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Asymptotic Distribution of the LM Test Statistic for the Nested Error Component Regression Model

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ABSTRACT

In this paper, we consider the panel data regression model in which the disturbances have nested error component. We derive a Lagrange Multiplier(LM) test which is jointly testing for the presence of random individual effects and nested effects under the normality assumption of the disturbances. This test extends the earlier work of Breusch and Pagan(1980) and Baltagi and Li(1991). Further, it is shown that this LM test has the same asymptotic distribution without normality assumption of the disturbances.

Keywords: Panel data; Nested error component; LM Test.

1. INTRODUCTION

In the context of error components model with panel data (Baltagi(1995) and Màtyàs and Sevestre(1996)), many researchers have provided the LM tests for testing the existence of the various error components. This paper considers the panel regression model for a data having a natural nested grouping. For example, data on firms may be grouped by industry, data on state by region and data on individuals by profession. In these cases, one can control for unobserved group and nested subgroup effects using a nested error component model(see Baltagi(1993)). In this paper, we derive a LM test statistic which jointly tests the presence of random individual effects and nested effects under the normality assumption of the disturbances.

Further, it is of some interest to investigate the behavior of this proposed test statistic under the assumption of non-normal disturbances, since Caroll and Ruppert(1981) and Koenker(1981) found that the asymptotic distribution of LM test statistics for heteroscedasticity under the normality assumption is different from that under the assumption of non-normal disturbances. The second purpose

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of this paper is to investigate the asymptotic property of the LM test statistic for the nested error components model with non-normal disturbances.

The outline of this paper is as follow. In section 2, we introduce the panel data regression model with nested error component. In section 3, we first derive a LM test which jointly tests the presence of random individual effects and nested effects under the normality assumption of the disturbances. Next, regardless of normality assumptions of the disturbances, it will be shown that the proposed test has a same asymptotic distribution of the LM test statistic evaluated at the normality assumption. Section 4 gives a conclusion.

2. THE MODEL

We consider the following panel regression model

$$y_{ijt} = x'_{ijt}\beta + u_{ijt}, \ i = 1, \cdots, M, \ j = 1, \cdots, N \text{ and } t = 1, \cdots, T, \tag{2.1}$$

where y_{ijt} be an observation on a dependent variable for the jth nested subgroup within the ith group for the tth time period. x_{ijt} denotes of k nonstochastic regressor vector. β is a $k \times 1$ unknown coefficient parameter vector. The disturbances term u_{ijt} in (2.1) are assumed that

$$u_{ijt} = \mu_i + \nu_{ij} + \varepsilon_{ijt}, \ i = 1, \cdots, M, \ j = 1, \cdots, N \text{ and } t = 1, \cdots, T, \tag{2.2}$$

where μ_i denote ith group specific effects which are assumed to be $i.i.d.$ $(0, \sigma_\mu^2)$, ν_{ij} denote the nested effects within the ith group which are $i.i.d.$ $(0, \sigma_\nu^2)$ and ε_{ijt} are the remainder disturbances which are also assumed to be $i.i.d.$ $(0, \sigma_\varepsilon^2)$. The μ_i's, ν_{ij}'s and ε_{ijt}'s are mutually independent. The model (2.1) can be rewritten in matrix notation as

$$y = X\beta + u, \tag{2.3}$$

where y and u are a $MNT \times 1$ and X is a $MNT \times k$ of rank k. The equation (2.2) is in vector form :

$$u = (I_M \otimes \iota_N \otimes \iota_T)\mu + (I_M \otimes I_N \otimes \iota_T)\nu + \varepsilon, \tag{2.4}$$

where $\mu' = (\mu_1, \cdots, \mu_M)$, $\nu' = (\nu_{11}, \cdots, \nu_{1N}, \cdots, \nu_{MN})$, $\varepsilon' = (\varepsilon_{111}, \cdots, \varepsilon_{MNT})$, ι_N and ι_T are vectors of ones of dimension N and T, respectively. I_M and I_N are identity matrices of dimension M and N, and $\otimes$ denotes the Kronecker product. The disturbance covariance matrix $E(uu')$ can be written as

$$\Omega = \sigma_\mu^2(I_M \otimes J_N \otimes J_T) + \sigma_\nu^2(I_M \otimes I_N \otimes J_T) + \sigma_\varepsilon^2(I_M \otimes I_N \otimes I_T), \tag{2.5}$$

where $J_N = \iota_N \iota_N'$ and $J_T = \iota_T \iota_T'$ are matrices of ones of dimension N and T.

To construct the log-likelihood function, we replace J_N by $N\bar{J}_N$, J_T by $T\bar{J}_T$, I_N by $E_N + \bar{J}_N$ and I_T by $E_T + \bar{J}_T$, where $E_N = I_N - \bar{J}_N$ and $E_T = I_T - \bar{J}_T$, and collect terms with the same matrices (Wansbeek and Kapteyn(1983)), we get the spectral decomposition of Ω.

$$\Omega = \sigma_\varepsilon^2 Q_1 + \sigma_2^2 Q_2 + \sigma_3^2 Q_3, \tag{2.6}$$

where $\sigma_2^2 = T\sigma_\nu^2 + \sigma_\varepsilon^2$, $\sigma_3^2 = NT\sigma_\mu^2 + T\sigma_\nu^2 + \sigma_\varepsilon^2$, and $Q_1 = (I_M \otimes I_N \otimes E_T)$, $Q_2 = (I_M \otimes E_N \otimes \bar{J}_T)$ and $Q_3 = (I_M \otimes \bar{J}_N \otimes \bar{J}_T)$. It is easy to show that σ_ε^2, σ_2^2 and σ_3^2 are the distinct characteristic roots of Ω with multiplicity $MN(T-1)$, $M(N-1)$ and M, respectively, and each Q_i, $i = 1, 2, 3$, is symmetric idempotent matrix. Moreover, the $Q_i's$ are pairwise orthogonal and sum to the identity matrix. Using the advantage of the spectral decomposition of Ω, we get

$$\Omega^{-1} = (\sigma_\varepsilon^2)^{-1} Q_1 + (\sigma_2^2)^{-1} Q_2 + (\sigma_3^2)^{-1} Q_3 \tag{2.7}$$

and

$$|\Omega| = (\sigma_\varepsilon^2)^{MN(T-1)} (\sigma_2^2)^{M(N-1)} (\sigma_3^2)^M. \tag{2.8}$$

Therefore, under the normality assumption of the disturbances, the log-likelihood function of (2.3) is given by

$$\begin{aligned} L(\beta, \sigma_\mu^2, \sigma_\nu^2, \sigma_\varepsilon^2) &= Const. - \frac{M}{2} \log \sigma_3^2 - \frac{M(N-1)}{2} \log \sigma_2^2 \\ &\quad - \frac{MN(T-1)}{2} \log \sigma_\varepsilon^2 - \frac{1}{2} u'\Omega^{-1} u. \end{aligned} \tag{2.9}$$

3. LM TEST

Engle(1984) and Godfrey(1989) demonstrated the wide applicability of Lagrange Multiplier(LM) test for various model specifications in econometrics and statistics. The LM test is much simpler to compute than the likelihood ratio test, since the LM approach is based on the estimation of the model under the null hypothesis. In our model, the null hypothesis is

$$H_0 : \sigma_\mu^2 = \sigma_\nu^2 = 0, \tag{3.1}$$

and the alternative H_1 is that at least one component is greater than zero.

Theorem 3.1. *The LM test statistic for the hypothesis (3.1) is given by*

$$LM = \frac{MN}{2(N-1)}\Big(A^2 - 2AB + \frac{NT-1}{T-1}B^2\Big), \tag{3.2}$$

where $A = \frac{\widehat{u}'(I_M \otimes J_N \otimes J_T)\widehat{u}}{\widehat{u}'\widehat{u}'} - 1$, $B = \frac{\widehat{u}'(I_M \otimes I_N \otimes J_T)\widehat{u}}{\widehat{u}'\widehat{u}'} - 1$, *and* $\widehat{u}$ *is OLS residuals.*

Proof: Let $\theta = (\sigma_\mu^2, \sigma_\nu^2, \sigma_\varepsilon^2)'$. Then the information matrix will be block diagonal between θ and β parameters, and the part of information matrix corresponding to β and the part of partial derivatives with respect to β will be ignored in computing the LM test statistic. In order to construct the LM test statistic for the hypothesis (3.1), we need $D(\theta) = (\partial L/\partial\theta)$ and information matrix $J(\theta) = E[-\partial^2 L/\partial\theta\partial\theta]$ evaluated at the restricted MLE $\tilde{\theta}$.

Following Hemmerle and Hartley(1973), we obtain

$$\partial L/\partial\theta_r = -\frac{1}{2}tr[\Omega^{-1}(\partial\Omega/\partial\theta_r)] + \frac{1}{2}[u'\Omega^{-1}(\partial\Omega/\partial\theta_r)\Omega^{-1}u] \tag{3.3}$$

and

$$E\Big[-\frac{\partial^2 L}{\partial\theta_r\partial\theta_s}\Big] = \frac{1}{2}tr\Big[\Omega^{-1}\frac{\partial\Omega}{\partial\theta_r}\Omega^{-1}\frac{\partial\Omega}{\partial\theta_s}\Big], \tag{3.4}$$

for $r, s = 1, 2, 3$ (see Harville(1977)). Under the null hypothesis, the Ω reduces to $\sigma_\varepsilon^2 I_{MNT}$ and the restricted MLE of β is $\widehat{\beta}_{OLS}$, so that $\widehat{u} = y - X\widehat{\beta}_{OLS}$ are the OLS residuals and $\widehat{\sigma}_\varepsilon^2 = \widehat{u}'\widehat{u}/MNT$. Using the equations (3.3) and (3.4), we can obtain the partial derivatives and information matrix, evaluated at the restricted MLE, given by

$$D(\widehat{\theta}) = \begin{bmatrix} \frac{MNT}{2\widehat{\sigma}_\varepsilon^2}\big(\frac{\widehat{u}'(I_M \otimes J_N \otimes J_T)\widehat{u}}{\widehat{u}'\widehat{u}} - 1\big) \\ \frac{MNT}{2\widehat{\sigma}_\varepsilon^2}\big(\frac{\widehat{u}'(I_M \otimes I_N \otimes J_T)\widehat{u}}{\widehat{u}'\widehat{u}} - 1\big) \\ 0 \end{bmatrix} \tag{3.5}$$

and

$$J(\widehat{\theta}) = \frac{MNT}{2\widehat{\sigma}_\varepsilon^4}\begin{bmatrix} NT & T & 1 \\ T & T & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

with

$$\widehat{J}^{-1} = \frac{2\widehat{\sigma}_\varepsilon^4}{MN(N-1)T^2(T-1)} \begin{bmatrix} T-1 & -(T-1) & 0 \\ -(T-1) & NT-1 & -T(N-1) \\ 0 & -T(N-1) & T^2(N-1) \end{bmatrix}. \quad (3.6)$$

Using (3.5) and (3.6), the LM test statistic is given by

$$LM = D(\widehat{\theta})'J(\widehat{\theta})^{-1}D(\widehat{\theta}) = \frac{MN}{2(N-1)}\Big(A^2 - 2AB + \frac{NT-1}{T-1}B^2\Big), \quad (3.7)$$

where $A = \dfrac{\widehat{u}'(I_M \otimes J_N \otimes J_T)\widehat{u}}{\widehat{u}'\widehat{u}} - 1$ and $B = \dfrac{\widehat{u}'(I_M \otimes I_N \otimes J_T)\widehat{u}}{\widehat{u}'\widehat{u}} - 1.$ □

These terms can be easily computed from the OLS residuals. Under the null hypothesis the LM test statistic in (3.7) is asymptotically distributed as χ_2^2.

The LM test statistic in (3.2) depends on the normality assumption of disturbances u. However, even if u does not have a normal distribution, the LM test statistic in (3.2) still has a same asymptotic distribution of the LM test statistic *evaluated at the normality assumption of* u. Using $\widehat{\sigma}_\varepsilon^2 = \widehat{u}'\widehat{u}/MNT$, the LM test statistic in (3.2) is revised as

$$\begin{aligned} LM &= \Big(\frac{\widehat{u}'(I_M \otimes J_N \otimes J_T)\widehat{u} - \widehat{u}'\widehat{u}}{\sqrt{2L_1}\widehat{\sigma}_\varepsilon^2}\Big)^2 + \Big(\frac{\widehat{u}'(I_M \otimes I_N \otimes J_T)\widehat{u} - \widehat{u}'\widehat{u}}{\sqrt{2L_2}\widehat{\sigma}_\varepsilon^2}\Big)^2 \\ &- \frac{1}{L_1}\Big(\frac{\widehat{u}'(I_M \otimes J_N \otimes J_T)\widehat{u} - \widehat{u}'\widehat{u}}{\widehat{\sigma}_\varepsilon^2}\Big)\Big(\frac{\widehat{u}'(I_M \otimes I_N \otimes J_T)\widehat{u} - \widehat{u}'\widehat{u}}{\widehat{\sigma}_\varepsilon^2}\Big), \quad (3.8) \end{aligned}$$

where $L_1 = MN(N-1)T^2$ and $L_2 = MN(N-1)(T-1)T^2/(NT-1)$. To show an asymptotic property of LM test statistic in (3.8) for the case of non-normal disturbances, we adopt the following classical device. Consider a following sequence of local alternative parameters $(\sigma_\mu^2, \sigma_\nu^2)$ converge to the null parameters $(0,0)$ at the rate of $1/\sqrt{L_1}$ and $1/\sqrt{L_2}$.

$$\sigma_\mu^2 = 0 + \delta_\mu/\sqrt{L_1} \geq 0, \qquad \sigma_\nu^2 = 0 + \delta_\nu/\sqrt{L_2} \geq 0 \quad (3.9)$$

and

$$\lim_{M\to\infty, N\to\infty, T\to\infty} \delta_\mu = \delta_1, \qquad \lim_{M\to\infty, N\to\infty, T\to\infty} \delta_\nu = \delta_2. \quad (3.10)$$

The δ_μ and δ_ν are parameters when sample sizes are M, N and T, and these parameters change as M, N and T increases, and the δ_1 and δ_2 are nonnegative fixed parameters. Finally, we assume $\lim_{M\to\infty, N\to\infty, T\to\infty} \Big(\frac{1}{MNT}X'X\Big) = Q$, where Q is a $k \times k$ positive definite matrix.

Under these assumptions, we have the following theorem :

Theorem 3.2. *As $M, N, T \to \infty$, the LM test statistic in (3.8) is asymptotically distributed as non-central χ_2^2 with non-centrality parameters $(\delta_1^2 + \delta_2^2)/2\sigma_\varepsilon^4$ for the non-normal disturbances. If both δ_1 and δ_2 are zero (under H_0), LM test statistic is asymptotically distributed as central χ_2^2.*

In order to prove Theorem 3.2, we provide some lemmas which will be used later.

Lemma 3.1. *The following two equations hold.*

$$(i)\quad \frac{\widehat{u}'(I_M \otimes J_N \otimes J_T)\widehat{u} - \widehat{u}'\widehat{u}}{\sqrt{2L_1}\sigma_\varepsilon^2} = \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j\neq k}^{N}\sum_{t\neq s}^{T}\varepsilon_{ijt}\varepsilon_{iks} + \frac{NT(NT-1)}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\mu_i^2 + o_p(1), \quad (3.11)$$

$$(ii)\quad \frac{\widehat{u}'(I_M \otimes I_N \otimes J_T)\widehat{u} - \widehat{u}'\widehat{u}}{\sqrt{2L_2}\sigma_\varepsilon^2} = \frac{1}{\sqrt{2L_2}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{t\neq s}^{T}\varepsilon_{ijt}\varepsilon_{ijs} + \frac{T(T-1)}{\sqrt{2L_2}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j=1}^{N}\nu_{ij}^2 + o_p(1). \quad (3.12)$$

Proof: First, we will show the holding of part (i) in Lemma 3.1. The left-hand side(LHS) of the equation (3.11) is given by

$$\begin{aligned}\frac{\widehat{u}'(I_M \otimes J_N \otimes J_T)\widehat{u} - \widehat{u}'\widehat{u}}{\sqrt{2L_1}\sigma_\varepsilon^2} &= \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\Big(\sum_{i=1}^{M}\Big[(\sum_{j=1}^{N}\sum_{t=1}^{T}\widehat{u}_{ijt})^2 - \sum_{j=1}^{N}\sum_{t=1}^{T}\widehat{u}_{ijt}^2\Big]\Big) \\ &= \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\Big(\sum_{i=1}^{M}\sum_{j\neq k \ or}^{N}\sum_{t\neq s}^{T}\widehat{u}_{ijt}\widehat{u}_{iks}\Big). \quad (3.13)\end{aligned}$$

Since $\widehat{u}_{ijt} = u_{ijt} - x_{ijt}(X'X)^{-1}X'u$, equation (3.13) can be rewritten as

$$\begin{aligned}&\frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\Big(\sum_{i=1}^{M}\sum_{j\neq k \ or}^{N}\sum_{t\neq s}^{T}\widehat{u}_{ijt}\widehat{u}_{iks}\Big) \\ &= \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j\neq k \ or}^{N}\sum_{t\neq s}^{T}u_{ijt}u_{iks} \\ &- \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j\neq k \ or}^{N}\sum_{t\neq s}^{T}u_{ijt}x_{iks}'(X'X)^{-1}X'u\end{aligned}$$

$$- \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j\neq k \text{ or } t\neq s}^{N}\sum^{T} u_{iks}x'_{ijt}(X'X)^{-1}X'u$$
$$+ \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j\neq k \text{ or } t\neq s}^{N}\sum^{T} x'_{ijt}(X'X)^{-1}X'ux'_{iks}(X'X)^{-1}X'u. \quad (3.14)$$

In the following we will show that the first term of the right-hand side(RHS) in (3.14) can be the RHS in (3.11), and the other terms of the RHS in (3.14) are $o_p(1)$.

(i) First, substituting (2.2) into the the first term of the RHS in (3.14), we get

$$\frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j\neq k \text{ or } t\neq s}^{N}\sum^{T} u_{ijt}u_{iks}$$
$$= \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j\neq k \text{ or } t\neq s}^{N}\sum^{T}(\mu_i+\nu_{ij}+\varepsilon_{ijt})(\mu_i+\nu_{ik}+\varepsilon_{iks})$$
$$= \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}NT(NT-1)\mu_i^2 + \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j=1}^{N}T(NT-1)\mu_i\nu_{ij}$$
$$+ \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{t=1}^{T}(NT-1)\mu_i\varepsilon_{ijt} + \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j=1}^{N}T(T-1)\nu_{ij}^2$$
$$+ \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j\neq k}^{N}T^2\nu_{ij}\nu_{ik} + \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j\neq k}^{N}\sum_{t=1}^{T}T\nu_{ij}\varepsilon_{ikt}$$
$$+ \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{t=1}^{T}T\nu_{ij}\varepsilon_{ijt} + \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j\neq k \text{ or } t\neq s}^{N}\sum^{T}\varepsilon_{ijt}\varepsilon_{iks}. \quad (3.15)$$

Except the first term and last term, the other terms in (3.15) converge to 0 as sample sizes get larger. For example, we consider the term $\frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j=1}^{N} T(T-1)\nu_{ij}^2$. Since ν_{ij} are independent for all i and j, $(L_2^{1/4}\nu_{ij})^2$ are $i.i.d.$ with finite mean. Therefore, using the Khinchine's theorem, $\sum_i\sum_j(L_2^{1/4}\nu_{ij})^2/MN$ converge in probability to $\lim\sqrt{L_2}\sigma_\nu^2 = \lim\delta_\nu = \delta_2$. Since, the fourth term in (3.15) can be written as $\frac{MNT(T-1)}{\sqrt{2L_1L_2}}\sum_i\sum_j(L_2^{1/4}\nu_{ij})^2/MN$ and $\lim\frac{MNT(T-1)}{\sqrt{2L_1L_2}} = 0$, the fourth term of (3.15) is $o_p(1)$. Also, the last term in (3.15) is devided as the following :

$$\frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j\neq k \text{ or } t\neq s}\sum\varepsilon_{ijt}\varepsilon_{iks} = \frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j}\sum_{t\neq s}\varepsilon_{ijt}\varepsilon_{ijs}$$

$$+\frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j\neq k}\sum_{t}\varepsilon_{ijt}\varepsilon_{ikt}+\frac{1}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_{i=1}^{M}\sum_{j\neq k}\sum_{t\neq s}\varepsilon_{ijt}\varepsilon_{iks}. \quad (3.16)$$

Let $p_{ij} = \sum_{t\neq s}\varepsilon_{ijt}\varepsilon_{ijs}/\sqrt{2\sigma_\varepsilon^2 T(T-1)}$, then p_{ij} are $i.i.d$ with mean 0 and variance 1. Hence, $\sum_i\sum_j p_{ij}/\sqrt{MN}$ converges in distribution to the standard normal distribution by CLT. Since, the first term of RHS in (3.16) becomes $\sqrt{\frac{T-1}{(N-1)T}}$ $(\sum_i\sum_j p_{ij}/\sqrt{MN})$, which is $o_p(1)$. By using the similar method, the second term of RHS in (3.16) is also $o_p(1)$. Therefore, the first term of RHS in (3.14) becomes to RHS in (3.11).

(*ii*) Next, consider the second term of RHS in (3.14) which can be expressed by

$$\Big(\sum_{i=1}^{M}\sum_{j\neq k}^{N}\sum_{or\ t\neq s}^{T}u_{ijt}x_{iks}/\sigma_\varepsilon^2\sqrt{2L_1MNT}\Big)\Big(X'X/MNT\Big)^{-1}\Big(X'u/\sqrt{MNT}\Big). \quad (3.17)$$

The second term in (3.17) converge to Q^{-1} by assumption, and the third term converges in distribution to a k-variate normal distribution with mean 0 and covariance matrix $\sigma_\varepsilon^2 Q$. To show this, we can rewrite the third term in (3.17) as

$$\begin{aligned}\frac{1}{\sqrt{MNT}}X'u &= \frac{1}{\sqrt{MNT}}\sum_i\sum_j\sum_t x_{ijt}\varepsilon_{ijt}+\frac{1}{\sqrt{MNT}}\sum_i\sum_j\sum_t x_{ijt}\mu_i\\ &+\frac{1}{\sqrt{MNT}}\sum_i\sum_j\sum_t x_{ijt}\nu_{ij}. \quad (3.18)\end{aligned}$$

The first term of RHS in (3.18) converges in distribution to a k-variate normal distribution with mean 0 and covariance matrix $\sigma_\varepsilon^2 Q$ (see, Greene(1997)), and the second and third term in (3.18) are $o_p(1)$. To show this, the second term in (3.18) can be written as $\frac{NT}{L_1^{1/4}}\sum_i \bar{x}_{i..}(L_1^{1/4}\mu_i)/\sqrt{M}$, where $\bar{x}_{i..} = \sum_i\sum_j\sum_t x_{ijt}/NT$. Assuming that the regression model (2.1) has a constant term, the condition $\lim(X'X/MNT) = \lim\sum\sum\sum x_{ijt}x'_{ijt} = Q$ implies that each element of $\bar{x}_{i..}$ is $O(1)$. As $L_1^{1/4}\mu_i$ are $i.i.d$ with mean 0 and variance δ_μ, $\sum_i \bar{x}_{i..}(L_1^{1/4}\mu_i)/\sqrt{M}$ converges in distribution to k-variate normal distribution. Hence, $\frac{1}{\sqrt{MNT}}\sum_i\sum_j\sum_t x_{ijt}\mu_i$ is $o_p(1)$. Using the similar method, it is easy to show that the third term of RHS in (3.18) is $o_p(1)$. Next, substituting (2.2) into the first term in (3.17), and using the similar method, we obtain that the first term in (3.17) is $o_p(1)$. Therefore, the second and third term of RHS in (3.14) is $o_p(1)$.

(*iii*) Finally, we can show that the fourth term of RHS in (3.14) is also $o_p(1)$. To show this, we rewrite it as

$$\Big[\frac{NT-1}{\sqrt{2MN(N-1)T^2\sigma_\varepsilon^2}}\Big]\Big[\Big(\frac{X'X}{MNT}\Big)^{-1}X'u/\sqrt{MNT}\Big]$$
$$\cdot\Big[\sum_{i=1}^{M}\sum_{j\neq k\ or}\sum_{t\neq s} x_{ijt}x'_{iks}/MNT(NT-1)\Big]\Big[\Big(\frac{X'X}{MNT}\Big)^{-1}X'u/\sqrt{MNT}\Big] \quad (3.19)$$

The second and fourth term in (3.19) converge in distribution to a k-variate normal distribution, and the third term in (3.19) converges to a $k\times k$ constant matrix, and the first term in (3.19) converges to zero. Therefore, the fourth term of RHS in (3.14) is also $o_p(1)$.

Using the similar technique, we can prove that the results of part (*ii*) hold. □

Lemma 3.2. *The following convergence results hold*

$$(i)\ \frac{\widehat{u}'(I_M\otimes J_N\otimes J_T)\widehat{u}-\widehat{u}'\widehat{u}}{\sqrt{2L_1}\sigma_\varepsilon^2} \xrightarrow{d} Z_1 \sim N(\delta_1/\sqrt{2}\sigma_\varepsilon^2, 1),$$
$$(ii)\ \frac{\widehat{u}'(I_M\otimes I_N\otimes J_T)\widehat{u}-\widehat{u}'\widehat{u}}{\sqrt{2L_2}\sigma_\varepsilon^2} \xrightarrow{d} Z_2 \sim N(\delta_2/\sqrt{2}\sigma_\varepsilon^2, 1), \qquad (3.20)$$

where d denotes convergence in distribution. Furthermore, Z_1 and Z_2 are independent.

Proof: Define $p_i = \frac{\sum_{j\neq k}^{N}\sum_{t\neq s}^{T}\varepsilon_{ijt}\varepsilon_{iks}}{\sqrt{2NT(N-1)(T-1)\sigma_\varepsilon^2}}$ and $w_{ij} = \frac{\sum_{t\neq s}^{T}\varepsilon_{ijt}\varepsilon_{ijs}}{\sqrt{2T(T-1)\sigma_\varepsilon^2}}$, then p_i and w_{ij} are *i.i.d* with mean 0 and variance 1, respectively. Therefore, the CLT implies that both $\frac{1}{\sqrt{M}}\sum_{i=1}^{M}p_i$ and $\frac{1}{\sqrt{MN}}\sum_{i=1}^{M}\sum_{j=1}^{N}w_{ij}$ converge in distribution to the standard normal distribution. Since,

$$\frac{\sum_{i=1}^{M}\sum_{j\neq k}^{N}\sum_{t\neq s}^{T}\varepsilon_{ijt}\varepsilon_{iks}}{\sqrt{2L_1}\sigma_\varepsilon^2} = \sum_{i=1}^{M}p_i/\sqrt{M}\Big(\sqrt{\frac{MN(N-1)T(T-1)}{MN(N-1)T^2}}\Big)$$

and $$\frac{\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{t\neq s}^{T}\varepsilon_{ijt}\varepsilon_{ijs}}{\sqrt{2L_2}\sigma_\varepsilon^2} = \sum_{i}\sum_{j}w_{ij}/\sqrt{MN}\Big(\sqrt{\frac{MNT(T-1)(NT-1)}{MN(N-1)T^2(T-1)}}\Big),$$

the term $\frac{\sum_{i=1}^{M}\sum_{j\neq k}^{N}\sum_{t\neq s}^{T}\varepsilon_{ijt}\varepsilon_{iks}}{\sqrt{2L_1}\sigma_\varepsilon^2}$ and $\frac{\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{t\neq s}^{T}\varepsilon_{ijt}\varepsilon_{ijs}}{\sqrt{2L_2}\sigma_\varepsilon^2}$ converge in distribution to the standard normal distribution, respectively.

Next, we can show the asymptotic behavior of the second term of RHS in (3.11) and (3.12). Since,

$$\frac{NT(NT-1)}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_i \mu_i^2 = \frac{MNT(NT-1)}{\sqrt{2}L_1\sigma_\varepsilon^2}\sum_i (L^{1/4}\mu_i)^2/M$$

and

$$\frac{T(T-1)}{\sqrt{2L_2}\sigma_\varepsilon^2}\sum_i\sum_j \nu_{ij}^2 = \frac{MNT(T-1)}{\sqrt{2}L_2\sigma_\varepsilon^2}\sum_i\sum_j (L^{1/4}\nu_{ij})^2/MN,$$

using the Khinchine's theorem $\frac{NT(NT-1)}{\sqrt{2L_1}\sigma_\varepsilon^2}\sum_i \mu_i^2$ and $\frac{T(T-1)}{\sqrt{2L_2}\sigma_\varepsilon^2}\sum_i\sum_j \nu_{ij}^2$ converge in probability to $\delta_1/\sqrt{2}\sigma_\varepsilon^2$ and $\delta_2/\sqrt{2}\sigma_\varepsilon^2$, respectively. Therefore, equation (3.11) converges in distribution to Z_1 which is distributed as $N(\delta_1/\sqrt{2}\sigma_\varepsilon^2, 1)$, and equation (3.12) also converges in distribution to Z_2 which is also distributed as $N(\delta_2/\sqrt{2}\sigma_\varepsilon^2, 1)$. Furthermore, the $\sum_i p_i/\sqrt{M}$ and $\sum_i\sum_j w_{ij}/\sqrt{MN}$ are uncorrelated, Z_1 and Z_2 are independent. □

Lemma 3.3. *The following convergence results hold*

$$(i)\ \Big(\frac{\widehat{u}'(I_M\otimes J_N\otimes J_T)\widehat{u}-\widehat{u}'\widehat{u}}{\sqrt{2L_1}\sigma_\varepsilon^2}\Big)^2 + \Big(\frac{\widehat{u}'(I_M\otimes I_N\otimes J_T)\widehat{u}-\widehat{u}'\widehat{u}}{\sqrt{2L_2}\sigma_\varepsilon^2}\Big)^2 \xrightarrow{d} \chi^2(2,(\delta_1^2+\delta_2^2)/2\sigma_\varepsilon^4), \tag{3.21}$$

$$(ii)\ \frac{1}{L_1}\Big(\frac{\widehat{u}'(I_M\otimes J_N\otimes J_T)\widehat{u}-\widehat{u}'\widehat{u}}{\sigma_\varepsilon^2}\Big)\Big(\frac{\widehat{u}'(I_M\otimes I_N\otimes J_T)\widehat{u}-\widehat{u}'\widehat{u}}{\sigma_\varepsilon^2}\Big) \xrightarrow{p} 0, \tag{3.22}$$

$$(iii)\ \widehat{\sigma}_\varepsilon^2 = \widehat{u}'\widehat{u}/MNT \xrightarrow{p} \sigma_\varepsilon^2. \tag{3.23}$$

Proof:

(i) From Lemma 3.2, we can show

$$\Big(\frac{\widehat{u}'(I_M\otimes J_N\otimes J_T)\widehat{u}-\widehat{u}'\widehat{u}}{\sqrt{2L_1}\sigma_\varepsilon^2}\Big)^2 \xrightarrow{d} Y_1 \sim \chi^2(1,\delta_1^2/2\sigma_\varepsilon^4)$$

and

$$\Big(\frac{\widehat{u}'(I_M\otimes I_N\otimes J_T)\widehat{u}-\widehat{u}'\widehat{u}}{\sqrt{2L_2}\sigma_\varepsilon^2}\Big)^2 \xrightarrow{d} Y_2 \sim \chi^2(1,\delta_2^2/2\sigma_\varepsilon^4).$$

Furthermore, from Lemma 3.2, Y_1 and Y_2 are independent. Therefore, we obtain the part (i) of Lemma 3.3.

(ii) Following Lemma 3.1, the equation (3.22) can be written as

$$\Big(\frac{\widehat{u}'(I_M \otimes J_N \otimes J_T)\widehat{u} - \widehat{u}'\widehat{u}}{\sqrt{L_1}\sigma_\varepsilon^2}\Big)\Big(\frac{\widehat{u}'(I_M \otimes I_N \otimes J_T)\widehat{u} - \widehat{u}'\widehat{u}}{\sqrt{L_2}\sigma_\varepsilon^2}\Big)\Big(2\sqrt{\frac{T-1}{NT-1}}\Big). \quad (3.24)$$

Using the result of lemma 3.1 and lemma 3.2, the first term and the second term in (3.24) converge in distribution to Z_1 and Z_2, respectively, and the third term in (3.24) converges to zero. Therefore, the entire term in (3.24) is $o_p(1)$.

(*iii*) Since, $\widehat{\sigma}_\varepsilon^2 = \sum_i \sum_j \sum_t \widehat{u}_{ijt}^2/MNT$, by the similar method of (3.14) we can rewrite $\widehat{\sigma}_\varepsilon^2$ as

$$\begin{aligned}\widehat{\sigma}_\varepsilon^2 &= \sum_i \sum_j \sum_t u_{ijt}^2/MNT - 2\sum_i \sum_j \sum_t u_{ijt}x_{ijt}(X'X)^{-1}X'u/MNT \\ &+ \sum_i \sum_j \sum_t (x_{ijt}(X'X)^{-1}X'u)^2/MNT. \quad (3.25)\end{aligned}$$

Using the similar technique to prove Lemma 3.1, it can be easily shown that the second and the third term of RHS in (3.25) are $o_p(1)$. Also, substituting (2.2) into the first term of RHS in (3.25), and applying Khinchine's theorem, we can show that it converges in probability to σ_ε^2. Therefore, we obtain the result of part (*iii*) of Lemma 3.3. □

Proof of Theorem 3.2: In order to prove Theorem 3.2, we revise the LM test statistic in (3.2) as

$$\begin{aligned}LM &= \Big\{\Big(\frac{\widehat{u}'(I_M \otimes J_N \otimes J_T)\widehat{u} - \widehat{u}'\widehat{u}}{\sqrt{2L_1}\sigma_\varepsilon^2}\Big)^2 + \Big(\frac{\widehat{u}'(I_M \otimes I_N \otimes J_T)\widehat{u} - \widehat{u}'\widehat{u}}{\sqrt{2L_2}\sigma_\varepsilon^2}\Big)^2\Big\}\Big(\frac{\sigma_\varepsilon^4}{\widehat{\sigma}_\varepsilon^4}\Big) \\ &- \frac{1}{L_1}\Big(\frac{\widehat{u}'(I_M \otimes J_N \otimes J_T)\widehat{u} - \widehat{u}'\widehat{u}}{\sigma_\varepsilon^2}\Big)\Big(\frac{\widehat{u}'(I_M \otimes I_N \otimes J_T)\widehat{u} - \widehat{u}'\widehat{u}}{\sigma_\varepsilon^2}\Big)\Big(\frac{\sigma_\varepsilon^4}{\widehat{\sigma}_\varepsilon^4}\Big) \quad (3.26)\end{aligned}$$

Combining the results from Lemma 3.1 to Lemma 3.3, the LM test statistic in (3.26) is asymptotically distributed as noncentral χ_2^2 with noncentrality parameter $(\delta_1^2 + \delta_2^2)/2\sigma_\varepsilon^4$. If $\delta_1 = 0$ and $\delta_2 = 0$ (under H_0) then the LM statistic in (3.26) is asymptotically distributed as central χ_2^2. □

4. CONCLUSION

In this paper, we derived the LM test which is jointly testing for the presence of random individual effects and nested effects. Further, even if the disturbances do not have a normal distribution, it is shown that the proposed test has a same asymptotic distribution of the LM test statistic evaluated at the normality assumption. Regardless of the normality assumption of the disturbances, the resulting test statistics should be proven useful for nested error component model specifications.

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