해저 관로의 일점 상승에 의한 대변형

Large Deflection of Subsea Pipeline due to One Point Lifting

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요 지

일반 해양 구조물이나 해저면에 설치되는 해저 관로는 외력에 의한 변형이 발생된다. 구조물 형상이 복잡하거나, 구성 요소의 개수가 많을 경우 응력해석 시 많은 초기값이 필요하고 해석 시간 또한 장 시간 소요된다. 해양 구조물에 작용하는 대표적인 외력은 파도, 조류, 바람이고 이런 외력은 구조물의 사용 기간(operation life)동안 계속적으로 작용하기 때문에 구조물의 변형율은 항상 허용치 안에서 발생되도록 설계 되야 한다. 허용 변형은 탄성범위 내에 존재해야 하며, 비교적 큰 변형을 일으키는 구조물이나 해저파이프라인의 응력해석을 수치적으로 접근하는 방법을 고찰하였다. 평행상태의 하중 벡터값만 직각 좌표계에서 인트린직(intrinsic) 좌표로 변환시킬 때 변형이 발생함으로, 본 논문에서 소개하는 이차 요소(quadratic element)방법을 사용할 경우 수치해석 시 많은 장점이 있다는 것을 보여준다. 본 방법을 도입함으로써 비교적 큰 변형이 발생되는 구조물 해석 시 일반 수치해석 방법과 그 결과는 같으나 해석 시간을 단축시킬 수 있다는 장점이 있다. 응력 해석 시 국부 강도 행열(element stiffness matrix)은 방향과 무관하며 이차요소 방법을 사용하여 각 요소 벡터를 발생시켰다. 해저관로 일점 상승 시 관로에 작용하는 변형과 상승력에 따른 휨 모멘트를 산출하여 일반적으로 사용되는 선형이론과 비교하였다.

핵심용어 : 인트린직 좌표 요소, 해저관로, 대변형, 이차행렬

Abstract

A numerical method for solving large deflections of elasticas to general structures and offshore pipelines is investigated. The method is based on finite element analysis using intrinsic coordinates in which element stiffness is independent of element orientation and the displacement vector is expressed in terms of nodal values of cross-sectional rotation. The generation of the vector of integrating coefficients for numerical integration adopts quadrature method, which is subsequently assembled into quadrature matrix. The transformations that are required from intrinsic to Cartesian coordinates affect only the load vector in the equilibrium equation. From this technique, significant computational advantages can be expected from the intrinsic coordinate formulation, particularly for large deflection problems. The method is applied to analyze the offshore pipeline when lifted by a single point.

Keywords : *Intrinsic coordinate elements, offshore pipelines, large deflection, quadrature matrix.*

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1. INTRODUCTION

Offshore structures including subsea pipeline are subjected to external loads mainly caused by environment in the installed site. Generally the major loads to offshore structures are from wave, current and wind. Since offshore structures are to be functional during design life, they are designed to sustain continuous stresses in the operation period. The wave load is characterized as cyclic load and the current and wind loads are not linear. Whatever the type of loads are, the structures are designed to permit the external loads within the allowable strain. The allowable strain should be inside the elastic deflection range.

Offshore pipelines are subjected to current and wave induced forces after installation. The stress and deflection of subsea pipeline in contact with seabed are not significant. However, offshore pipelines are experienced with large deflections during installation. Large deflections also can occur to large offshore structures like platforms, mono-pods in relatively deep water. The pipelines under large deflection and with small stain are investigated in the paper that are defined as a classical elastica problem where axial and shear deformations are neglected. The equations for large deflection are nonlinear and the length of the suspended pipeline during installation is not known a priori. Numerical FEM (finite element method) are often used to solve these problems. FEM usually refers the load-displacement interaction to a Cartesian coordinate system. In this paper, the displacement field specified in terms of cross sectional rotation Ψ(s) is applied. The s is measured along the deformed axis of an element. This method is explored for a variety of elastica examples, including pipeline problems.

The vector of integrating coefficients for numerical integration is generated with quadrature method, which is subsequently assembled into quadrature matrix. Considering the fact that the load vector in the equilibrium equation is affected in the transformations from intrinsic to Cartesian coordinates, a quadrature matrix can be originated. From this technique, significant computational advantages can be obtained from the intrinsic coordinate formulation, especially for large deflection problems. The technique is applied to analyze the offshore pipeline as lifted by a single point.

2. THE FINITE ELEMENT FORMULATION AND EQUILIBRIUM EQUATION

The present development is based on the assumption that the element is elastic in bending and can undergo large displacements with small strain. In this approach, axial and shear deformations are neglected by Frisch-Fay[1]. The displacement field of an element is specified in terms of the cross sectional rotation Ψ(s), where s is measured along the deformed axis of the element. The coordinates (s, Ψ) are referred herein as an intrinsic coordinates by Wang[2].

The strain energy due to bending, U^e of an element along length is :

$$U^e = \int_0^l \frac{EI}{2}\left[\frac{d\Psi}{ds}\right]^2 ds \quad (1)$$

In equation (1), EI represents a flexural rigidity. The displacement field Ψ(s) is defined by a continuous polynomial, so that the rotations within any element may be interpolated from the nodal values on that element. Nodes are positioned at each end of an element. The quality of nodal values will satisfy inter-element compatibility. A two-node element is therefore the minimum configuration to satisfy compatibility. That element may be called a linear element, since the rotation varies linearly. Additional internal nodes of rotation can also be generated within the element for more accurate representation of displacement field. In this paper, a three-node element is called as a quadratic element and a four-node element is a cubic element.

The polynomial $\Psi(s)$ may be expressed as a Lagrangian interpolation polynomial $\Psi(s)=[\beta]\{\bar{\theta}\}$, where $[\beta]$ is the coefficient function and $\{\bar{\theta}\}$ the vendor of nodal rotations. Substituting Lagrangian interpolation polynomial into (1) gives:

$$U^e \approx \frac{1}{2}\{\bar{\theta}\}^T\left[\int_0^l EI[\beta]^T[\beta]ds\right]\bar{\theta} \approx \frac{1}{2}\{\bar{\theta}\}^T[\overline{K}]\{\bar{\theta}\} \quad (2)$$

where the element matrix $[\overline{K}]$ represents the integral term. The $[\overline{K}]$ matrices, for the linear, quadratic and cubic elements, respectively, are :

$$\frac{EI}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\frac{EI}{3l}\begin{bmatrix} 7 & -8 & 1 \\ & 16 & -8 \\ sym & & 7 \end{bmatrix}$$

$$\frac{EI}{l}\begin{bmatrix} 3.7 & -4.725 & 1.35 & -0.325 \\ & 10.8 & -7.425 & 1.35 \\ & & 10.8 & -4.725 \\ sym & & & 3.7 \end{bmatrix} \quad (3)$$

The strain energy of an elastica is the sum of the strain energies of component elements, i.e $U=\Sigma U^e=\frac{1}{2}\{\theta\}^T[K]\{\theta\}$, where $\{\theta\}$ and $[K]$ are assembled from the element displacement vectors and $[\overline{K}]$ matrices respectively. The work product, W, due to external point loads, P_i, uniformly distributed load, w and a moment, M_r, may be expressed as:

$$W=\Sigma P_i d_i + \int w\nu\, ds + M_r\theta_r \quad (4)$$

where, d_i is the conjugate displacement and ν the vertical displacement. If the load P_i is resolved horizontally and vertically, with ϕ the angle of inclination of P_i, equation (4) can be expressed as,

$$W=\Sigma P_i\cos\phi u_i + \Sigma P_i\sin\phi\nu_i + w\int\nu ds + M_r\theta_r \quad (5)$$

where u_i is the conjugate horizontal displacement. Now,

$$u_i=\int_0^{s_i}(\cos\Psi-1)ds=\int_0^{s_i}\cos\Psi\, ds - s_i \quad (6)$$

and

$$\nu_i=\int_0^{s_i}\sin\Psi\, ds \quad (7)$$

Such integrals can be evaluated numerically, using nodal values of the integrand.

$$\int_0^{s_i} f(s)ds \approx [C(0,\ i)]\{f\} \quad (8)$$

where $[C(0,\ i)]$ is a vector of coefficient, depending on i, and $\{f\}$ is the vector of nodal values of $f(s)$. The calculation of the vector $[C(0, i)]$ for the quadratic element is discussed as follows:

For the case of quadratic elements discussed in this paper, consider nodes O to n where n is even. Let

$$s_{2j-1}-s_{2j-2}=s_{2j}-s_{2j-1}=\frac{l_j}{2} \tag{9}$$

Based on a parabolic approximation of the integral $f(s)$,

$$\int_{s_{2j-2}}^{s_{2j-1}} f(s)ds=I(2j-2,2j-1)$$

$$\approx\frac{l_j}{24}[5f_{2j-2}+8f_{2j-1}-f_{2j}]$$

$$\int_{s_{2j-2}}^{s_{2j}} f(s)ds=I(2j-2,2j)$$

$$\approx\frac{l_j}{6}[f_{2j-2}+4f_{2j-1}+f_{2j}] \tag{10}$$

If $I(O, k)=\int_{s_0}^{s_k} f(s)ds$, then the integral approximations are,

$$I(O,2j-1)=I(O,2j-2)+(2j-2,2j-1)$$

$$\approx[C(0,2j-1)]\{f\}$$

$$I(O,2j)=I(O,2j-2)+I(2j-2,2j)$$

$$\approx[C(0,\ 2j)]\{f\} \tag{11}$$

Details on generating the integrating coefficients for linear and cubic elements can be found in Ngiam[3]. Using these integrating vectors and letting $[D_i]=[C(0, i)]$, equations (6) and (7) become:

$$u_i=[D_i]\{\cos\theta\}-s_i$$

$$\nu_i=[D_i]\{\sin\theta\} \tag{12}$$

The term $\int \nu\, ds$ in (5) can also be evaluated numerically,

$$\int_0^{s_N} \nu ds=[C(0,\ N)]\ \{\nu_i\}$$

$$=[C(0,\ N)]\{[D_i]\{\sin\theta\}\}$$

$$=[E]\{\sin\theta\} \tag{13}$$

Substituting (12) and (13) into (5),

$$W=\Sigma P_i\cos\phi_i([D_i]\{\cos\theta\}-s_i)$$

$$+\Sigma P_i\sin\phi_i[D_i]\{\sin\theta\}+w[E]\{\sin\theta\}+M_r\theta_r$$

$$=[G]\{\cos\theta\}+[H]\{\sin\theta\}+M_r\theta_r+constant \tag{14}$$

where $[G]$ and $[H]$ are coefficient vectors. It will be noted that W is a linear function of $\{\sin\theta\}$ and $\{\cos\theta\}$ for any loading. The total potential energy of the system is $\Pi=U-W$. Considering a variation $\delta\Pi$, it can be shown that for an elastic system, the equilibrium state corresponds to a minimum in the total potential energy as shown in Bathe et al.[4]. Hence, after partial differentiation the equilibrium equation is:

$$[K]\{\theta\}-\{Q\}=0 \tag{15}$$

where $\{Q\}$ is the load vector which is obtained by partial differentiation of (14) with respect to θ_i.

$$Q_i=G_i\sin\theta_i+H_i\cos\theta_i+M_r\delta_{ri} \tag{16}$$

For a structural system where lengths of members are usually given and fixed, the $[K]$ in (15) is constant and symmetric, and only the load vector is a function of nodal rotations, $\{\theta\}$. Consequently, this will offer significant reduction of computational effort in the repeated solutions required for non-linear problems.

3. NUMERICAL EXAMPLE

Single Point Lift of a Continuous Pipeline

Lifting a continuous pipeline is very important and significant problem in repair, tie-in and installation of pipelines. Depending on water depth and pipe specification, the allowable lifting height is to be decided. At any case, the applied stress should be within allowable range. For tie-in operation, the pipeline should be lifted enough to provide sufficient clearance for pipeline welding.

In the analysis, a pipeline resting on a level at rigid seabed is lifted by a vertical force, $2P$. It is assumed that the pipeline will slide without frictional resistance. Th curvature of the pipeline at seabed lift-off point equals zero. For small deflections, the pipeline lifted length, $2L$, the lifted height at lift point, d and the corresponding moment, M_n can be shown to be as follows:

$$L=\frac{3P}{2w}\ ;\ d\frac{wL^4}{72EI}\ ;\ M_n=\frac{wL^2}{6} \tag{17}$$

For large deflections, the numerical solution based on intrinsic coordinate elements considers only half of lifted pipeline due to symmetry. Boundary conditions of $\Psi_0=\Psi_n=0$ can be used as constraint equations. Corresponding Lagrangian multipliers are M_0 and M_n. Using the lift off point at seabed of 0 as the origin, equilibrium equations are obtained after partial differentiation of total potential energy function with respect to nodal rotations as,

$$\sum_j K_{ij}\Psi_i-(wE_i+PC_i)\cos\Psi_i$$
$$+M_0\delta_{oi}+M_n\delta_{ni}=0 \tag{18}$$

$$\Psi_n=0 \tag{19}$$

$$\Psi_n=0 \tag{20}$$

where $i=j=0,\ 1,\ 2,\ \ldots,\ n$. Equilibrium equations plus a boundary condition of $M_0=0$ constitute a set of n+4 equations, represented by

$$R_k(\Psi,\ M_0,\ M_n,\ L)=0\ ;$$
$$k=0,\ 1,\ 2,\ \ldots,\ n+3 \tag{21}$$

For an approximate solution, the values of R_k are known as residuals. The Newton-Raphson procedure is applied for the solution of residual equations and it requires starting values of the variables. Since the pipeline is lifted gradually from a flat seabed, the starting values for nodal rotations may be assumed zero. The starting values of lifted length, L and the moment, M_n can be established based on the small deflection solutions in (17).

The dimensionless terms of characteristic length, lifted length, lift force, moment and lift height are considered in the solution,

$$\hat{L}=\left(\frac{EI}{W}\right)^{1/3}$$

$$\overline{L}=\frac{L}{\hat{L}}$$

$$\overline{P}=\frac{P}{w\hat{L}}$$

$$\overline{M}=\frac{M}{w\hat{L}^2}$$

$$\overline{Y}=\frac{y}{\hat{L}} \qquad (22)$$

Large deflection problems have been previously solved by Kooi and Kuipers[5] using a finite difference method. Some of solutions using quadratic elements are presented in Table 1 and compared with theirs. In Table 1, the results presented are based on a dimensionless lift force $2\overline{P}=20$, which is the greatest load adopted by Kooi and Kuipers[5]. The deflection due to this load is extremely large. The results are obtained by quadratic elements of equal length and four incremental load steps.

Kooi and Kuipers[5] obtained the values quoted above by Richardson extrapolation, from solutions using 333 and 653 unknowns. Very similar results have been obtained here

Table 1 Singlf point lift of a pipeline (dimensionless lift force of 20)

number of elements	half length $\overline{L}$	lifted height $\overline{Y}$	bending moment $\overline{M}_n$
4	11.258732	9.763271	5.914088
8	11.199737	9.733902	4.491175
16	11.202644	9.806287	4.346398
32	11.202700	9.801202	4.416090
Kooi & Kuipers[5]	11.202701	9.811415	4.431179
Linear theory	15	703.125	37.5

using only 32 elements or 68 equations. Their values of 11.20300 and 11.202701 for the dimensionless half length using 333 and 653 equations respectively, a difference of 0.0003. It can be noted that the difference is less than 0.0001 for the solutions using 16 and 32 elements (see Table 1) or 36 and 68 unknowns respectively. Also it was reported by Ngiam[3] that for the dimensionless lift forces smaller than 4.0, the solutions are reasonably consistent with those based on linear theory equations.

The schematic diagram of applied forces on subsea pipeline segment is as shown in Figure 1 which shows moment, force, angle, weight and segment length. The notation in Figure 1 is not explained in detail since it is self-explainable. Figure 2 shows the modeling

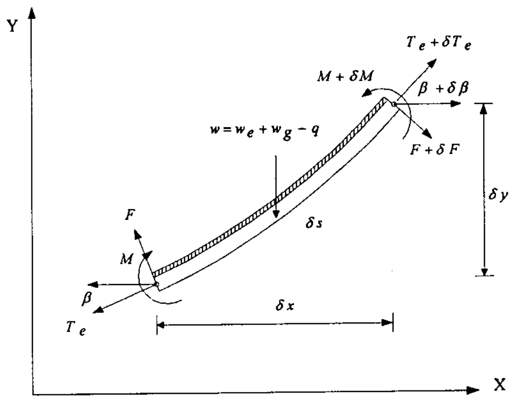

Fig. 1 Applied forces on subsea pipeline

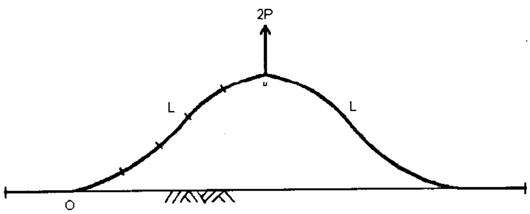

Fig. 2 Single point lift of a continuous pipeline

scheme for continuos pipeline being lifted by a single point with a 2P force. The deflected profiles of the pipeline under the dimensionless lift force of 5, 10, 15 and 20 are presented in Figure 3. Plots of the dimensionless lift force against the dimensionless bending moment at the lift point are shown in Figure 4 where P stands for lifting force and Mn represents bending moment. It shows clearly the difference between large deflection solution and linear solution. It explains that to obtain the accurate solution for non-linear case such as large deflection problems, the linear theory is not recommended. The lifting lengths subjected to lifting forces are indicated in Figure 5. As the lifting force increase the difference in lifted length also grows.

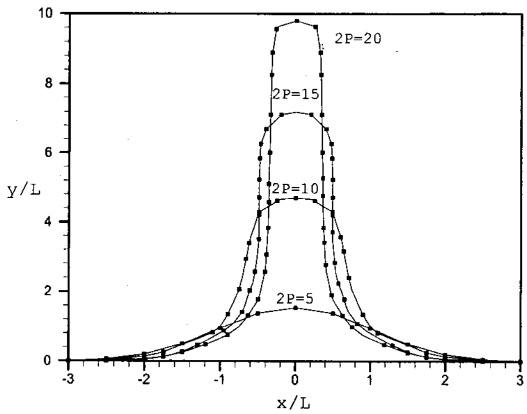

Fig. 3 Deflected gometry of a pipeline

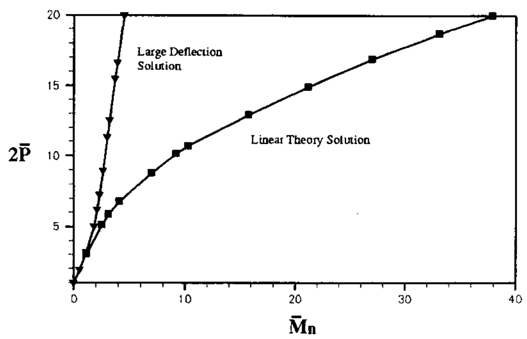

Fig. 4 Dimensionless lift force vs. bending moment at lift point

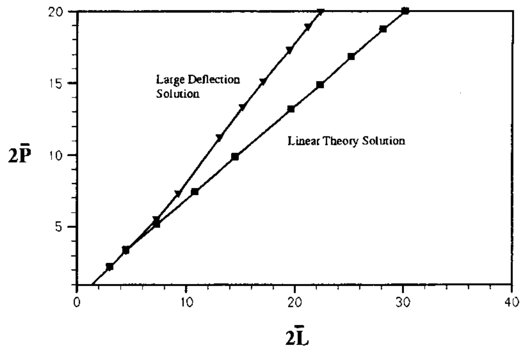

Fig. 5 Dimensionless lift force vs. lifted length

4. CONCLUSIONS

External forces acting on offshore structures create stress and strain to the members of the structure. The strain and stress should be within allowable values in elastic range. For offshore pipelines, the large deflection is generated during installation. The large deflection also occurs to long and large offshore platforms. To resolve this type of large deflection problems, a new finite element procedure has been developed and introduced in the paper. This technique can be effectively applied to offshore pipelines with the deformation defined in terms of nodal rotations.

Since the element stiffness matrix is independent of element orientation, it proved that the introduced technique could obtain a significant computational advantage. It was found that the only load vector in the equilibrium equation is affected in the transformation from Cartesian to intrinsic coordinates.

It shows clearly that there is significant difference between linear solution and large deflection solution. As introduced the paper, in the range of small lifting force, the difference in moment and lifting length is relatively small. However, as the lifting force increases, the gab also grows. This tendency agrees well with the results from established solutions for large deflection of elastica problems.

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