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The Behavior of Several Measures of Burn-In Reliability When Weak Components are Present

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ABSTRACT

Often, a small proportion of components may be "weak" because of some imperfection in the control of the production process or in the design of the components. The weak components will exhibit an increasing and then a decreasing hazard function. The remaining components will exhibit the usual "bathtub" form for the hazard function. The time-to-failure pattern can be modeled by a two-mixed Weibull distribution. This paper considers the case when the time-to-failure pattern resembles a modified bathtub curve where the weak proportion creates an increasing hazard rate (failure rate) to some time, t. Thereafter the hazard rate follows the usual "bathtub" curve. The relationship among three common measures of reliability; namely failure rate ($h(t)$), mean residual life ($m(t)$), and conditional reliability ($R(x|t)$) has not been previously discussed under this modified bathtub curve. Not surprisingly, knowledge concerning these relationships is important in order to formulate an effective burn-in procedure. New comparisons are discussed and new mathematical analyses are performed for the relationship. The results of mathematical analyses show that the optimal burn-in times for the three reliability measures are different. Thus, identifying which measure should be considered for a particular product must be the first step to plan a burn-in procedure

Keywords: Two-mixed Weibull distribution; Conditional reliability; Failure rate; Mean residual life; Modified bathtub curve; Optimal Burn-in.

1. INTRODUCTION

n today's market, early failures of products can seriously impair customer satisfaction and damage the reputation of a company even though most products carry some type of warranty to protect customers. Customer do not wish

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to spend time dealing with failures even when these are covered by warranties. Therefore, burn-in is needed to prevent delivering of defective products to customers. Burn-in is a screening method to detect and weed out early failures of products/components. Historically, some industrial products (electronical components/systems, mechanical equipments, automobiles, etc.) exhibit a "bathtub" curve for their failure rate functions as shown in Figure 1.1 when the products have weak parts and strong parts for their life. This curve is called a modified bathtub curve and is discussed by Jensen and Petersen (1982). The modified bathtub curve consists of four stages (see Figure 1.1). The first stage has an increasing failure rate (hazard rate) to some time, $t(a)$. Thereafter, the failure rate follows the traditional "bathtub" curve with early failure region, useful region, and wearout region. The early failure region has a decreasing failure rate. This is often observed in electronic components or systems. The failure rate decreases and the surviving components are more reliable as weak components fail and are rejected. In most products, the early failures correspond to a small percentage of the population. The useful region has a constant rate in which failures occur randomly. Many products exhibit this pattern. The wearout region has an increasing failure rate. Wearout occurs toward the end of product life when deterioration and weakening effects of accumulated stresses lead to an increasing failure rate function.

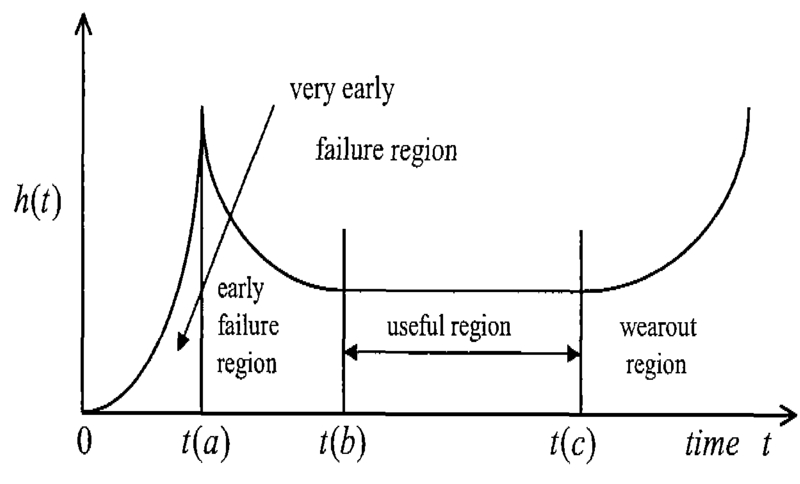

Figure 1.1: Modified Bathtub Curve, suggested by Jensen and Peterson. (1982,p.12)

Burn-in has been used to improve reliability measures such as failure rate($h(t)$), mean residual life($m(t)$), or conditional reliability ($R(x|t)$). Are the optimal burn-in times for reliability measures same? Chandrasekaran (1977) showed the case when the optimal burn-in time for mean residual life and failure rate are different. Then, Park (1985) examined the effect of burn-in on the mean residual life and showed that the optimal burn-in times for failure rate and mean residual life are different for the traditional bathtub curve. For some cases, Guess and Park (1988) showed that the burn-in times for minimizing failure rate and maximizing mean residual life are the same. Recently, Guess, Walker, and Gallant (1992) discussed the optimal burn-in times for the three reliability measures. They showed by use of examples that the optimal burn-in times for the reliability measures are not the same. However, the relationship among three reliability measures has not been previously discussed under the modified bathtub curve. In this paper, the characteristics of these three measures will be examined assuming weak components are presented. Also, graphical data-analysis will be presented for studying the relationship between failure rate ($h(t)$), mean residual life ($m(t)$), and conditional reliability ($R(x|t)$).

Two-mixed Weibull distribution is a good model to describe the failure rate function of many components or systems if they have weak parts and main parts for stress. Kao (1959) introduced a two-mixed Weibull distribution to describe the failure time of electronic tubes. Stitch et al. (1975) found that the failure time of microcircuits follows a mixed distribution law. Reynolds and Stevens (1978) also found that a two-mixed Weibull distribution describe the time-to-failure patterns of electronic components. For electro-mechanical device, Boardman and Colvert (1978) found that the failure time of oral irrigators follows a two-mixed Weibull distribution. By focusing our attention on early failure for a component, interest necessarily narrows to the first three stages in the modified bathtub curve of Figure 1.1. The two-mixed Weibull distribution can be used to model these portions of the curve.

2. TWO-MIXED WEIBULL DISTRIBUTION

A two-mixed Weibull distribution is composed of two cumulative density functions (CDF). Let $F_1(t)$ be the CDF of the weak population (small proportion of susceptible components), $F_2(t)$ be the CDF of the main population (the rest of the components), and $F(t)$ be the total CDF for the entire population. Then, $F(t)$ is constructed by taking a weighted average of the CDFs for the weak and main subpopulations. The weights are the proportions of each type of subpopulation. Thus, if the weak population has p proportion, and the main population has $(1-p)$ proportion, then

$$F(t) = pF_1(t) + (1-p)F_2(t) \tag{2.1}$$

Typically, $F_1(t)$ has a high early failure rate while $F_2(t)$ has a low early failure rate that either stays constant or increases very late in life in the equation (2.1). Assume that $f_1(t)$ is the probability density function (pdf) for $F_i(t)$ for $i = 1, 2$. Then, the failure rate of the two-mixed distribution is expressed as

$$h(t) = \frac{pf_1(t) + (1-p)f_2(t)}{1 - [pF_1(t) + (1-p)F_2(t)]} \tag{2.2}$$

where $h(t)$ is interpreted as the instantaneous failure rate at time t.

Now, let us consider the two-mixed Weibull distribution with two parameters for each population. From the equation (2.1), the CDF of the two-mixed Weibull distribution is as follow:

$$\begin{aligned} F(t) &= pF_1(t) + (1-p)F_2(t) \\ &= 1 - p(\exp([-(t/\eta])^{\beta_1}) - (1-p)(\exp[-(t/\eta_2)^{\beta_2}]) \end{aligned} \tag{2.3}$$

where

β_1 : shape parameter of the weak population,

β_2 : shape parameter of the main population

η_1 : scale parameter of the weak population,

η_1 : scale parameter of the main population

p : proportion of the weak population

When the shape parameter is less than one, we observe a decreasing failure rate function. When the shape parameter is equal to one, we get constant failure rate. When the shape parameter is greater than one, an increasing failure rate function results. Therefore, the shape parameter determines in which failure region a product belongs. The scale parameter is also called the characteristic life; the point at which 63.2% of units will have failed.

From equations (2.2) and (2.3), the failure density function, reliability function, and failure rate function can be expressed as follow:

$$
\begin{aligned}
f(t) &= p\left(\frac{\beta_1}{\eta_1}\right)\left(\frac{t}{\eta_1}\right)^{\beta_1-1} \exp\left[-(t/\eta_1)^{\beta_1}\right] \\
&\quad +(1-p)\left(\frac{\beta_2}{\eta_2}\right)\left(\frac{t}{\eta_2}\right)^{\beta_2-1} \exp\left[-(t/\eta_2)^{\beta_2}\right] \\
R(t) &= p\left(\exp\left[-(t/\eta_1)^{\beta_1}\right]\right) + (1-p)\left(\exp\left[-(t/\eta_2)^{\beta_2}\right]\right) \\
h(t) &= \frac{p\left(\frac{\beta_1}{\eta_1}\right)\left(\frac{t}{\eta_1}\right)^{\beta_1-1} \exp\left[-(t/\eta_1)^{\beta_1}\right]}{p\left(\exp\left[-(t/\eta_1)^{\beta_1}\right]\right) + (1-p)\left(\exp\left[-(t/\eta_2)^{\beta_2}\right]\right)} \\
&\quad + \frac{1-p)\left(\frac{\beta_2}{\eta_2}\right)\left(\frac{t}{\eta_2}\right)^{\beta_2-1} \exp\left[-(t/\eta_2)^{\beta_2}\right]}{p\left(\exp\left[-(t/\eta_1)^{\beta_1}\right]\right) + (1-p)\left(\exp\left[-(t/\eta_2)^{\beta_2}\right]\right)}
\end{aligned}
$$

The two-mixed Weibull distribution will be used to compare the reliability measures. We will revisit the model in the example section of this paper.

3. THREE IMPORTMANT RELIABILITY MEASURES

Three reliability measures are usually considered for assessment during burn-in. These are failure rate($(h(t))$, mean residual life($m(t)$), and conditional reliability ($R(x|t)$). Failure rate ($h(t)$) is interpreted as the instantaneous failure rate at time t. Mean residual life($m(t)$) is the average lifetime remaining after burn-in to time t. The conditional reliability determines the probability of survival of any unit in undertaking a new mission given the unit already survived the burn-in period to time t. Let X be the random life of a component, sub-system, or system with ($f(t)$) its density. Then, the common definitions are:

mean residual life:

$$m(t) = \int_t^{\infty} \frac{R(u)}{R(t)} du \tag{3.1}$$

failure rate:

$$h(t) = \frac{f(t)}{R(t)} = -\frac{1}{R(t)}\left[\frac{d}{dt}R(t)\right] \tag{3.2}$$

conditional reliability:

$$R(x|t) = \exp\left[-\int_t^{x+t} h(u)du\right] \tag{3.3}$$

where reliability: $R(t) = \int_x^\infty f(u)du = \exp\left[-\int_0^\infty h(u)du\right]$, $R(x) > 0$ and $R(t) > 0$.

Traditionally, burn-in is used to maximize the mean residual life of devices. Since we are interested in finding the burn-in time t at which $m(t)$ attains its maximum, let us study same properties of mt which will be needed later. From equation (3.1), the first derivative of $m(t)$ is obtained as follows:

$$\begin{aligned} m'(t) &= \frac{\partial}{\partial t}\left[\frac{1}{R(t)}\int_t^\infty R(u)du\right] \\ &= h(t)\int_t^\infty \frac{R(u)}{R(t)}du - 1 \\ &= \frac{1}{R(t)}\left[h(t)\int_t^\infty R(u) - R(t)\right] \end{aligned} \tag{3.4}$$

To simplify equation (3.4), let

$$L(t) = h(t)\int_t^\infty R(u)du - R(t) \tag{3.5}$$

Then, the first derivative of m(t) can be expressed as follow:

$$m'(t) = \frac{1}{R(t)}L(t) \tag{3.6}$$

Thus, the first derivative of mean residual life depends on the sign of $L(t)$ since the reliability function is always greater than (or equal to) zero. In other words, if $L(t) > 0$, then $m'(t) > 0$ which implies that mean residual life ($m(t)$) is an increasing function. $L(t)$ is a just function defined in equation (3.5). We introduce the function, $L(t)$, here to use it for examining the relation between mean residual life and failure rate under the modified bathtub curve. In equation (3.5), if $t \to \infty$, we have

$$L(t=\infty) = Lim_{t\to\infty}\left[h(t)\int_t^\infty R(t)du - R(t)\right] = 0 \tag{3.7}$$

Let us define the property of $L(t)$. From equation (3.5), the first derivative of $L(t)$ is

$$\begin{aligned} L'(t) &= \frac{\partial}{\partial t}\left[h(t)\int_t^\infty R(u)du - R(t)\right] \\ &= h'(t)\int_t^\infty R(u)du + h(t)(-R(t)) + f(t) \\ &= h'(t)\int_t^\infty R(u)du \end{aligned} \quad (3.8)$$

From equation (3.8), we see that the sign of $L'(t)$ is the same as that of $h'(t)$. Thus, if $h'(t) > 0$, then $L'(t) > 0$ and so $L(t)$ is an increasing function. If $h'(t) < 0$, then $L'(t) < 0$ and so $L(t)$ is a decreasing function.

In the next section, we will define the relationship between mean residual life ($m(t)$) and failure rate ($h(t)$) under the modified bathtub curve using equations (3.6), (3.7), and (3.8).

4. $M(t)$ AND $h(t)$ UNDER MODIFIED BATHTUB CURVE

Now, let us compare mean residual life($m(t)$) and failure rate($h(t)$) for the modified bathtub curve. Figure 4.1 presents the relationship between $m(t)$ and $h(t)$. We consider the curve in Figure 1.1 from right to left.

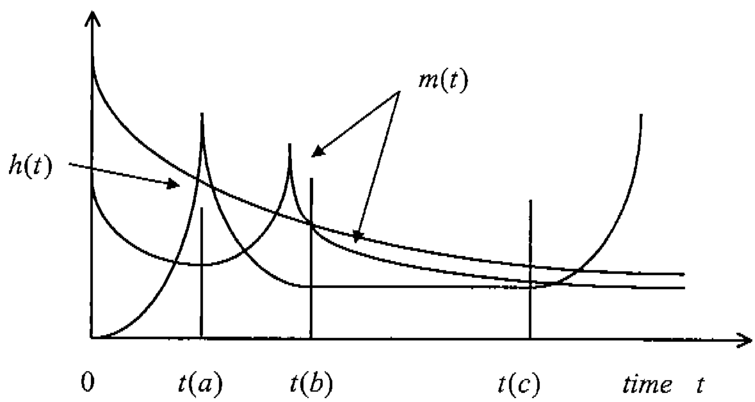

Figure 4.1: The Relationship between and under the Modified Bathtub Curve.

Note: If there is not one solution of $L(t) = 0$ in the early failure region, the maximum $m(t)$ is attained at burn-in time $t = 0$ and $m(t)$ is decreasing function.

- In wearout region (between $t(c)$ and ∞ in Figure 1.1), the failure rate is increasing. Thus, $h'(t) > 0$. This implies that $L'(t) > 0$ from equation (3.8) and so $L(t)$ is also increasing, however, $L(t) < 0$ where $t \in [t(c), \infty]$ since $L(\infty)$ by equation (3.7). Then, we have $m'(t) < 0$ from equation (3.6). This implies that $m(t)$ is a decreasing function in wearout region.

- In the useful region (between $t(b)$ and $t(c)$ in Figure 1.1), the failure rate is constant. This implies that $h'(t)$. Then, we have $L'(t) = 0$ from equation (3.8). Since $L'(t) = 0$, $L(t)$ where $t \in [t(b), t(c)]$ is constant but smaller than 0 because $L(t(c)) < 0$. Then, we have $m'(t) < 0$ from equation (3.6). This implies that $m(t)$ is a decreasing function in the useful region.

- In the early failure region (between $t(a)$ and $t(b)$ in Figure 1.1), the failure rate is decreasing. Thus, $h'(t) < 0$ which implies that $L'(t) < 0$ by equation (3.8). Then, $L(t)$ where $t \in [t(a), t(b)]$ is a decreasing function. Since $L(t(b)) < 0)$, it might be possible to have one solution of $L(t) = 0$ or L(t)¡0.

  First, consider the case when $L(t) = 0$. Then, $m'(t) = 0$ from equation (3.6). There is a t satisfying $m'(t) = 0$ and $m''(t) < 0$ since $m''(t) = m(t)h'(t) + m'(t)h(t)$. Thus, $m(t)$ is quasi-concave in this region and the relative maximum mean residual life $(m(t))$ lies between $t(a)$ and $t(b)$ in Figure 1.1.

  For the second case when $L(t) < 0$, $m'(t) < 0$ from equation (3.6). Then, $m(t)$ is a decreasing function in the early failure region.

- In the very early failure region (between 0 and $t(a)$ in Figure 1.1), the failure rate is increasing. Thus, $h'(t) > 0$. This implies that $L'(t) > 0$ and so $L(t)$ where $t \in [0, t(a)]$ is increasing function. Since $L(t(a)) > 0$ or $L(t(a)) < 0$, it might be possible to have one solution of $L(t) = 0$ or $L(t(a)) < 0$. Then, we have two possible cases.

  For the first case when $L(t) = 0$, $m'(t) = 0$ from equation (3.6), there is a t satisfying $m'(t) = 0$ and $m''(t) > 0$ since $m''(t) = m(t)h'(t) + m'(t)h(t)$. Thus, $m(t)$ is quasi-convex in this region. Then, the relative minimum $m(t)$ lies between 0 and $t(a)$.

  For the second case, $L(t) < 0$, $m'(t) < 0$ from equation (3.6). Then, $m(t)$ is a decreasing function in the very early failure region.

From the above results, we conclude that the burn-in times for optimum $m(t)$ and $h(t)$ are different. As shown in Figure 4.1, the maximum $m(t)$ is attained at burn-in time between $t(a)$ and $t(b)$ (or at burn-in time $t = 0$) while minimum failure rate is attained at the time $t(b)$ (or $t \in [t(b), t(c)]$). The relation between mean residual life ($m(t)$) and failure rate ($h(t)$) is presented in Figure 4.1.

5. . $R(x|t)$ AND $h(t)$ UNDER THE MODIFIED BATHTUB CURVE

The optimal burn-in times for conditional reliability ($R(x|T)$) and failure rate ($h(t)$) are different for the modified bathtub curve. If a unit has already accumulated t hours of burn-in and its reliability for a new mission of x hours duration is desired, this reliability is a conditional reliability, i.e., conditional to having surviving the first t hours to be able to undertake the new mission. The conditional reliability in equation (3.3) is defined as follow:

$$\begin{aligned} R(x|t) &= \exp\left(-\int_t^{x+t} hu)du\right) = \exp\left(-\int_t^0 h(u)du - \int_0^{x+t} h(u)du\right) \\ &= \left(\frac{1}{\exp\left(-\int_t^0 h(u)du\right)}\right)\left(\exp\left(-\int_0^{x+t} h(u)du\right)\right) \\ &= \left(\frac{1}{R(t)}\right)(R(x+t)) \qquad (5.1) \end{aligned}$$

Equation (5.1) implies that the reliability of a new mission of x hours duration, having already operated successfully for t hours of burn-in is equal to the probability of surviving of $x + t$ hours of operation divided by the probability of surviving t hours. Thus, conditional reliability is defined by two reliability functions, $R(t)$ and $R(x + t)$.

Now, we need to study some property of $R(x|t)$ to find a burn-in time t at which $R(x|t)$ attains its maximum. The first derivative of the conditional reliability in equation (5.1) is defined as follow:

$$\begin{aligned} R'(x|t) &= \left(\frac{1}{R(t)}\right)^2 (R(t)R'(x+t) - R(x+t)R'(t)) \\ &= \left(\frac{1}{R(t)}\right)^2 \left(\frac{1}{h(x+t)} - \frac{1}{h(t)}\right) R'(t)R'(x+t) \qquad (5.2) \end{aligned}$$

From equation (5.2), we see that the sign of first derivative of the conditional Reliability is the same as the sign of $\left(\frac{1}{h(x+t)} - \frac{1}{h(t)}\right)$ since $\left(\frac{1}{R(t)}\right)^2 > 0$. Then, we have the following results for when the mission time is fixed:

a. If $\left(\frac{1}{h(x+t)} - \frac{1}{h(t)}\right) > 0$, $R(x|t)$is monotone increasing function of t

b. If $\left(\frac{1}{h(x+t)} - \frac{1}{h(t)}\right) < 0$, $R(x|t)$is monotone decreasing function of t

c. The optimal $R(x|t)$is attained at $\left(\frac{1}{h(x+t)} - \frac{1}{h(t)}\right) = 0$

From the above results, we see that the conditional reliability ($R(x|t)$) is depend on the failure rate at time t ($h(t)$) and the failure rate at time $x + h$ ($h(x+t)$). If $h(t)$ is less than $h(x+t)$, the conditional reliability ($R(x|t)$) function is decreasing. This implies that no burn-in is necessary to improve the conditional reliability ($R(x|t)$). Under the modified bathtub curve, the conditional reliability ($R(x|t)$) varies depending on the mission duration x. However, the optimal burn-time for failure rate ($h(t)$) is the time $t(b)$ in Figure 1.1. Therefore, the optimal burn-in times for $h(t)$ and $R(x|t)$ are different under the modified bathtub curve.

6. $M(t)$ AND $R(x|t)$ UNDER MODIFIED BATHTUB CURVE

Under the modified bathtub curve, the optimal burn-in times for $m(t)$ and $R(x|t)$ are also different. Conditional reliability ($R(x|t)$) depends on mission time, say x, the failure rate at time t($h(t)$), and the failure rate at time $x + t(h(x+t))$. On the other hand, mean residual life ($m(t)$) does not depend on the mission time x. The mean residual life is only depending on the burn-in time t. If the mission time $x \in [0.t(c)]$ in Figure 1.1, there may exist one solution that optimal burn-in times for mean residual life and conditional reliability may be the same. However, the optimal burn-in times for mean residual life and conditional reliability are not the same for most cases when $x \in [0.t(c)]$.

7. AN ILLUSTRATIVE EXAMPLE

Suppose that the time-to-failures for 100 CMOS components are in Table 7.1. Then, graphical analysis techniques can be applied to the failure data to obtain estimates of the parameter values.

First, median ranks for the failures in Table 7.1 are calculated using following equation.

$$\text{Median Rank} = \frac{i - 0.3}{n + 0.4} \times 100$$

where $i = order\ number$ and $n = 150$.

Table 7.1: Time-To-Failures in Hours for CMOS Components

21.67	41.52	43.02	46.55	61.63	62.73	67.98	70.87	72.17	72.52
75.12	75.72	76.78	78.44	79.69	81.50	82.34	85.89	87.31	87.87
93.22	93.96	98.12	99.97	102.19	102.45	105.97	108.54	110.55	111.33
117.13	120.18	124.61	133.07	140.97	509.73	655.56	690.85	737.16	753.06
770.26	792.36	793.53	807.18	808.17	809.69	815.41	832.10	832.21	835.54
838.67	841.25	847.13	847.90	861.95	864.44	869.94	877.15	889.54	907.17
907.83	907.96	919.28	921.46	922.57	938.55	939.41	948.82	955.62	959.12
964.96	967.72	967.96	970.11	984.23	984.51	994.86	996.83	1005.88	1009.37
1014.32	1016.01	1019.67	1026.29	1031.53	1034.74	1037.58	1038.34	1041.07	1041.18
1049.78	1063.44	1063.69	1065.11	1111.81	1114.84	1121.76	1125.69	1150.24	1166.91

Next, using time-to-failures and median ranks, a Weibull probability plot is prepared in Figure 7.1. Since the time-to-failure pattern follows S-curve, we say that the components have weak components and main components (for more detail, see Jensen and Petersen (1982). The advantage of using a Weibull plot is that we can identify the proportion of weak population approximately just by looking at the Weibull plot. This is accomplished by determining the flat part of the curve on Weibull plot. The flat part of the curve indicates the proportion of weak population. When the curve shows truly flat part, the proportion of weak population can be read off on the Weibull ordinate as a percentage.

Applying Jensen and Petersen (1982) method to the Weibull plot in Figure 7.1, we find that the time-to-failure pattern of the data follows two-mixed Weibull distribution law and estimated parameter values as following:

$$\hat{p} = 0.3,\ \hat{\beta}_1 = 3.75,\ \hat{\eta}_1 = 100,\ \hat{\beta}_2 = 9.5,\ \hat{\eta}_2 = 1000.$$

Form the estimated parameter values, we can estimate the reliability measures. Table 7.2 presents the estimated reliability measures for selected time steps. The optimal values and corresponding burn-in times for the reliability measures are determined from the Table 7.2 directly; i.e., find the optimal values of the reliability measures, then read the corresponding burn-in time from the first column. These are shown in bold type.

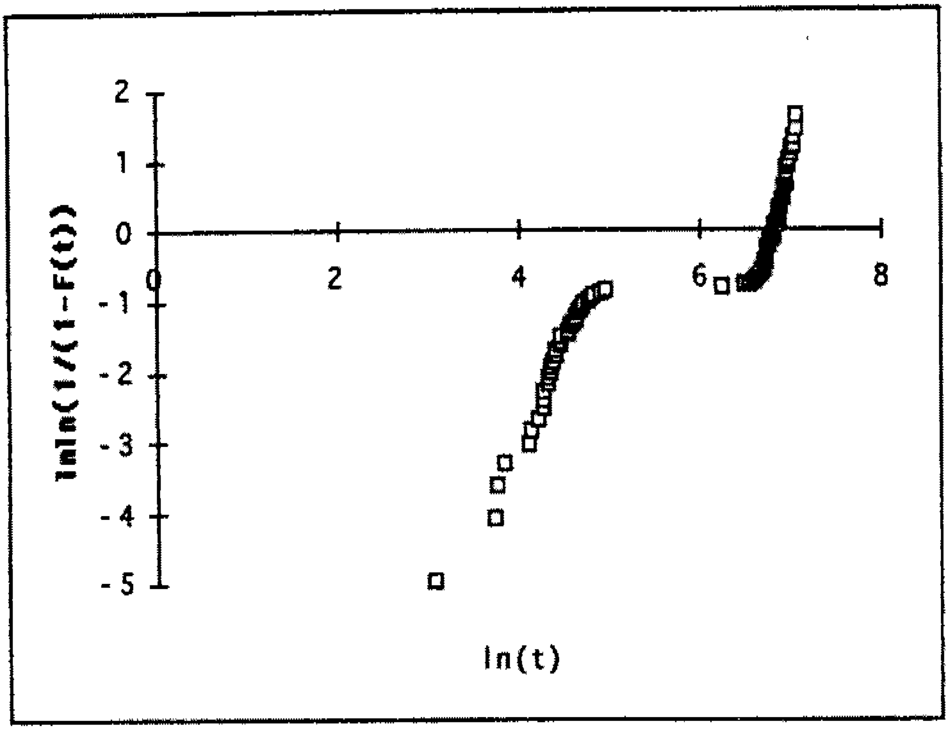

Figure 7.1: Weibull Probability Plot for the Example

Table 7.2: The Estimated Values of $h(t)$, $m(t)$ and $R(x|t)$ for the Example Problem (truncated at burn-in time 400hours)

t	$h(t)$	$R(x=200\|t)$	$R(x=500\|t)$	$R(x=900\|t)$	$m(t)$
0	0	0.6500	0.6491	**0.4501**	636.23
50	0.0018578667	0.6667	0.6644	0.3607	611.15
100	0.0062001531	0.8346	0.8281	0.3070	714.03
150	0.0006314794	0.9944	**0.9780**	0.2028	**782.14**
200	0.0000002058	**0.9998**	0.9668	0.0843	732.14
250	**0.00000007**	0.9994	0.9370	0.0229	682.14
300	0.0000003413	0.9986	0.8868	0.0035	632.14
350	0.0000012656	0.9966	0.8077	0.0002	582.14
400	0.0000039376	0.9923	0.6925	0	532.14

From Table 7.2, the optimal value for the failure rate ($h(t)$) is the 0.00000007. Thus, the corresponding burn-in time is the 250 hours. For the mean residual life ($m(t)$), we have the optimum value at burn-in time 150 hours. The optimal burn-in times for the conditional reliability ($R(x|t)$) do not coincide since $R(x|t)$ depend on the new mission time x. $R(x = 200|t)$ has its optimum at burn-in time 200 hours, $R(x = 500|t)$ has its optimum at burn-in time 150 hours, and $R(x = 900|t)$ has its optimum at burn-in time zero. Therefore, optimal burn-in times for the three reliability measures are different for this case and are different in all cases where the modified bathtub curve represent the time-to-failures.

The relationship among three reliability measures for this example is displayed graphically in Figure 7.2.

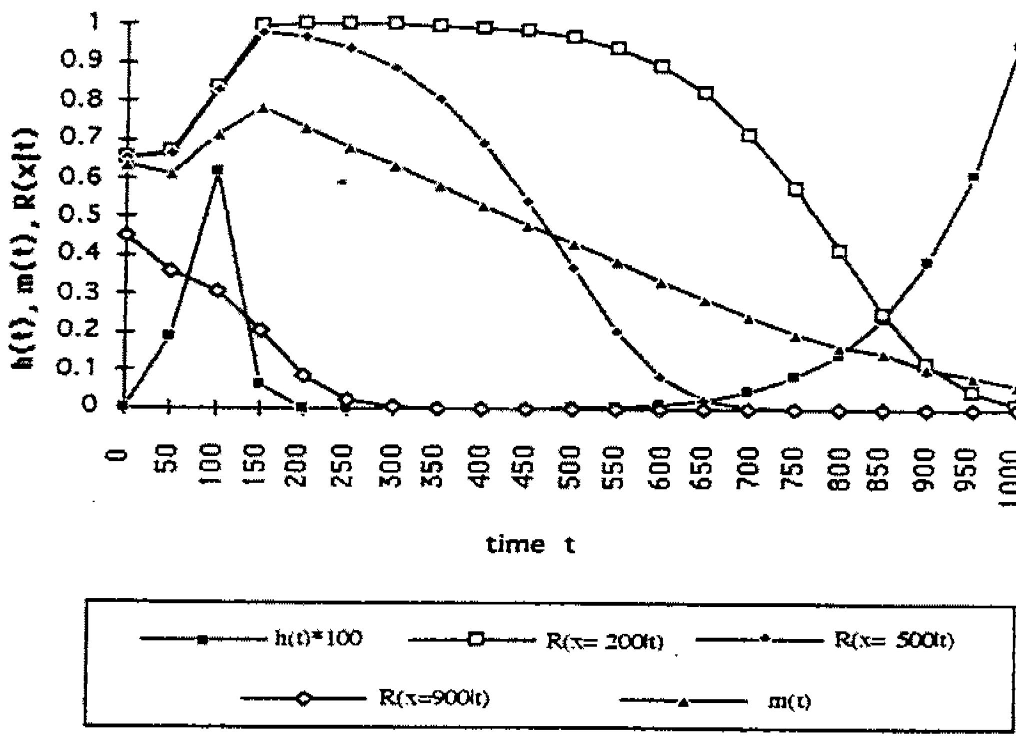

Figure 7.2: The Relation among Three Reliability Measures for the Example

At the beginning, the failure rate ($h(t)$) is increasing up to 100 hours in Figure 7.2. Thereafter the failure rate follows the usual "bathtub" curve. Therefore, the failure rate function of two-mixed Weibull distribution follows a modified bathtub curve. The mean residual life is increasing up to 150 hours of burn-in time. Thereafter, it is decreasing. The conditional reliabilities in Figure 7.2 shows different curves by depending on the mission time x. For $R(x = 900|t)$, the conditional reliability has a decreasing function from the beginning. For $R(x = 200|t)$ and $R(x = 500|t)$, the conditional reliabilities are increasing up to burn time 350 hours. Then, the conditional reliabilities are decreasing.

From Figure 7.2, we also can see that the optimal burn-in times for three measures are not the same for the failure rate function that follows the modified bathtub curve.

8. DISCUSSION

The relationship among three common measures of reliability; namely failure rate ($h(t)$), mean residual life ($m(t)$), and conditional reliability ($R(x|t)$) has been discussed and mathematical graphical analyses are performed for the relationship under the modified bathtub curve. The results of mathematical and graphical analyses show that the burn-in time that optimizes one reliability measure does not yield an optimal value for another measure when a failure rate function follows the modified bathtub curve. Therefore, identifying the right measure for a particular product should be done as a first step before planning a burn-in procedure. If the objective is to improve the average life of a product, then the mean residual life ($m(t)$) is the right choice for burn-in. If a manufacturer wants to minimize the early failures, then the failure rate ($h(t)$) is the relevant measure. If the goal is to improve reliability for mission time x (fixed length), then the conditional reliability ($R(x|t)$) is the relevant measure to be considered for the burn-in procedure.

In practice, it is not unusual to have mixed population (weak population and main (strong) population) for many electrical or electro-mechanical products for their life. However, products have different reliability objectives. For example, failure rate ($h(t)$) or conditional reliability ($R(x|t)$) will be a required objective for most CMOS components. For the electro-mechanical product such as an oral irrigator (Boardman and Colvert, 1978), the mean residual life ($m(t)$) is the objective to be maximized. For a process with weak population, we recommend that the first step of burn-in should be identifying the most appropriate reliability measure.

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