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자기상관함수의 비선형 유추 해석

Nonlinear Analog of Autocorrelation Function

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Abstract

Autocorrelation function is widely used as a tool measuring linear dependence of hydrologic time series. However, it may not be appropriate for choosing decorrelation time or delay time τ_d which is essential in nonlinear dynamics domain and the mutual information have recommended for measuring nonlinear dependence of time series. Furthermore, some researchers have suggested that one should not choose a fixed delay time τ_d but, rather, one should choose an appropriate value for the delay time window $\tau_w = \tau(m-1)$, which is the total time spanned by the components of each embedded point for the analysis of chaotic dynamics. Unfortunately, the delay time window cannot be estimated using the autocorrelation function or the mutual information. Basically, the delay time window is the optimal time for independence of time series and the delay time is the first locally optimal time. In this study, we estimate general dependence of hydrologic time series using the $C-C$ method which can estimate both the delay time and the delay time window and the results may give us whether hydrologic time series depends on its linear or nonlinear characteristics which are very important for modeling and forecasting of underlying system.

Keywords: autocorrelation function, correlation integral, delay time, delay time window

요 지

자기상관함수는 수문시계열의 선형상관 관계를 나타내는 척도로 널리 이용되고 있다. 그러나 비선형 동역학에서 필수적인 지체시간 또는 무상관시간 τ_d를 산정하는데는 적합하지 않을수도 있기 때문에 비선형 상관관계의 척도로 상호정보이론이 추천되어 왔다. 최근에 일부 학자들은 카오스 동역학 분석을 위하여 지체시간 τ_d 대신에 상태 공간상에 구축된 각 상태 벡타의 성분들의 총시간을 표시하는 지체시간창을 제안하였다. 그러나 지체시간창은 자기상관함수나 상호정보이론에 의해 추정될 수 없다. 기본적으로 지체시간창은 시계열 자료의 상관관계가 가장 작은 최적시간이며 지체시간은 국지적인 최소값 중 첫번째의 최적시간이다. 본 연구에서는 수문시계열의 지체시간과 지체시간창을 구하기 위하여 C-C방법이라는 기법을 이용하고, 여기에서 산정된 값들을 근거로 수문시계열의 모형화와 예측에 중요한 선형 또는 비선형 종속성을 파악하고자 한다.

핵심용어 : 자기상관함수, 상관적분, 지체시간, 지체시간창

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1. INTRODUCTION

Analysis of chaotic time series is common in many fields of science and engineering, and the method of delays has become popular for attractor reconstruction from scalar time series. From the attractor dynamics, one can estimate the correlation dimension and other quantities to see whether the scalar time series is chaotic or stochastic. Therefore, attractor reconstruction is the first stage in chaotic time series analyses. Since the choice of the delay time τ_d for attractor reconstruction using the method of delays has not been fully developed, many researchers use the autocorrelation function (ACF), which is computationally convenient and does not require large data sets. However, it has been pointed out that the ACF is not appropriate for nonlinear systems, and, instead, τ_d should be chosen as the first local minimum of the mutual information (Fraser and Swinney, 1986; Moon et al., 1995). Unfortunately, this approach is cumbersome computationally and requires large data sets (Tsonis, 1992).

According to Packard et al. (1980) and Takens (1981), the method of delays can be used to embed a scalar time series $\{x_i\}$, $i=1, 2, \ldots,$ into an m-dimensional space as follows :

$$\vec{x}_i = (x_i, x_{i+t}, \ldots, x_{i-(m-1)t}), \quad \vec{x}_i \in R^m, \tag{1}$$

where t is the index lag and R is real line. If the sampling time is τ_s, the delay time is $\tau_d = t\tau_s$. Takens' theorem assumes that we have an infinite noise-free data set, in which case, we can choose the delay time almost arbitrarily. However, since real data sets are finite and noisy, the choice of the delay time plays an important role in the reconstruction of the attractor from the scalar time series. If τ_d is too small, the reconstructed attractor is compressed along the identity line, and this is called *redundance*. If τ_d is too large, the attractor dynamics may become causally disconnected, and this is called *irrelevance* (Casdagli, 1991). However, Wu (1995) found that the embedding with the delay time which corresponds to a reconstructed phase diagram does not necessarily lead to a good convergence of the correlation dimension.

The alternative of fixing the delay time window $\tau_w = \tau(m-1)$ can be used for chaotic time series analysis, but the estimation of τ_w is less well developed. Martinerie et al. (1992) examined the delay time window and compared it with the delay times estimated using the ACF and the mutual information. They concluded that τ_w could not be estimated by either of these two methods. Basically, τ_w is the optimal time for independence of the data, but these methods estimate the first locally optimal time, which is τ_d. From this distinction between τ_d and τ_w, Kim et al. (1999a) developed a technique using the BDS statistic (Brock et al., 1991, 1996) originated from the correlation integral, which they called the C-C method, that can estimate both τ_d and τ_w. Brock et al. (1991, 1996), in their development of a test for nonlinearity in a time series, used the statistic $S(m,N,r) = C(m,N,r) - C^m(1,N,r)$, where $C(m,N,r)$ is the correlation integral (see Eq. (2)). Kim et al. (1999a) used the similar statistic $S(m,N,r,t) = C(m,N,r,t) = C^m(1,N,r,t)$, and examined its dependence on the index lag t.

Hydrologists and environmentalists have invoked low dimensional chaos for understanding the nature of hydrologic variables (Rodriguez-Iturbe et al., 1989; Sharifi et al., 1990; Wilcox et al., 1991; Sangoyomi et al., 1996; Puente and Obregon, 1996; Jeong and Rao, 1996), wastewater flows (Angelbeck and Minkara, 1994), and other technical issues (Ghliard and Rosso, 1990; Bormann and Kincanon, 1996; Fernandez and Garbrecht, 1996; Angelbeck and Minkara, 1996). Most of hydrologists and environmentalists have been used the ACF except for the study of Sangoyomi et al. (1996) which used the mutual information for the estimation of the delay time. However, Kim et al. (1998a) showed that, especially for small data sets, as the embedding dimension m is increased, the correlation dimension converges more rapidly for the case of τ_w held fixed than for the case of τ_d held fixed.

2. MEASURE OF GENERAL DEPENDENCE

2.1 Correlation Integral and BDS Statistic

After the attractor has been reconstructed using Eq. (1), quantitative properties of the chaotic system can be determined. The correlation dimension introduced by Grassberger and Procaccia (1983) is widely used in many fields for the quantitative characterization of strange attractors. The correlation integral for the embedded time series is the following function:

$$C(m,N,r,t) = \frac{2}{M(M-1)} \sum_{1 \le i < j \le M} \Theta(r - \| \vec{x}_i - \vec{x}_j \|), \qquad r > 0, \tag{2}$$

where

$$\Theta(a) = 0, \quad \text{if} \quad a < 0,$$
$$\Theta(a) = 1, \quad \text{if} \quad a \ge 0,$$

N is the size of the data set, $M = N - (m-1)t$ is the number of embedded points in m-dimensional space, and $\| \cdot \|$ denotes the sup norm. $C(m,N,r,t)$ measures the fraction of the pairs of points $\vec{x}_i$, $i = 1, 2, \ldots, M$, whose sup norm separation is no greater than r. If the limit of $C(m,N,r,t)$ as $N \to \infty$ exists for each r, we write the fraction of all state vector points that are within r of each other as $C(m,r,t) = \lim_{N \to \infty} C(m,N,r,t)$, and the correlation dimension is defined as $D_2(mt) = \lim_{N \to \infty} [\log C(m,r,t) / \log r]$. In practice, N remains finite, and thus, r cannot go to zero; instead, a linear region of slope $D_2(m,t)$ can be found in the plot of $C(m,N,r,t)$ vs. $\log r$.

Brock et al. (1991, 1996) studied the BDS statistic, which is based on the correlation integral, to test the null hypothesis that the data are independently and identically distributed (*iid*). This test has been particularly useful for chaotic systems and nonlinear stochastic systems. Under the *iid* hypothesis, the BDS statistic for $m > 1$ is defined as

$$\text{BDS}(m,M,r) = \frac{\sqrt{M}}{\sigma} [C(m,M,r) - C^m(1,M,r)], \tag{3}$$

this converges to a standard normal distribution as $M \to \infty$, and $\sigma^2 = \sigma^2(m,M,r)$ is the variance of BDS statistic. Note that the asymptotic variance $\sigma^2(m,M,r)$ can be estimated as

$$\sigma^2(m,M,r) = 4[m(m-1)C^{2(m-1)}(K - C^2) + K^m - C^{2m} + 2\sum_{i=1}^{m-1}\{C^{2i}(K^{m-i} - C^{2(m-i)}) - mC^{2(m-i)}(K - C^2)\}], \tag{4}$$

$$C(m,M,r) = \frac{2}{M(M-1)} \sum_{1 \le i < j \le M} \Theta(r - \| \vec{x}_i - \vec{x}_j \|), \tag{5}$$

$$K(m, M, r) = \frac{6}{M(M-1)(M-2)} \sum_{1 \le i < j < k \le M} [\Theta(r - \| \vec{x}_i - \vec{x}_j \|)\Theta(r - \| \vec{x}_j - \vec{x}_k \|)], \tag{6}$$

where $C = C(m, M, r)$ and $K = K(m, M, r)$. In addition, the t is varied for Eq. (2) but $t = 1$ for Eq. (5). The BDS statistic originates from the statistical properties of the correlation integral, and it measures the statistical significance of calculations of the correlation dimension. Even though the BDS statistic cannot be used to distinguish between a nonlinear deterministic system and a nonlinear stochastic system, it is a powerful tool for distinguishing random time series from the time series generated by chaotic or nonlinear stochastic processes. Its statistical properties, along with proofs, can be found in the literature (Brock et al, 1991, 1996).

2.2. *C-C* Method

The present study is concerned with the properties of $S(m, N, r, t) = C(m, N, r, t) - C^m(1, N, r, t)$. Brock et al. (1991) made a comment in their work: "If a stochastic process $\{x_i\}$ is *iid*, it will be shown that $C(m, r) = C^m(1, r)$ for all m and r. That is to say, the correlation integral behaves much like the characteristic function of a serial string in that the correlation integral of a serial string of independent random variables is the product of the correlation integrals of component substrings." This led us to interpret the statistic $S(m, N, r, t)$ as the serial correlation of a nonlinear time series. Therefore, it can be regarded as a dimensionless measure of nonlinear dependence, and it can be used to determine an appropriate index lag t. For fixed m, N, and r, a plot of $S(m, N, r, t)$ versus t is a nonlinear analog of the plot of autocorrelation function versus t.

In order to study the nonlinear dependence and eliminate spurious temporal correlations, the time series $\{x_i\}$, $i = 1, 2, \ldots, N$, are subdivided into t disjoint time series of size N/t. $S(m, N, r, t)$ is then computed from the t disjoint time series as follows: For $t = 1$, there exists single time series $\{x_1, x_2, \ldots, x_N\}$, and

$$S(m, N, r, 1) = C_1(m, N, r, 1) - C_1^m(1, N, r, 1). \tag{7}$$

For $t = 2$, two disjoint time series $\{x_1, x_3, \ldots, x_{N-1}\}$ and $\{x_2, x_4, \ldots, x_N\}$ are obtained, each of length $N/2$, and

$$S(m, N, r, 2) = \frac{1}{2}[\{C_1(m, N/2, r, 2) - C_1^m(1, N/2, r, 2)\} + \{C_2(m, N/2, r, 2) - C_2^m(1, N/2, r, 2)\}]. \tag{8}$$

For general t, this becomes

$$S(m, N, r, t) = \frac{1}{t}\sum_{s=1}^{t}[C_s(m, N/t, r, t) - C_s^m(1, N/t, r, t)]. \tag{9}$$

Finally, as $N \to \infty$,

$$S(m, r, t) = \frac{1}{t}\sum_{s=1}^{t}[C_s(m, r, t) - C_s^m(1, r, t)]. \quad m = 2, 3, \ldots \tag{10}$$

For fixed m and t, $S(m, r, t)$ will be identically equal to zero for all r if the data is *iid* and $N \to \infty$. However, real data sets are finite, and data may be serially correlated, so, in general, $S(m, r, t) \neq 0$. Thus, the locally optimal times may be either the zero crossings of $S(m,r,t)$ or the times at which $S(m,r,t)$ shows the least variation with r, since this indicates a nearly uniform distribution of points. Hence, several representative values r_j are selected, and the quantity $\Delta S(m,t)$ is defined as

$$\Delta S(m,t) = \max\{S(m,r_j,t)\} - \min\{S(m,r_j,t)\}. \tag{11}$$

The locally optimal times t are then the zero crossings of $S(m,r,t)$ and the minima of $\Delta S(m,t)$. In the first case, the zero crossings should be nearly the same for all m and r, and, in the second case, the minima should be nearly the same for all m (otherwise, the time is not locally optimal). The delay time τ_d will correspond to the first of these locally optimal times.

In determining the nonlinear dependence of a finite time series by using the statistic $S(m,N,r,t)$, one must have criteria for selecting the values of m and r. In addition, one must know the role of the sample size N. For a fixed value of N, as m becomes large, the data become very sparse, so that $C(m,N,r,t)$ becomes vanishingly small. Also, if r exceeds the size of the attractor, then $C(m,N,r,t)$ saturates, since most pairs of points are within the distance r. Thus, neither m nor r should be too large.

Brock et al. (1991) investigated the BDS statistic for time series generated from six asymptotic distributions in order to determine what values of m and r are appropriate. Time series with three sample sizes, $N=100$, 500, and 1000, were generated by Monte Carlo simulation from six asymptotic distributions: standard normal, student t with 3 degrees of freedom, double exponential, Chi square with 4 degrees of freedom, uniform distribution, and bimodal mixture of normals. These studies led to the conclusion that m should be between 2 and 5 and r should be between $\sigma/2$ and 2σ. In addition the asymptotic distributions were well approximated by finite time series with $N \geq 500$.

Thus, four values of r in the range $\sigma/2 \leq r \leq 2\sigma$, $r_1=(0.5)\sigma$, $r_2=(1.0)\sigma$, $r_3=(1.5)\sigma$, and $r_4=(2.0)\sigma$, are selected as representative values. Then the following averages of the quantities given by Eqs. (10) and (11) are defined as

$$\bar{S}(t) = \frac{1}{16}\sum_{m=2}^{5}\sum_{j=1}^{4} S(m,r_j,t), \tag{12}$$

$$\Delta\bar{S}(t) = \frac{1}{4}\sum_{m=2}^{5}\Delta S(m,t). \tag{13}$$

The first zero crossing of $\bar{S}(t)$ or the first local minimum of $\Delta\bar{S}(t)$ for finding the first locally optimal time for independence of the data are found, and this gives the time delay $\tau_d = t\tau_s$. The optimal time is found from the index lag t for which $\bar{S}(t)$ and $\Delta\bar{S}(t)$ are both close to zero. If the equal importance to these two quantities are assigned, then the minimum of the quantity

$$S_{cor}(t) = \Delta\bar{S}(t) + |\bar{S}(t)|, \tag{14}$$

can simply be found and this optimal time gives the delay time window $\tau_w = t\tau_s$.

3. APPLICATIONS TO HYDROLOGIC TIME SERIES

Researchers have suggested obtaining τ_d from the autocorrelation function, which is practically convenient, since the ACF exhibits both periodic trends and information dissipation revealed by its decrease with time. When the ACF decays exponentially with index lag, Tsonis and Elsner (1988) suggested selecting the index lag t as the time at which the ACF drops to $1/e$. Otherwise t should be chosen as either the first zero crossing or the first local minimum of the ACF, whichever occurs first (Holzfuss and Mayer-Kress, 1986, Graf and Elbert, 1990).

We apply the ACF and the C-C method to three hydrologic time series and a chaotic system for measuring general dependence and choosing τ_d and τ_w of underlying systems. Informations and time series plots on data sets are shown in Table 1 and Fig. 1. The ACFs and the estimated delay times for used data sets are shown in Fig. 2 and Table 2. As we can see the ACF in Fig. 2, a daily streamflow, GSL volume, and Lorenz time series show long persistence which may be associated with Hurst phenomenon. We generate the time series for the Lorenz system of three coupled differential equations (Lorenz, 1963):

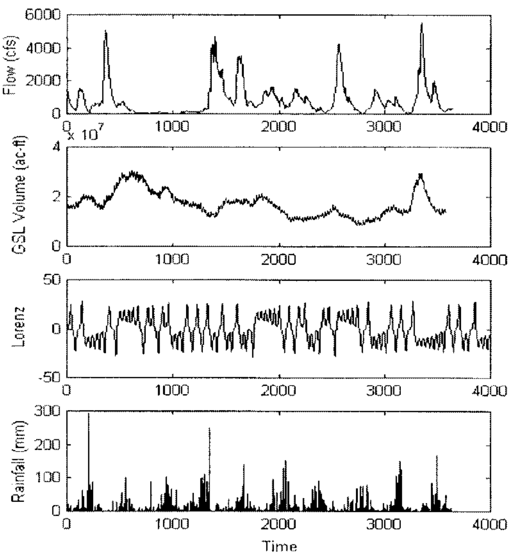

Figure 1. Time series plots for a streamflow, GSL volume, Lorenz system, and a rainfall.

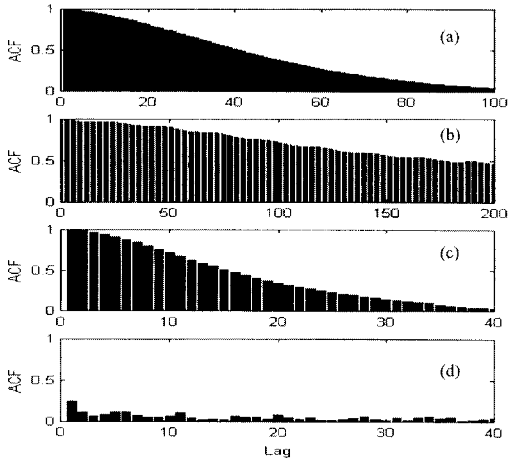

Figure 2. ACFs for (a) a streamflow, (b) GSL volume, (c) Lorenz system, and (d) a rainfall

Table 2. Estimated delay times using the ACF

Time series	Delay time	Selection point
Daily streamflow	51	1/e
GSL volume	14	First minimum
Lorenz x	19	1/e
Daily rainfall	3	First minimum

$$
\begin{aligned}
dx/dt &= -a(x-y),\\
dy/dt &= -xz + cx - y,\\
dz/dt &= xy - bz,
\end{aligned}
\qquad (15)
$$

where a, b, and c are constants. We solve this system of equations for $a = 16.0$, $b = 4.0$, and $c = 45.92$ to generate a time series of the

Table 1. Summary of three hydrologic and a chaotic time series

Country	Station	Time series	Record period	Data size
USA	St. Johns river near Cocoa, FL	Daily streamflow	1979-1988	3650
USA	Great Salt Lake, Utah	15 day volume	1847-1992	3578
	Lorenz system	x-variable		3650
Korea	Seoul	Daily rainfall	1987-1996	3650

variable x with $\tau_s = 0.01$. Lorenz time series is from chaotic system, the evidence of chaotic behavior for a daily streamflow and GSL volume data were proved by Kim et al. (1999b) and Sangoyomi et al. (1996), and these time series may be related to Hurst phenomenon (Kim et al., 1998b). The ACF of a daily rainfall fluctuates around zero value.

The values of $S(m,N,r)$ for a daily streamflow and various values of m and r are shown in Fig. 3. The circles indicate the vicinity of τ_d, which is where the first zero crossing occurs, and $\tau_d = 89\tau_s$ is obtained by averaging these four zero crossings. Fig. 4 indicates $\Delta S(m,t)$, $\Delta\bar{S}(t)$, $\bar{S}(t)$, and $S_{cor}(t)$ for a daily streamflow and the minimum of $S_{cor}(t)$ gives $\tau_w = 194\tau_s$. The value of $\tau_d = 11\tau_s$ for GSL volume data is obtained from the first minimum of $\Delta\bar{S}(t)$ and $\tau_w = 223\tau_s$ is estimated from $S_{cor}(t)$ in Fig. 5. The value of $\tau_d = 10\tau_s$ is estimated from

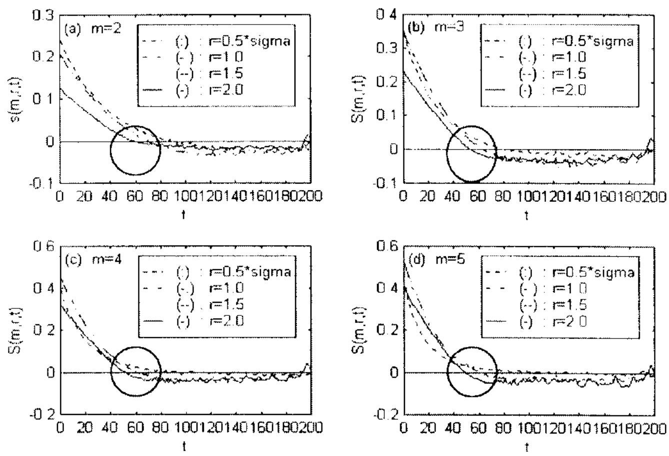

Figure 3. $S(m,r,t)$ **for a daily streamflow.**

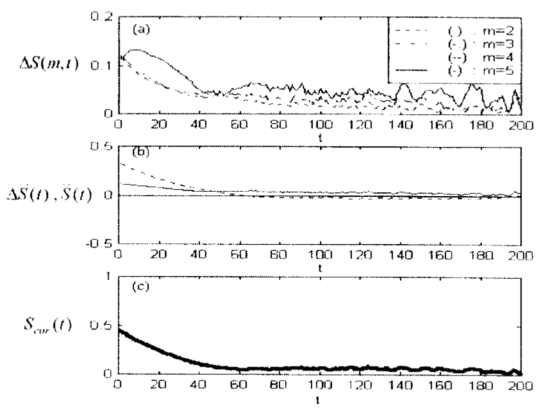

Figure 4. **(a)** $\Delta S(m,t)$, **(b) (--):** $\Delta\bar{S}(t)$, **(-):** $\bar{S}(t)$, **and (c)** $S_{cor}(t)$ **for a daily streamflow**

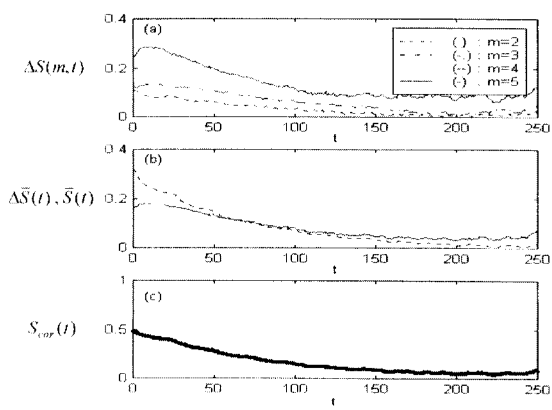

Figure 5. **(a)** $\Delta S(m,t)$, **(b) (--):** $\Delta\bar{S}(t)$, **(-):** $\bar{S}(t)$, **and (c)** $S_{cor}(t)$ **for GSL volume time series**

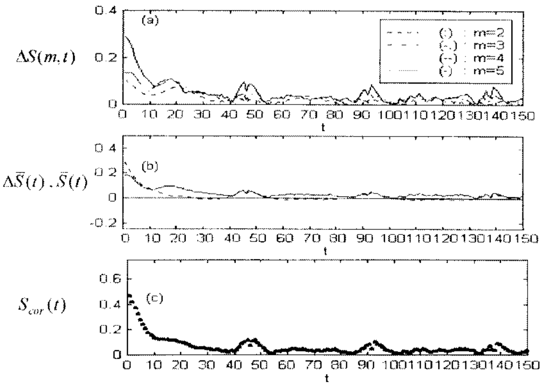

Figure 6. (a) $\Delta S(m,t)$, (b) (—): $\Delta\bar{S}(t)$, (-): $\bar{S}(t)$, and (c) $S_{cor}(t)$ for Lorenz system

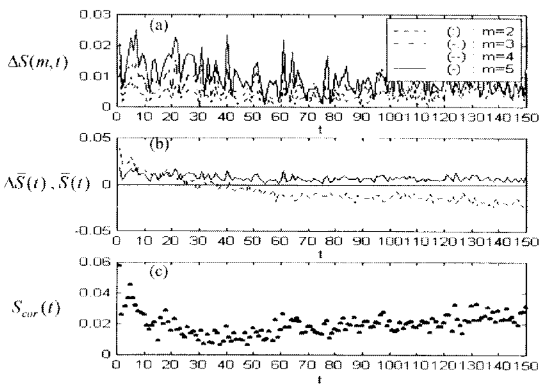

Figure 7. (a) $\Delta S(m,t)$, (b) (—): $\Delta\bar{S}(t)$, (-): $\bar{S}(t)$, and (c) $S_{cor}(t)$ for a daily rainfall

the first minimum of $\Delta\bar{S}(t)$, and $\tau_w = 100\tau_s$ from $S_{cor}(t)$ in Fig. 6 for Lorenz time series. For a daily rainfall, we also obtain $\tau_d = 3\tau_s$ from the first minimum of $\Delta\bar{S}(t)$, and $\tau_w = 48\tau_s$ from $S_{cor}(t)$ in Fig. 7. Summary of the delay times and the delay time windows estimated using the C-C method for each data sets are shown in Table 3.

If we compare the delay times estimated from the ACF and the C-C method, we can see a daily streamflow, GSL volume, and

Table 3. Estimated delay times and delay time windows using the C-C method.

Time series	Delay time	Delay time window
Daily streamflow	89	194
GSL volume	11	223
Lorenz	10	100
Daily rainfall	3	48

Lorenz time series have different results (see Table 2 and 3). This may be due to nonlinearity of the time series. However, the delay times estimated from the ACF and the C-C method for a daily rainfall have same results (see Table 2 and 3). It may be due to random fluctuations of time series having small autocorrelations. As proved by Martinere et al. (1992) the ACF and the mutual information could not estimate the delay time window but the C-C method do. From the results, the C-C method may be better than the ACF or the mutual information for describing nonlinear dependence of the time series.

CLOSING REMARKS

Martinerie et al. (1992) compared the delay time windows τ_w for the Lorenz system, a three-torus, and the Rossler system with the delay times τ_d found from the autocorrelation function and the mutual informaion. They concluded that the autocorrelation function and the mutual information could not give the value of τ_w. We have used a new method, called the C-C method, and shown that it can be used to find both τ_d and τ_w for some hydrologic time series and Lorenz system. It has shown that the C-C method can be used for measuring nonlinear dependence of the time series. Also the value of $S_{cor}(t)$ may represent nonlinear analog of linear autocorrelation function and it may be useful for describing the nonlinear dependence of the time series.

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