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〈Original Paper〉

Reactive Acoustic Filter based on the Phase Cancellation Effect

위상 반전 현상을 이용한 덕트 소음 제거기

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Key Words : Reactive Noise Filter(리액티브형 소음 휠터), Duct Noise(덕트소음), Webster Horn Equation(웹스터 혼 공식), Transmission Loss(전달 손실), Double Cone(이중 원추), Phase Cancellation(위상 상쇄 효과)

ABSTRACT

A reactive type acoustic filter is developed based on the phase cancellation effect which is occurring in the plane wave propagation through the two paths where the cross sectional areas are reversely changing. The theory is experimentally validated by the use of a cylindrical duct and an inserted hollowed cone of which vertex part is eliminated. Noise attenuation and the filtered frequency are dependent on the area variation and the effective length of the filter. Experimental comparison shows that the filtered frequencies of 1st and 2nd mode are lower than the analytical prediction due to the mass loading effects, and the 3rd mode is in good agreement. The proposed filter can be applied as an in-duct noise filter for improving the sound quality in a narrow space for various industrial applications.

요 약

본 논문은 길이 방향에 따라 단면적이 감소하는 경로와 증가하는 경로 2개가 존재할 경우 길이 방향에 따른 단면적 변화율과 경로의 길이에 의하여 특정 주파수에서 음파의 위상이 반대되는 현상이 발생하여 전달 경로 하단에서 해당 주파수의 소음이 상쇄되는 현상을 규명하였다. 단면적 변화를 고려한 1차 파동 방정식을 적용하여 매우 간단한 해석적인 해를 구하였으며, 원형 덕트 내의 속이 빈 콘 형상의 단면적 변화를 적용하여 실험적으로 검증하였다. 질량 효과에 의하여 필터링 주파수는 1, 2차 모드에서는 이론보다 더 낮은 주파수에서 결정되나, 3차 이상 고차 모드에서는 이론식으로 그 해당 주파수를 예측할 수 있다. 본 연구의 결과는 좁은 공간에서 덕트의 특정 소음의 주파수를 제거하여 음질을 개선할 필요가 있는 경우 이를 덕트 내부에 적용하는 콤팩트한 필터로 적용할 수 있다.

1. Introduction

Reactive type acoustic filters are based on the wave reflection at the area of acoustic impedance change. The most common examples are expansion chamber, side branch, Quincke tube, and Helmortz resonator[1]. Rim and Kim

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introduced an in-duct acoustic screen which was easily applicable to the space-constrained installation. By applying the two dimensional wave equation, they claimed that the 1st cross mode generated at the junctions was responsible for the out-of-phase cancellation in the two asymmetric paths[2]. However, the frequency of the cross mode itself is much higher than the characteristic frequency of the acoustic screen in a duct.

In this paper, a relative simple theory is developed for explaining the out-of phase noise noise attenuation based on the one-dimensional Webster Horn equation[3], and experimentally validated by the use of the 4-point method in a duct with the inserted noise filter. It is proved that the reverse area variation of two paths is a major contributor to the noise cancellation. For the effective noise attenuation, a double-cone type duct noise filter is proposed. The proposed noise filter is readily applicable to reduce the flow-induced noise and to improve the sound quality such as HVAC noise of a cabin and building, industrial compressor noise, and fluid-borne noise of hydraulic machinery.

2. Theoretical Background

2.1 Fundamentals

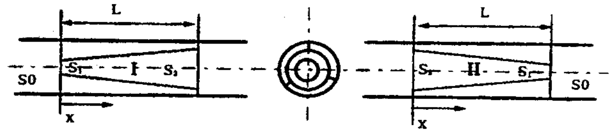
For a plane wave traveling through a varying sectional area, the wave equation, called Webster Horn equation, is given as (by suppressing $e^{i\omega t}$, and $\omega = ck$)

$$\frac{d^2p}{dx^2} + \left(\frac{1}{S}\frac{dS}{dx}\right)\frac{dp}{dx} + k^2p = 0 \qquad (1)$$

where $S(x)$ is a varying cross-sectional area. For simplicity of the derivation, consider the varying cross-sectional area inserted in an infinitely long duct as shown in Fig. 1. One can consider many kinds of the cross-sectional area variations:

i) Exponential-shape Section: $S = S_0 x^{mx}$ (2)

ii) Cone-shape Section: $S = S_0 x^2$ (3)



(a) $S = S_1$ Exp[mx] (b) $S - S_1$ Exp[$-mx$]

Fig. 1 Ideal model of varying cross sectional areas in an infinitely long duct (a) expanding area variation, (b) diminishing area variation

iii) Sine-shape Section: $S = S_0(1 + r_1 \sin Lx)^2$ (4)

For mathematical brevity, consider the exponential type of the expanding cross section of Eq. (2), as shown in the path I of Fig. 1(a). Since the derivative multiplied by the inverse of the area is constant, Eq. (1) then becomes

$$\frac{d^2p}{dx^2} + m\frac{dp}{dx} + k^2 p = 0 \qquad (5)$$

By substituting the $e^{-i\gamma x}$ type solution, the characteristic equation becomes

$$\gamma^2 + im\gamma - k^2 = 0 \qquad (6)$$

The solution becomes

$$p = Ae^{-\alpha x}e^{i\beta x} + Be^{-\alpha x}e^{-i\beta x},$$
$$\alpha = \frac{m}{2},$$
$$\beta = \sqrt{k^2 - \alpha^2} \qquad (7)$$

where A and B are the constants determined by the boundary conditions, and 1st component is the wave propagating in $-x$ direction and the second component is the wave propagating in $+x$ direction. If k is smaller than α, then the waves are exponentially decreased, called evanescent waves. The special case of $m = 4\pi / \lambda$ occurs at a frequency called cut-off frequency such that

$$f_c = \frac{mc}{4\pi} \qquad (8)$$

since no power can be transmitted for an acoustic wave lower than this frequency[4].

For acoustic waves traveling through the section, the acoustic impedance inside the path I is given by

$$Z_{AI}=\frac{p}{U}=\frac{p}{Su}=\frac{-i\omega\rho}{S}\frac{p}{(dp/dx)} \qquad (9)$$

when the Euler's equation is applied.
Substituting Eq. (7) into Eq. (9)

$$Z_{AI}(x)=\frac{-i\omega\rho}{S}\frac{Ae^{i\beta x}+Be^{-i\beta x}}{(-\alpha+i\beta)Ae^{i\beta x}-(\alpha+i\beta)Be^{-i\beta x}} \qquad (10)$$

The wave propagating through the path I satisfies the acoustic impedance boundary conditions:

$$Z_{AI}=\frac{\rho c}{S_0}=Z_A(x=0;S=S_1) \quad for \quad x=0,$$

$$Z_{AI}=\frac{\rho c}{S_0}=Z_A(x=L;S=S_2) \quad for \quad x=L, \qquad (11)$$

since the acoustic impedance of an incoming wave through the circular duct must be equal to the acoustic impedance of an outgoing wave through the path I at $x=0$, and the same is applicable to the location at $x=L$. Here, S_1 and S_2 are the cross sectional area of the path I at $x=0$ and L, respectively.

Substituting the 1st equation of Eq. (11) into Eq. (10), one can derive the relation:

$$A=\frac{\alpha+i(\beta-k_1)}{-\alpha+i(\beta+k_1)}B \qquad (12)$$

and the wavenumber satisfies the following equation:

$$e^{2i\beta L}=\frac{-\alpha+i(\beta+k_1)}{\alpha+i(\beta-k_1)}\frac{\alpha+i(\beta-k_2)}{-\alpha+i(\beta+k_2)} \qquad (13)$$

where

$$k_1=\frac{S_0}{S_1}k,\quad k_2=\frac{S_0}{S_2}k,\quad and \quad S_1k_1=S_2k_2. \qquad (14)$$

Thus, the wavenumber inside the duct depends on both the length and the variation of cross-sectional area.

By the use of Eqs. (12) and (13) in Eq. (10), one can derive the equation of the acoustic impedance inside of the path I as

$$Z_{AI}=\frac{i\rho ck}{S}\times\frac{-\alpha+i(\beta+k_1)+[\alpha+i(\beta-k_1)]e^{i2\beta x}}{(\alpha+i\beta)[-\alpha+i(\beta+k_1)]+(\alpha-i\beta)[\alpha+i(\beta-k_1)]e^{i2\beta x}} \qquad (15)$$

For example, if the mouth area becomes 3 times of the throat area (S_1) at $x=L$, then $m=ln(3)/L$. When $L=225$ mm, m=4.88 ; $\alpha=2.44$. Let also the radii of mouth and throat are $r_0=0.035$, $r_1=0.0175$. When the reactance, the imaginary part of the impedance, at $x=L$ is given as a function of the frequency, the peak frequencies correspond to the waves pass through the duct without reflections, in this example,

$f=800$, 1536, 2282 Hz.

On the other hand, when the variation of the cross-sectional area is reversed, i.e., if the area is exponentially diminishing, the path II in Fig. 1(b), the impedance inside the duct can be derived by the same way, and it is found that

$$Z_{AII}=\frac{-i\rho ck}{S}\times\frac{-\alpha-i(\beta+k_2)+[\alpha-i(\beta-k_2)]e^{i2\beta x}}{(\alpha-i\beta)[-\alpha-i(\beta+k_2)]+(\alpha+i\beta)[\alpha-i(\beta-k_2)]e^{i2\beta x}} \qquad (16)$$

which is the complex conjugate of the Eq. (15) with the substitution of k_1 into k_2, such that the phase difference of the interested waves in two paths is π. Since the passing frequencies are same in the two paths and they are out-of-phase, the corresponding waves are efficiently attenuated at the end of the cross-sectional area variation, here, it is called 'anti-phase mechanism' of the two separate paths.

For given wavenumbers satisfying Eq. (13), the reflection coefficient, the ratio of the incident wave to the reflected wave at $x=L$ can be given from Eqs. (7) and (12)

$$R=\frac{B}{A}e^{-2i\beta L}=\frac{-\alpha+i(\beta+k_1)}{\alpha+i(\beta-k_1)}e^{-2i\beta L} \qquad (17)$$

and by substituting Eq. (13), one can derive the reflection coefficient at $x=L$

$$R=\frac{-\alpha+i(\beta+k_2)}{\alpha+i(\beta-k_2)} \tag{18}$$

From this, the power transmission coefficient is expressed as[5]

$$T_\pi=1-|R|^2=1-\frac{\sqrt{\{\beta^2-\alpha^2-k_2^2\}^2+2\alpha^2\beta^2}}{\alpha^2+(\beta-k_2)^2} \tag{19}$$

2.2 Noise Cancellation Mechanism

To maximize the cancellation effects at the end of the noise filter, the transmitted power must be same between two paths; however, to generate the anti-phase mechanism, the cross-sectional area of one path must be expanding and the other be diminishing. To verify the noise cancellation based on the anti-phase mechanism, consider the waves propagating through the exponentially increasing and decreasing paths as shown in Fig. 2. For a given incident acoustic wave I, the problem is to determine 6 unknowns: R, A_I, B_I, A_{II}, B_{II}, and T. However, the ratio A_i/B_i ($i=I,II$) is given by Eq. (12), the unknowns are four, and one can solve this problem by using both the continuity of acoustic pressure and that of velocity at $x=0$, L. For simplicity, the two paths are expressed as

$$\begin{aligned} S_I(x)&=S_1e^{mx} \quad 0\le x\le L, \\ S_{II}(x)&=S_2e^{-mx} \quad 0\le x\le L, \\ S(x)&=S_I(x)+S_{II}(x). \end{aligned} \tag{20}$$

where S, S_I and S_{II} are the areas of the duct, path I and path II, respectively. At $x=0$ and $x=L$, S equals to S_0, which is the cross-sectional area of the circular duct.

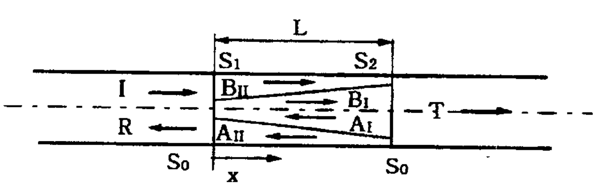


Fig. 2 Acoustics waves in a duct with the noise filter, a single stage configuration

The acoustic waves are expressed as

$$\begin{aligned} I&=I_0e^{-ik_0x}e^{i\omega t} \\ R&=R_0e^{i\beta x}e^{i\omega t} \\ P_I&=e^{-\alpha x}(A_Ie^{i\beta x}+B_Ie^{-i\beta x})e^{i\omega t} \\ P_{II}&=e^{\alpha x}(A_{II}e^{i\beta x}+B_{II}e^{-i\beta x})e^{i\omega t} \\ T&=T_0e^{-i\beta x}e^{i\omega t} \end{aligned} \tag{21}$$

where the k_0 is the wavenumber of the incident wave.

The continuity equations of the pressure and the velocity both at $x=0$ and $x=L$ provide the following algebraic equation:

$$\begin{bmatrix} -S_0 & S_I & S_I & S_2 & S_2 & 0 \\ i\beta & \alpha-i\beta & \alpha+i\beta & 0 & 0 & 0 \\ i\beta & 0 & 0 & -(\alpha+i\beta) & -(\alpha-i\beta) & 0 \\ 0 & S_1e^{-(\alpha-i\beta)L} & S_1e^{-(\alpha+i\beta)L} & S_2e^{(\alpha+i\beta)L} & S_2e^{(\alpha+i\beta)L} & -S_0e^{-i\beta L} \\ 0 & -(\alpha-i\beta)e^{-(\alpha-i\beta)L} & -(\alpha+i\beta)e^{-(\alpha+i\beta)L} & 0 & 0 & i\beta e^{-i\beta L} \\ 0 & 0 & 0 & (\alpha+i\beta)e^{(\alpha+i\beta)L} & (\alpha-i\beta)e^{(\alpha-i\beta)L} & i\beta e^{-i\beta L} \end{bmatrix} \begin{Bmatrix} R_0 \\ A_I \\ B_I \\ A_{II} \\ B_{II} \\ T_0 \end{Bmatrix} = \begin{Bmatrix} S_0I_0 \\ ik_0I_0 \\ ik_0I_0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{22}$$

or

$$C_x=y \tag{23}$$

Here, C is the coefficient matrix, and x and y are the complex amplitude vector and the incoming wave vector, respectively. From this equation, the frequencies for the effective wave cancellation are provided by the determinant zero condition. For the filtered frequencies, the behaviors of the waves inside the duct and the noise cancellator are given by the equation:

$$x=C^{-1}y \tag{24}$$

The transmission loss is then given by

$$TL = 20Log\left|\frac{I_0}{T_0}\right| \quad (25)$$

and the reflection coefficient due to the noise cancellator is given as

$$C_R = \left|\frac{R_0}{I_0}\right| \quad (26)$$

Fig. 3 shows the transmission loss and the

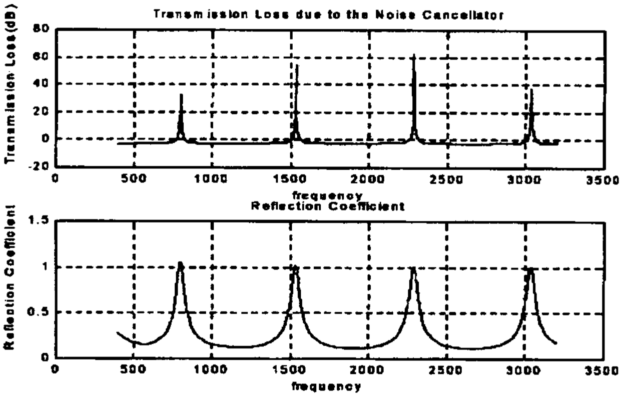


Fig. 3 Transmission loss and reflection coefficient of the noise filter, $L=0.225$ m, $m=4.88$

reflection coefficient of the previous example. Since the damping effects are neglected, the transmission loss is spiky, and the reflection coefficients are 1 for the passing frequencies since the waves are fully cancellated; thus, the acoustic energy moves backward. By the use of Eq. (25), the waves inside the two exponential horn-type paths are shown in Fig. 4 for a given unit magnitude of an incoming wave. As shown in Fig. 4, the wave cancellation effects result from the anti-phase mechanism of the cross-sectional area variation of the ducts. Using the analytical model, it can be shown that as the cross-sectional area variation is larger, the transmission loss is greater and the cancellation frequency is lower.

2.3 Cylindrical Cone

The wave equation inside a hollowed cylindrical cone is

$$x^2\frac{d^2p}{dx^2} + 2x\frac{dp}{dx} + x^2k^2p = 0 \quad (27)$$

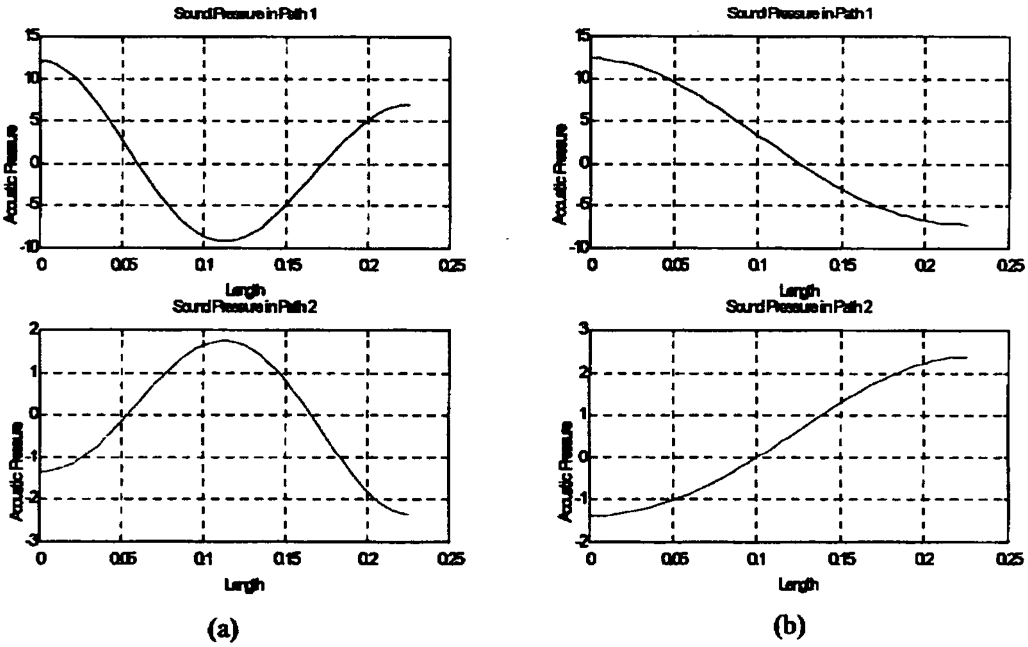


Fig. 4 Spacial distribution of the waves inside the noise filter for a given unit magnitude of an incoming wave; Path I is an expanding duct; Path II is a diminishing duct (a) $f=$ 1,536 Hz (one wavelength); (b) $f=$800 Hz (half wavelength)

The solution is given by

$$p = x^{-1}\left[Ae^{ikx} + Be^{-ikx}\right] \tag{28}$$

and for calculating the velocity, the derivative of the pressure is given as

$$\frac{dp}{dx} = -\frac{1}{x^2}\left[Ae^{ikx} + Be^{-ikx}\right] + \frac{ik}{x}\left[Ae^{ikx} - Be^{-ikx}\right] \tag{29}$$

The acoustic impedance inside the noise cone is

$$Z_A = \frac{p}{U} = \frac{p}{Su} = \frac{-i\omega\rho}{S}\frac{p}{(dp/dx)}$$
$$= \frac{-i\omega\rho}{S}\frac{Ae^{ikx} + Be^{-ikx}}{-x^{-1}\left[Ae^{ikx} + Be^{-ikx}\right] + ik\left[Ae^{ikx} - Be^{-ikx}\right]} \tag{30}$$

and two unknowns are determined by the boundary conditions.

Analytical derivations are not pursued in this paper; however, the results are experimentally verified in the following section since this type of a filter is easily fabricated.

3. Experimental Validation

To verify the analytical results, a conical noise filter, a hollowed cone of which vertex part is eliminated, is inserted in a circular duct, and the transmission loss is calculated by the 4 point method. Horn driver is used as an acoustic source at the one end of the duct, and the other end is terminated by a sound absorbing wedge. The experimental set up is shown in Fig. 5. For the sine sweep test, the sampling frequency is 1 Hz, the integration time is set as 15 cycles, and the settle time is 5 cycles. The inner radius of the duct is 35.0 mm, the area ratio of the noise filter is 3; the radius of throat is 17.5 mm (area = 240 mm^2), and the radius of mouth is 30.3 mm (area = 720 mm^2). For 35 mm duct, the 1st cross mode is 1.84c / (2*3.14*0.035), or 2,846 Hz. The results are shown in Fig. 6 and Fig. 7 for a single stage noise filter and for a double stage filter, respectively. For a single stage filter, most of the passing frequencies are equally cancellated, and the bandwidth are much broader than the analytical predictions. However, only the passing frequency correspond to the half wavelength is effectively canceled for the double stage filter, called a double cone, thus the geometric shape is well matched with the pressure variation. The results for various multi-stage filters are summarized by Table 1.

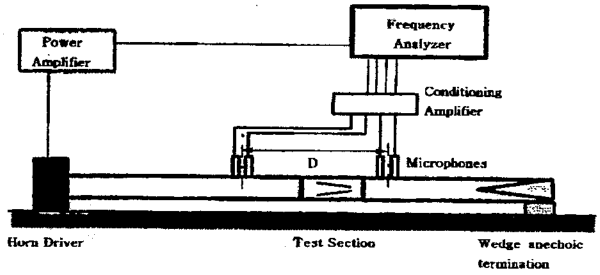
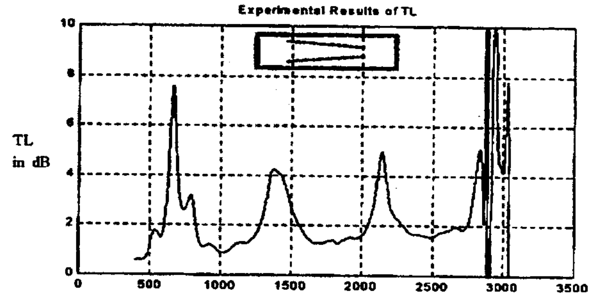
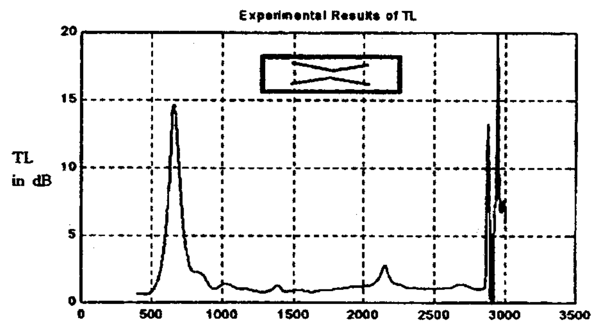


Fig. 5 Experimental setup for measuring the transmission loss of the noise filter, D=700 mm, distance between the two microphones=24.5 mm



Frequency in Hz

Fig. 6 Experimental result for the transmission loss of a single stage filter, L=0.225 m, a hollow cone type. The frequency for the 1st cross mode is 2,846 Hz



Frequency in Hz

Fig. 7 Experimental result for the transmission loss of a double stage filter, L=0.225 m, a hollow cone type

Table 1 Experimental results for hollowed-cone type noise filters, sampling frequency = 1 Hz, Frequency range = 400 ~ 3100 Hz. Description of the sectional shape :
1 stage: , 2 stage: , 3 stage: , 4 stage:

No	Section	Length (mm)	Comparison	1 st		2 nd		3 rd		4 th	
				Freq	TL	Freq	TL	Freq	TL	Freq	TL
1	1 stage	225	Test	667	7.58	1,380	4.27	2,138	6.97	3,030	5.1
			Theory	800	32.9	1,536	55.0	2,282	63.5	3,036	39.3
2	1 stage	225	Test	657	14.72	-	-	2,148	2.85		
3	1 stage	223	Test	634	3.19	1,474	16.69	2,124	6.10		
4	1 stage	239	Test	608	1.9	1,234	22.6	2,074	2.0	2,575	2.66

The magnitude of TL shows a huge difference between the theory and experiment in the No 1, since the theory assumes a wave propagation through an ideal filter in a lossless medium. The results confirm that when the waves are well matched with the cross sectional variation of the filters, the corresponding waves are effectively eliminated. The cancellation frequency is mainly depends on the length and the slope of the cross-sectional area variation along the duct.

4. Conclusion

Theoretical foundation is developed for explaining the wave cancellation mechanism at the two concentric paths whose cross sectional areas are reversely varying. An effective product realization is suggested and experimentally validated. By applying the suggested type of noise filter one can easily reduce 10 dB for a narrow band noise. The advantages of the proposed filter are compact and simple compared with a side branch. Also, the flow resistance is relatively low compared with a wedge type in-duct screen since the cross-sectional area variation is negligible through the duct and the filter shape can be configured to be well matched with the stream line. The practical application of the proposed filter is investigated for eliminating the 1st blade passing frequency of the multiblade fan noise of an air conditioner for improving the sound quality inside the cab.

In the future, the methods for widening the bandwidth must be developed for industrial applications. Also, an optimal geometric shape design of the sectional area for increasing TL with a minimum flow resistance should be investigated.

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