L-1. \tag{3.4}$$

These two equations (3.3) and (3.4) entail that

$$\mathrm{E}(C_{j+n(N_1+J)}) = \sum_{i=1}^{N_1+J} \mathrm{E}(T_i).$$

From (3.4) we also have

$$\mathrm{E}(C) = \sum_{n=0}^{L-1} \sum_{i=1}^{N_1+J} \mathrm{E}(T_{i+n(N_1+J)}) = L \sum_{i=1}^{N_1+J} \mathrm{E}(T_i).$$

Thus we can deduce that

$$\mathrm{E}(C_{j+n(N_1+J)}) = \mathrm{E}(C)/L. \tag{3.5}$$

Finally from (2.3), (2.4) and (3.5) we have

$$\begin{aligned}\mathrm{E}(J_{j+n(N_1+J)}) &= \{1-\rho_{I(j+n(N_1+J))}\}\mathrm{E}(C_{j+n(N_1+J)}) \\ &= \{1-\rho_{I(j)}\}\mathrm{E}(C)/L.\end{aligned} \tag{3.6}$$

The expected value of C can be computed by the formula

$$\mathrm{E}(C) = \mathrm{E}(D)/(1-\rho) \tag{3.7}$$

where $D = \sum_{k=1}^{M} D_k$.

Low-priority stations ($J_{N_1+j+n(N_1+J)}$'s with $1 \le j \le J$, $0 \le n \le L-1$)

Here again the equations (2.2) and (3.2) entail

$$\begin{aligned}\mathrm{E}(C_{N_1+j+n(N_1+J)}) &= \sum_{l=0, l\ne n}^{L-1} \sum_{i=1}^{N_1+J} h_{N_1+j+n(N_1+J), i+l(N_1+J)} \mathrm{E}(T_{i+l(N_1+J)}) \\ &\quad + \sum_{i=1, i\ne N_1+j}^{N_1+J} h_{N_1+j+n(N_1+J), i+n(N_1+J)} \mathrm{E}(T_{i+n(N_1+J)}) \\ &\quad + \mathrm{E}(T_{N_1+j+n(N_1+J)}) \\ &= \sum_{l=0, l\ne n}^{L-1} \sum_{i=1}^{N_1+J} \mathrm{E}(T_{i+l(N_1+J)}) + \sum_{i=1, i\ne N_1+j}^{N_1+J} \mathrm{E}(T_{i+n(N_1+J)}) \\ &\quad + \mathrm{E}(T_{N_1+j+n(N_1+J)}) \\ &= \sum_{l=0}^{L-1} \sum_{i=1}^{N_1+J} \mathrm{E}(T_{i+l(N_1+J)}) \\ &= \mathrm{E}(C).\end{aligned}$$

Thus we have

$$\mathrm{E}(J_{N_1+j+n(N_1+J)}) = \{1-\rho_{I(N_1+j)}\}\mathrm{E}(C). \tag{3.8}$$

The equations (3.6) and (3.8) with use of (3.7) give explicit formula for computing the expected values of J_k's so that there is no need to solve the system of M equations as is given in (2.5).

3.2 Second moments of J_k's

We consider here the system of M^2 equations for computing $r_{k\ell}$'s. We note first that by the symmetry property of phases it follows that

$$r_{k+n(N_1+J),\,\ell} = r_{k,[M+\ell-n(N_1+J)]_M} \tag{3.9}$$

for $k=1,\dots,N_1+J$, $n=1,\dots,(L-1)(N_1+J)$ and $\ell=1,\dots,M$. Thus one only needs to calculate $r_{k\ell}$'s for $1\le k\le N_1+J$, $1\le \ell\le M$. The rest of r's can be obtained by (3.9). Below we simplify the equations (2.7), (2.8) and (2.9) for calculating these $r_{k\ell}$'s for $1\le k\le N_1+J$, $1\le \ell\le M$.

<u>High-priority stations</u> ($r_{k\ell}$'s with $1\le k\le N_1$)

Let $K_1(k)=k+1+(L-1)(N_1+J)$. One can see from (3.1) that

$$h_{kn}=\begin{cases}1 & \text{if } 1\le n\le k-1 \text{ or } K_1(k)\le n\le M\\ 0 & \text{otherwise}\end{cases}.$$

Thus for $1\le \ell < k$ the three summations in (2.7) turn out to be

$$\sum_{n=k+1}^{M} r_{\ell n}h_{kn} = \sum_{n=K_1(k)}^{M} r_{\ell n}$$

$$\sum_{n=1}^{\ell-1} r_{\ell n}h_{kn} = \sum_{n=1}^{\ell-1} r_{\ell n}$$

$$\sum_{n=\ell}^{k-1} r_{n\ell}h_{kn} = \sum_{n=\ell}^{k-1} r_{n\ell},$$

and for $\ell = k$ the one in (2.9) turns out to be

$$\sum_{n\ne k}^{M} r_{kn}h_{kn} = \sum_{n=1}^{k-1} r_{kn} + \sum_{n=K_1(k)}^{M} r_{kn}.$$

Now for $\ell > k$ the first and the third summations in (2.8) include r's with first index exceeding N_1+J. However, by the symmetry property of phase we have

$$r_{n\ell} = r_{[n-m(N_1+J)]_M,\,[\ell-m(N_1+J)]_M}$$

for any integer m. Let $K_2(\ell)=[\ell-M+N_1+J]_M$ and $K_3(\ell)=\ell-M+N_1+J$. Then, for $k<\ell\le K_1(k)$, it follows that

$$\sum_{n=\ell}^{M} r_{n\ell}h_{kn} = \sum_{n=K_1(k)}^{M} r_{n\ell} = \sum_{n=i+1}^{N_1+J} r_{n,K_2(\ell)}$$

$$\sum_{n=1}^{k-1} r_{n\ell}h_{kn} = \sum_{n=1}^{k-1} r_{n\ell}, \qquad \sum_{n=k+1}^{\ell-1} r_{\ell n}h_{kn} = 0,$$

and for $K_1(k) < \ell \leq M$, we have that

$$\sum_{n=\ell}^{M} r_{n\ell} h_{kn} = \sum_{n=\ell}^{M} r_{n\ell} = \sum_{n=K_3(\ell)}^{N_1+J} r_{n,K_3(\ell)}$$

$$\sum_{n=1}^{k-1} r_{n\ell} h_{kn} = \sum_{n=1}^{k-1} r_{n\ell}$$

$$\sum_{n=k+1}^{\ell-1} r_{\ell n} h_{kn} = \sum_{n=K_1(k)}^{\ell-1} r_{\ell n} = \sum_{n=k+1}^{K_3(\ell)-1} r_{K_3(\ell),n}.$$

By these simplifications the system of the equations (2.7), (2.8) and (2.9) is reduced to

(i) $1 \leq \ell < k$

$$r_{k\ell} = \frac{\rho_{I(k)}}{1-\rho_{I(k)}}\left(\sum_{n=K_1(k)}^{M} r_{\ell n} + \sum_{n=1}^{\ell-1} r_{\ell n} + \sum_{n=\ell}^{k-1} r_{n\ell} \right)$$

(ii) $k < \ell \leq K_1(k)$

$$r_{k\ell} = \frac{\rho_{I(k)}}{1-\rho_{I(k)}}\left(\sum_{n=k+1}^{N_1+J} r_{n,K_2(\ell)} + \sum_{n=1}^{k-1} r_{n\ell} \right)$$

(iii) $K_1(k) < \ell \leq M$

$$r_{k\ell} = \frac{\rho_{I(k)}}{1-\rho_{I(k)}}\left(\sum_{n=K_3(\ell)}^{N_1+J} r_{n,K_3(\ell)} + \sum_{n=1}^{k-1} r_{n\ell} + \sum_{n=k+1}^{K_3(\ell)-1} r_{K_3(\ell),n} \right)$$

(iv) $\ell = k$

$$r_{kk} = \frac{\mathrm{var}(D_k)}{(1-\rho_{I(k)})^2} + \frac{\lambda_{I(k)}\{\mathrm{E}(S_{I(k)})\}^2\{\mathrm{E}(m^2_{I(k)}) - \mathrm{E}(m_{I(k)})\}\mathrm{E}(J_k)}{(1-\rho_{I(k)})^2}$$
$$+\lambda_{I(k)}\mathrm{E}(J_k)\{\mathrm{E}S^2_{I(k)}\mathrm{E}m_{I(k)} + \lambda_{I(k)}(\mathrm{E}S_{I(k)})^3(\mathrm{E}m_{I(k)})(\mathrm{E}m^2_{I(k)} - \mathrm{E}m_{I(k)})\}$$
$$\div (1-\rho_{I(k)})^3 + \frac{\rho_{I(k)}}{1-\rho_{I(k)}}\left(\sum_{n=1}^{k-1} r_{kn} + \sum_{n=K_1(k)}^{M} r_{kn} \right).$$

Furthermore the formula (2.5) is now rewritten as

$$\mathrm{var}(J_k) = \frac{1-\rho_{I(k)}}{\rho_{I(k)}}\left(\sum_{n=1}^{k-1} r_{kn} + \sum_{n=K_1(k)}^{M} r_{kn} \right) + \mathrm{var}(D_k).$$

Low-priority stations ($r_{k\ell}$'s with $N_1 < k \leq N_1 + J$)

In this case it is clear that $h_{kn}=1$ if $n \neq k$ and 0 otherwise. Define $K_4(n) = [n]_{N_1+J}$, $K_5(n, \ell) = [\ell - n + K_4(n) + M]_M$, $K_6(\ell) = [\ell]_{N_1+J}$, $K_7(n, \ell) = [K_6(\ell) - \ell + n + + M]_M$. By similar arguments as in the case of high-priority stations we conclude

(i) $1 \le \ell < k$

$$r_{k\ell} = \frac{\rho_{I(k)}}{1-\rho_{I(k)}} \left(\sum_{n=k+1}^{M} r_{\ell n} + \sum_{n=1}^{\ell-1} r_{\ell n} + \sum_{n=\ell}^{k-1} r_{n\ell} \right)$$

(ii) $k < \ell \le M$

$$r_{k\ell} = \frac{\rho_{I(k)}}{1-\rho_{I(k)}} \left(\sum_{n=\ell}^{M} r_{K_4(n), K_5(n,\ell)} + \sum_{n=1}^{k-1} r_{n\ell} + \sum_{n=k+1}^{\ell-1} r_{K_6(\ell), K_7(n,\ell)} \right)$$

(iii) $\ell = k$

$$r_{kk} = \frac{\mathrm{var}(D_k)}{(1-\rho_{I(k)})^2} + \frac{\lambda_{I(k)}\{E(S_{I(k)})\}^2\{E(m^2_{I(k)}) - E(m_{I(k)})\}E(J_k)}{(1-\rho_{I(k)})^2}$$

$$+\lambda_{I(k)} E(J_k)\{ES^2_{I(k)} Em_{I(k)} + \lambda_{I(k)}(ES_{I(k)})^3 (Em_{I(k)})(Em^2_{I(k)} - Em_{I(k)})\}$$

$$\div (1-\rho_{I(k)})^3 + \frac{\rho_{I(k)}}{1-\rho_{I(k)}} \sum_{n=1, n\neq k}^{M} r_{kn}.$$

And, the formula (2.5) is rewritten as

$$\mathrm{var}(J_k) = \frac{1-\rho_{I(k)}}{\rho_{I(k)}} \sum_{n=1, n\neq k}^{M} r_{kn} + \mathrm{var}(D_k).$$

3.3 An application

We consider a data communication processing system with 14 stations, where 2 of them each carry 25 percent of the traffic and the other 50 percent is evenly distributed among the rest of the stations. Specifically, we set $\lambda_i = 0.048/\mu s$ (here, μs means microsecond) for the high traffic stations and $\lambda_i = 0.008/\mu s$ for the low traffic stations. We consider a HSSF server with transmission rate of 640 Mbits / second. We assume that all messages are of fixed length of 64 bytes. This means that the service time for a message is fixed to be $64 \times 8 \ / \ 640 \times 10^6 (s) = 0.8(\mu s)$. Also, we consider fixed switch-over time of $0.1(\mu s)$. For the distribution of the batch size we assume that $P(m_i = k) = 2^{-k}$, $k = 1, 2, \ldots$. Thus $E(m_i) = 2$ and $E(m_i^2) = 6$. In this case, $\rho_i = 0.048 \times 2 \times 0.8 = 0.0768$ for the high traffic stations, $\rho_i = 0.0128$ for the low traffic stations, and the total system intensity is $\rho = 2 \times 0.0768 + 12 \times 0.0128 = 0.3072$.

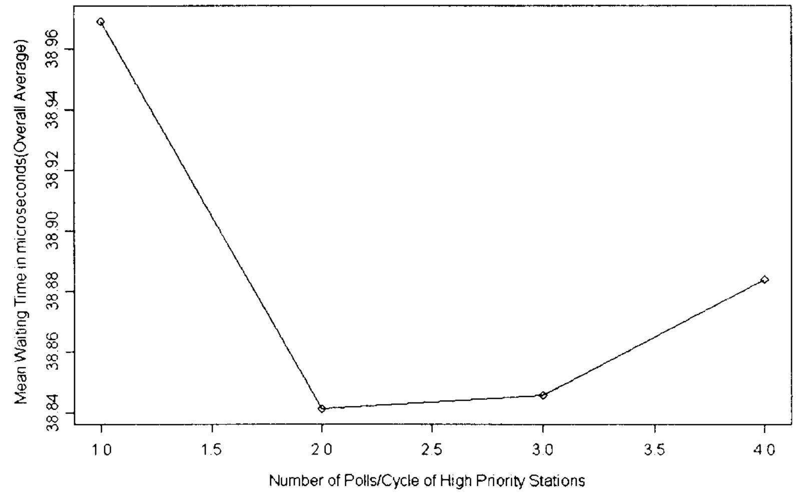

Figure 1

We consider the effect on message waiting times of polling the two high traffic stations 1, 2, 3, or 4 times per cycle. In Figure 1, we plot the mean message waiting time averaged over all stations. In fact, though not reported here, as the number of polls per cycle of the high priority stations increases, the mean waiting times at the high priority stations have large reductions in return for a smaller increase at the low priority stations. Figure 1 shows that the overall system performance is optimized when the server polls the high priority stations twice as frequent as the low priority stations.

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