Journal of KSNVE, Vol. 9, No. 6, pp. 1259~1266, 1999.

〈Original Paper〉

Dynamic Analysis of A High Mobility Tracked Vehicle Using Compliant Track Link Model

유연성 궤도 모델을 사용한 고기동성 궤도차량의 동역학 해석

W. K. Baek*, J. H. Choi** and D. S. Bae***

백 운 경 · 최 진 환 · 배 대 성

(Received May 24, 1999 : Accepted November 10, 1999)

Key Words : Track Link(궤도 링크), Compliant Forces Element(유연성 힘요소), Multibody Tracked Vehicle(다물체 궤도차량), Variable Step(가변 구간), Explicit Integration(외연적 적분), Contact Forces(접촉력), Numerical Integration(수치 적분)

ABSTRACT

The objective of this investigation is to develop a compliant track link model and apply this model to the multi-body dynamic analysis of high mobility tracked vehicles. Two major difficulties encountered in developing the compliant track models. The first one is that the integration step size must be kept small in order to maintain the numerical stability of the solution. This solution deals with high oscillatory signals resulting from the impulsive contact forces and stiff compliant elements to represent the joints between the track links. The second difficulty is due to the large number of the system equations of motion of the three dimensional multibody tracked vehicle model. This problem was solved by decoupling the equations of motion of the chassis subsystem and the track subsystems. Recursive methods are used to obtain a minimum set of equations for the chassis subsystem. Several simulation scenarios were tested for the high mobility tracked vehicle including acceleration, high speed cruising, braking, and turning motion in order to demonstrate the effectiveness and validity of the methods proposed in this investigation.

요 약

본 연구의 목적은 유연성 궤도 모델을 개발하여 고기동성 궤도차량의 다물체 동역학 해석에 응용하는 것이다. 유연성 궤도 모델을 개발하는데는 대체로 두 가지 어려운 문제가 따른다. 첫째로, 해의 안정성을 유지하기 위해 적분구간이 충분히 작아야 한다는 것이다. 즉, 궤도 링크 사이의 유연성 조인트 모델과 충격적인 접촉력에 따른 고진동 입력을 처리해야 한다. 둘째로, 3차원 다물체 궤도차량 모델에 대한 수 많은 운동 방정식을 풀어야 한다는 것이다. 따라서 궤도차량을 샤시와 궤도 부시스템으로 나누고 회귀적인 방법을 사용하여 운동방정식의 수를 최소화하였다. 본 연구에서 개발된 방법을 검증하기 위하여 차량의 가속, 고속주행, 제동, 선회 등의 시뮬레이션을 수행하였다.

* Member, Pukyong Nat'l University

** Kyunghee University

*** Hanyang University

1. Introduction

High mobility tracked vehicles are subjected to impulsive dynamic loads resulting from the interaction of the track chains with the vehicle components and the ground. These dynamic loads can have an adverse effect on the vehicle performance and cause high stress levels that limit the operational life of the vehicle components. For this reason, high mobility tracked vehicles have sophisticated suspension system, more elaborate design of the links of the track chains, and improved vibration characteristics that allow the vehicle to perform efficiently in hostile operating environments.

Bando et al[1] developed a planar computer model for rubber tracked bulldozers. Steel and fiber molded continuous rubber track is discretized into several rigid bodies connected by compliant force elements. Characteristics of track damage, vibration, and noise are investigated using the simulation results. Nakanishi and Shabana[2] developed a two dimensional contact force model for planar analysis of multibody tracked vehicle systems. The stiffness and damping coefficients in this contact force model were determined based on experimental observations of the overall vibration characteristics of the tracked vehicle. The generalized contact forces associated with the system generalized coordinates were obtained using the virtual work. Choi[3~5] presented a large scale multibody dynamic model of construction tracked vehicle in which the track is assumed to consist of track links connected by single degree of freedom pin joints. In this detailed three dimensional dynamic model, each track link, sprocket, roller, and idler is considered as a rigid body that has relative rotational degrees of freedom. The three dimensional tracked vehicle model developed by Choi[3] may have hundreds or thousands differential and algebraic equations. These equations are highly non-linear and can only be solved using efficient computational methods. In addition to this dimensionality problem, tracked vehicles are subjected to impulsive forces due to the contacts between the track links and the vehicle components as well as the ground.

Newmark[6] presented an absolutely stable second-order numerical integrator in the area of structural dynamics. The Newmark integrator was modified by Wilson[7] so that highly oscillatory state variables are numerically damped out. The numerical damping algorithms are extended and generalized in implicit and explicit forms with a constant step size by Chung.[8,9]

The objective of this investigation is to develop a computational procedure for the nonlinear dynamics of high mobility tracked vehicles equipped with double pin track links. In the model developed in this paper, compliant force elements are used to model the connectivity between the links of the track chains. The application of the numerical integration scheme developed by Chung[8,9] to tracked vehicle dynamics is investigated in this paper using several simulation scenarios that include vehicle acceleration, high speed cruising, braking and turning motion.

2. A High Mobility Tracked Vehicle Model

The vehicle model in the Fig. 1 consists of a chassis and two track subsystems. The chassis subsystem includes a chassis, sprockets, support rollers, idlers, road arms, road wheels and the suspension units. The sprockets, support rollers, and road arms are connected to the chassis by revolute joints.

The suspension system includes a hydro-pneumatic suspension unit(HSU)[10], and torsion bar that are modeled as force elements whose compliance characteristics are evaluated using analytical and empirical methods. The stiffness coefficient of the torsion bar spring is approximately

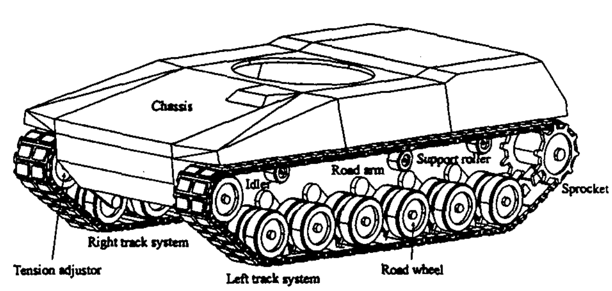

Fig. 1 A multibody tracked vehicle model

5×10^4 N-m/rad.

Each track subsystem is modeled as a series of bodies connected by rubber bushings around the link pins which are inserted into a shoe plate with some radial pressure in order to reduce the nonlinear effect of the rubber. When the vehicle runs over rough surface, the track chains are subjected to extremely high impulsive contact forces as the result of their interaction with the vehicle components such as road wheels, idlers, and sprocket teeth, as well as the ground. The rubber bushings and double pins tend to reduce the high impulsive contact forces by providing cushion and reducing the relative angle between the track links. About 20 % of the vehicle weight is given as the pre-tension for the track to prevent frequent separations of the track when the vehicle runs at a high speed. About 14 degrees of a pre-torsion is also provided in order to reduce the fluctuation of the torque in the rubber bushing when the track links contact the sprocket and idler. The vehicle model consists of 189 bodies, 36 revolute joints and 152 bushing elements and has 954 degrees of freedom.

3. Equations of Motion

The relative generalized coordinates are employed in order to reduce the number of equations of motion and to avoid the difficulty associated with the solution of differential and algebraic equations. Since the track chains interact with the chassis components through contact forces and adjacent track links are connected by compliant force elements, each link in the track chain has six degrees of freedom which are represented by three translational coordinates and three Euler angles[11]. Recursive kinematic equations of tracked vehicles using absolute Cartesian velocities of the chassis components can be expressed in terms of the independent joint velocities as follows:

$$\dot{\boldsymbol{q}} = \boldsymbol{B}\,\dot{\boldsymbol{q}}_i^r \qquad (1)$$

where $\dot{\boldsymbol{q}}_i^r$, $\boldsymbol{B}$ and $\dot{\boldsymbol{q}}$ are relative independent velocities, velocity transformation matrix, and Cartesian velocities of the chassis subsystem, respectively. Differentiating Eq. (1) gives the acceleration equation as follows.

$$\ddot{\boldsymbol{q}} = \boldsymbol{B}\,\ddot{\boldsymbol{q}}_i^r + \dot{\boldsymbol{B}}\,\dot{\boldsymbol{q}}_i^r \qquad (2)$$

The equations of motion for the chassis subsystem can be written as follows.

$$\boldsymbol{M}^c\,\ddot{\boldsymbol{q}} = \boldsymbol{Q}^c \qquad (3)$$

where $\boldsymbol{M}^c$ is the mass matrix, $\ddot{\boldsymbol{q}}$ is Cartesian acceleration, and $\boldsymbol{Q}^c$ is the forces on the chassis subsystem.
Substituting Eq. (2) into Eq. (3) and arranging,

$$\boldsymbol{M}^c\boldsymbol{B}\ddot{\boldsymbol{q}}_i^r = \boldsymbol{Q}^c - \boldsymbol{M}^c\,\dot{\boldsymbol{B}}\dot{\boldsymbol{q}}_i^r \qquad (4)$$

Multiplying $\boldsymbol{B}^T$ on the both side of the Eq. (4), the equations of motion of the chassis that employs the velocity transformation can be given as follows :

$$\boldsymbol{B}^T\boldsymbol{M}^c\boldsymbol{B}\ddot{\boldsymbol{q}}_i^r = \boldsymbol{B}^T(\boldsymbol{Q}^c - \boldsymbol{M}^c\,\dot{\boldsymbol{B}}\dot{\boldsymbol{q}}_i^r) \qquad (5)$$

where $\boldsymbol{M}^c$ is the mass matrix, and $\boldsymbol{Q}^c$ is the generalized external force, and $\dot{\boldsymbol{q}}_i^r$ is the velocity vectors of the chassis subsystem.

Since there is no kinematic coupling between the chassis subsystem and the track subsystems, the equations of motion of the track subsystems can be obtained as follows :

$$\boldsymbol{M}^t\,\ddot{\boldsymbol{q}}^t = \boldsymbol{Q}^t \qquad (6)$$

where $\boldsymbol{M}^t$, $\ddot{\boldsymbol{q}}^t$, and $\boldsymbol{Q}^t$ denote mass matrix, generalized coordinates, and force vectors for the track subsystem, respectively. The motion of the chassis and the track links can be obtained by solving Eqs. (5) and (6).

4. A Compliant Track Model

Fig. 2 shows the details of the link, pin and bushing connections of a single pin track link. In this investigation, a continuous force model is used to define the pin joint connections. This force model is a non-linear function of the coordinates of the two links. In order to define the generalized compliant bushing forces, several coordinate systems are introduced: two centroidal body coordinate systems($X_b^i\ Y_b^i\ Z_b^i$ and $X_b^j\ Y_b^j Z_b^j$ for the track links i and j, respectively), a joint coordinate system ($X^i\ Y^i\ Z^i$) whose origin is assumed to be located at the geometric center of the circular groove containing the pin and the bushing, and a pin coordinate system ($X^j\ Y^j Z^j$) whose origin is rigidly attached to the center of the pin. Because of the bushing effect, the origins of the joint and pin coordinate systems do not always coincide. The displacement of the pin coordinate system ($X^j\ Y^j Z^j$) with respect to the joint coordinate system ($X^i\ Y^i\ Z^i$) is a function of the bushing stiffness. Also the location and orientation of the joint coordinate system ($X^i\ Y^i\ Z^i$) can be determined as a function of the generalized coordinates of link i. For simplicity, it is assumed in this investigation that the location and orientation of the pin coordinate system can be defined in terms of the coordinates of link j. The deviation $\boldsymbol{d}_R=[d_x\ d_y\ d_z]^T$ shown in Fig. 2 can be used to determine the generalized forces acting on the two links i and j as the result of the bushing effect. The bushing force and torque applied to the frame j can be given as follows:

$$\begin{bmatrix} \boldsymbol{Q}_R^j \\ \boldsymbol{Q}_\theta^j \end{bmatrix} = -\begin{bmatrix} \boldsymbol{K}_R & 0 \\ 0 & \boldsymbol{K}_\theta \end{bmatrix}\begin{bmatrix} \boldsymbol{d}_R \\ \boldsymbol{d}_\theta \end{bmatrix} - \begin{bmatrix} \boldsymbol{C}_R & 0 \\ 0 & \boldsymbol{C}_\theta \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{d}}_R \\ \dot{\boldsymbol{d}}_\theta \end{bmatrix} \tag{7}$$

where $\boldsymbol{K}_R, \boldsymbol{K}_\theta, \boldsymbol{C}_R$ and $\boldsymbol{C}_\theta$ are the 3×3 matrices that contain the stiffness and damping coefficients of the bushing, and $\boldsymbol{Q}_R^j$ is translational force vector and $\boldsymbol{d}_R$ is the vector of translational deformations of the frame j relative to the frame i. Similarly, $\boldsymbol{Q}_\theta^j$ is the rotational force vector and $\boldsymbol{d}_\theta$ is the vector of relative rotational deformations of the frame j relative to the frame i. The force and torque applied to the frame i are assumed to be equal in magnitude and opposite in direction to the force and torque acting on frame j. Once these forces are determined, the generalized bushing forces associated with the generalized coordinates of the track links i and j can be determined.

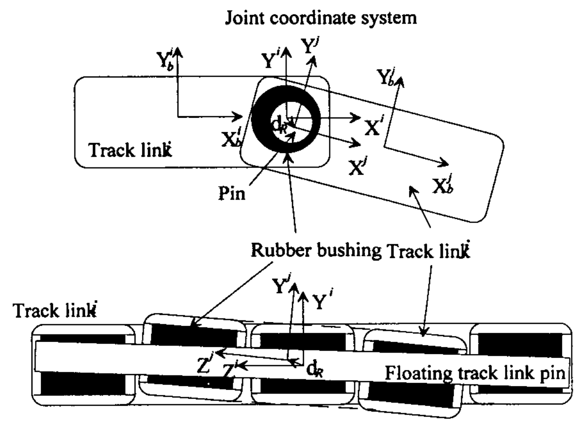

Fig. 2 Single pin connection

In order to determine the stiffness and damping coefficients of the compliant force models, an experimental study was conducted to examine the interaction between the track links. The dynamics of the compliant interaction can be considered in the measurement process[12]. While a viscous damping force is proportional to the velocity, in many cases, analytic expressions for the damping forces are not directly available. It is, however, possible to obtain an equivalent viscous damping coefficient by equating energy expressions before and after the contact. The effective stiffness and damping coefficient can be obtained by employing the hysteresis loop method[13]. Track compliance properties were measured using a test rig as shown in Fig. 3 and the detailed experimental study for the track links is described in reference 14. A LVDT sensor is attached between two adjacent track links to measure the relative displacement. For a static test, the actuator force is increased gradually up to 10 ton with 2 mm/min velocity. In this investigation, for the sake of simplicity, the stiffness and damping coefficients used in the force models are determined using empirical methods based on the results of the static test only. A spline curve fitting is used to obtain the compliant characteristics from measurement data.

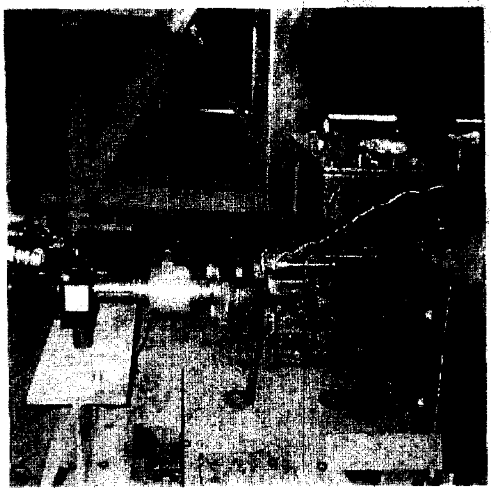

Fig. 3 Test rig for track compliance properties measurement

5. Contact Forces between Suspension Components

In this section, the methods used for developing the contact force models are briefly discussed. The scenarios of the contacts between the track links and the road wheels, rollers, sprockets, and the ground are explained. A more detailed discussion on the formulation of the contact forces is presented by Choi, et al.[4,5], and Nakanishi and Shabana[2].

5.1 Interaction between Track and Road Wheel, Idler, and Support Roller

Each roller of the vehicle model consists of two wheels which are rigidly connected. There are four different possibilities for the roller and track interaction. The first possibility occurs when a track link and one wheel of the roller are in contact. In this case, a concentrated contact force is used at the center of the contact surface of the wheel. The contact force acting on the link is assumed to be equal in magnitude and opposite in direction to the force acting on the roller. The second possibility occurs when both wheels of the roller are in contact with the track link. The third and fourth possibilities occur, respectively, when either one wheel or both wheels are in contact with the edges of track link.

In order to monitor track center guide and road wheel interactions, several algorithms are developed for road wheel and track link contact. Since the road wheel of the vehicle model used in this investigation consists of two rigidly connected wheels, there are four possibilities for the track center guide and wheel interactions. The first possibility is that the right side plate of the wheel can contact with the left side wall of the track center guide. In the second possibility, the left side plate of the wheel can contact with the right side wall of the track center guide. The third possibility is that one bottom surface of wheel and the top surface of track center guide are in contact. The fourth possibility occurs when the two road wheels are not in contact with the track center guide.

5.2 Interaction between the Sprocket Teeth and Track Link Pins

Five tooth surfaces are used to represent the spatial contact between the sprocket teeth and the track link pins. During the engagement between the sprocket teeth and the track links, several sprocket teeth can be in contact with several track link pins. The sprocket used in this model has ten teeth, and each tooth has five contact surfaces.

A computer algorithm was developed to determine whether or not the track link pin is in contact with one of surfaces of the sprocket teeth, utilizing the relative position of the track link pins with respect to the sprocket teeth. The interactions between the track link pins and the sprocket base circle are also considered in this investigation. To this end, the distance between the center of the track link pin and the center of sprocket is monitored. When this distance is less than the sum of the pin radius and the sprocket base circle radius, the contact is occurred and a concentrated force is applied to the sprocket and the track link pin.

5.3 Ground and Track Shoe Interactions

The track link used in this investigation has double shoe plate, and therefore, there are two surfaces on each track link that can come into contact with the ground. The global position vectors that define the location of points on the shoe plates are expressed in terms of the generalized coordinates of the track links and are used to predict whether or not the track link is in contact with the ground. In this investigation, contact forces are applied at selected six points on the track link shoe when it comes into contact with the ground. The normal force components are used with the coefficient of friction to define the tangential friction forces.[4,5]

6. Numerical Integration and Simulation Results

The dynamics of tracked vehicles is characterized by high impulsive forces resulting from the contact between the track chains and the vehicle components as well as the ground. Because of the high frequency impulsive forces, the numerical integration routine is forced to take a small time step. Therefore, the simulation of a complex tracked vehicle model involves a challenging computational task. Nonetheless, the high frequency oscillations may have little influence on the low frequency motion. In this case, the high oscillations can be damped out to obtain the gross motion of the track link. Various dissipation algorithms for time integration of structural systems have been proposed[7~9,15]. In this investigation, the method proposed by Chung and Lee[9] was used because it can be easily implemented and computationally efficient. Accuracy and stability conditions must be considered in carrying out a numerical integration of the tracked vehicle equations. The accuracy and stability conditions are obtained by using the truncation error and the error propagation analyses. These methods are used for the dynamic analysis of the high mobility tracked vehicle model.

A variable step integration algorithm is employed in this investigation. The resulting integration step size is shown in Fig. 4, when the vehicle runs with maximum acceleration, steady state velocity

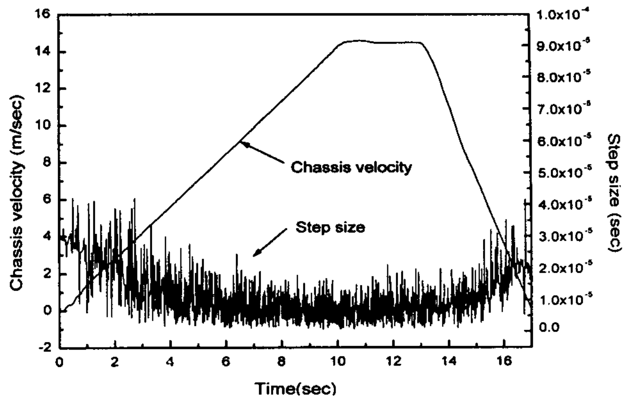

Fig. 4 Step size of variable step integration algorithm

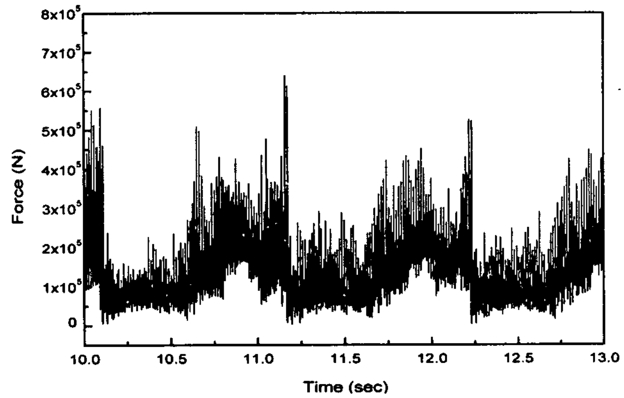

Fig. 5 Tension of track link

at 50 km/h, and stiff deceleration of braking. This figure shows that the integration step size depends on the vehicle speed changes. The increment of vehicle speed will increase impulsive contact forces and oscillation of track links, and the integration step size should be smaller, accordingly. Figure 5 shows the longitudinal track tension in the bushing between track links.

In the simulation of acceleration, high speed motion, and braking of the vehicle, the same angular velocity is used for both left and right sprockets in order to obtain straight line motion. The angular velocities of the sprockets are increased linearly up to -45 rad/sec in 10 sec, kept constant for 3 sec, and then decreased linearly to 0 rad/sec in 4 sec. The coefficient of friction between the track links and ground is assumed to be 0.7 in the case of rubber and concrete contact(16). The double pin track link is used in the numerical

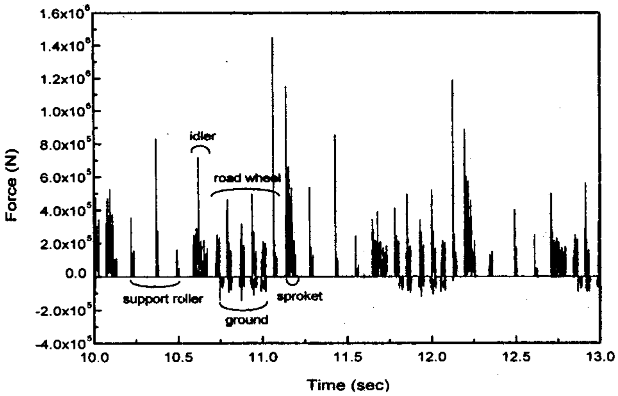

Fig. 6 Contact forces of track link

study presented in this section. In general, heavy duty high mobility tracked vehicles, as one used in this study, have double pin track links. One of the main advantages of using double pin track is that the shear stress on the rubber bushings can be significantly reduced as compared to the single pin track link. The simulation results showed the significant reduction of the load on the rubber bushings when a double pin track is used.

Figure 6 shows the contact forces exerted on one of the links of the right track chain as the result of its interaction with the road wheels, support rollers, idler, sprocket, and ground. The turning motion is obtained by providing two different values for the angular velocities of the sprockets. The angular velocity of the right sprocket is decreased linearly to -9 rad/sec and the angular velocity of the left sprocket is increased linearly to 9 rad/sec in 2 sec. The angular velocities are then kept constant velocities.

7. Summary and Conclusions

The dynamics of a high mobility multibody tracked vehicle is investigated in this paper. Compliant force elements are used to define the connectivity between the links of the track chains instead of an ideal pin joint. Double pin track link models are considered in this study. In the double pin track model, two pins are used with a connector element to connect two links of the track chain. Rubber bushings are used between the track links and the pins. The stiffness and damping characteristics of the contact forces are obtained empirically. By using experimental data, the generalized contact and bushing forces associated with the generalized coordinates of the tracked vehicle are developed. The vehicle model is assumed to consist of 189 bodies, 36 pin joints, and 152 bushing elements. The model has 954 degrees of freedom.

An explicit numerical integration method that has a large stability region is employed in this study. The method employs a variable time step size in order to achieve better computational efficiency. It was observed that the time step size significantly decreases as the vehicle speed increases. Several simulation scenarios are examined in this investigation. These include accelerated motion, high speed motion with a constant velocity, braking, and turning motion. The simulation results demonstrate the significant effect of the bushing stiffness on the dynamic response of the multibody tracked vehicle.

후 기

본 연구는 국방과학연구소의 지원(98-5-1)에 의한 연구결과의 일부이며, 관계자 여러분께 감사드립니다.

References

(1) Bando, K., Yoshida, K., and Hori, K., 1991, "The Development of the Rubber Track for Small Size Bulldozers", SAE, International off-Highway Power Plants Congress and Exposition, Milwaukee, WI, Sept. 9~12.

(2) Nakanishi, T., and Shabana A., 1994, "Contact Forces in the Nonlinear Dynamic Analysis of Tracked Vehicles", International Journal for Numerical Methods in Engineering, Vol. 37, pp. 1251~1275.

(3) Choi, J. H., 1996, "Use of Recursive and Approximation Methods in The Dynamic Analysis of Spatial Tracked Vehicle", Ph. D. Thesis, The University of Illinois at Chicago.

(4) Choi, J. H., Lee, H. C. and Shabana, A., 1998, "Spatial Dynamics of Multibody Tracked Vehicles:

Spatial Equations of Motion", International Journal of Vehicle Mechanics and Mobility, Vehicle System Dynamics, Vol. 29, pp. 27~49.

(5) Lee, H. C., Choi, J. H. and Shabana, A. A., 1998, "Spatial Dynamics of Multibody Tracked Vehicles: Contact Forces and Simulation Results", International Journal of Vehicle Mechanics and Mobility, Vehicle System Dynamics, Vol. 29, pp. 113~137.

(6) Newmark, N. M., 1959, "A Method of Computation for Structural Dynamics", Journal of the Engineering Mechanics Division, ASCE, 85, NO.EM3, pp. 67~94.

(7) Wilson, E. L., 1968, "A Computer Program for the Dynamic Stress Analysis of Underground Structures", SESM Report No. 68-1, Division of Structural Engineering and Structural Mechanics, University of California, Berkeley.

(8) Chung, J., 1992, "Numerically Dissipative Time Integration Algorithms for Structural Dynamics", Ph.D. Dissertation, University of Michigan, Ann Arbor.

(9) Chung, J. and Lee, J. M., 1994, "A New Family of Explicit Time Integration Methods for Linear and Non-linear Structural Dynamics", International Journal for Numerical Methods in Engineering, Vol. 37, pp. 3961~3976.

(10) Bruce Maclaurin, 1983, "Progress in British Tracked Vehicle Suspension Systems", SAE, 830442, 1983

(11) Shabana, A., Dynamics of Multibody Systems, John Wiley & Sons, New York.

(12) Trusty, R. M., Wilt, M. D., Carter, G. W. and Lesuer, D. R., 1988, "Field Measurement of Tension in a T-142 Tank Track", Experimental Techniques.

(13) Shabana, A., 1996, Theory of Vibration, An Introduction, Second Edition, Springer-Verlag, New York.

(14) Park, D. C., Seo, I. S. and Choi, J. H., 1999, "Experimental Study on the Contact Stiffness and Damping Coefficients of the High Mobility Multibody Tracked Vehicle", Journal of Korea Society of Automotive Engineers, Vol. 7, No. 3, pp. 348~357.

(15) Park, K. C. and Underwood, P. G., 1980, "A Variable-step Central Difference Method for Structural Dynamic Analysis - Part 1, Theoretical Aspects", Computer Methods in Applied Mechanics and Engineering, Vol. 22, pp. 241~258.

(16) Wong, J. Y., 1993, Theory of Ground Vehicles, John Wiley & Sons, Inc., pp. 26~29.