Fuzzy Scheme for Extracting Linear Features

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ABSTRACT

A linear feature often provides sufficient information for image understanding and coding. An objective of the research reported in this paper is to develop and analyze the reliable methods of extracting lines in gray scale images. The Hough Transform is known as one of the optimal paradigms to detect or identify the linear features by transforming edges in images into peaks in parameter space. The scheme proposed here uses the fuzzy gradient direction model and weighs the gradient magnitudes for deciding the voting values to be accumulated in parameter space. This leads to significant computational savings by restricting the transform to within some support region of the observed gradient direction which can be considered as a fuzzy variable and produces robust results.

선형적 특징을 추출하기 위한 퍼지 후프 방법

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요 약

특정 이미지에서의 선형적 특징은 이미지를 분석하고 이해하는데 충분한 정보를 제공하기도 한다. 본고에서는 이미지에서 선형적 특징을 추출하기 위한 신뢰성 있는 방법을 제시한다. 일반적으로 후프 변형 방법은 이러한 선형적 특징을 추출하는 최적의 방법 중의 하나로 인식되어 왔다. 대부분의 후프 기반 방법들은 특정 edge 모델을 선택하고, 인식된 edge 픽셀의 속성을 반영하는 변형식을 활용하여 파라미터 공간에 그 발생빈도를 기록하는 과정을 거치게 된다. 주로 edge 픽셀의 gradient 크기와 방향이 선형적 특징을 결정하는데 사용되지만, 본고에서는 그 값들이 퍼지변수로 활용될 수 있음을 보이고 파라미터 공간에 누적값을 계산하는데 활용한다. 이 방법을 기존의 방법과 비교하기 위하여 에러 측정 방식을 제안하고, 실험을 한 결과, 기존의 방법과 비교하여 우수한 성능을 보인다.

1. Introduction

From a mathematical perspective, finding the collinear points is equivalent to searching for concurrent lines passing through those points. We can approach this problem in two ways. The first way is to draw a probe line and examine the collinear feature points on that line. Or secondly, to draw the concurrent lines on each feature point and to examine the frequency of each line where frequency implies the number of points lying on that line. To implement this idea parametric representations have been used to describe a line and the discrete parameter spaces are created to accumulate the frequencies. The Hough Transform (HT) has been considered as an effective means to implement this idea. The HT method has been known to have many desirable features. The independent treatment to each pixel allows for parallel implementation and makes it possible to recognize partial, deformed, or occluded shapes. Also, it is possible to simultaneously accumulate evidence of several examples of a particular shape class occurring in the same image. Above all, it

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is very robust to noise, since noise is very unlikely to contribute coherently to a particular cell of parameter space. However, HT has many drawbacks. First, it needs intensive computation and large storage. Second, it is non-trivial task to detect proper peaks in parameter space. Third, false line problems and false point problems may be issues under certain conditions[1]. To cope with these drawbacks many modified HTs have been devised[2-8]. In this paper the conventional HT is modified by using the concepts from fuzzy set theory[9]. The Fuzzy HT(FHT) is proposed to extract lines from images by using gradient direction as a fuzzy variable and by weighing the gradient magnitude based on its membership function in computing voting values. This algorithm saves a significant amount of computation by restricting the transform to within some support region of each fuzzy variable which is observed gradient direction of a pixel. It is also more robust and flexible since they operate directly on the gray scale image without binarization.

2. Standard Hough Transform(SHT)

Edges are usually defined as local discontinuities or rapid changes in image feature, such as image luminance or texture. These changes are detected by a local operator that measures the magnitude of the change and its direction. Let's assume that discrete gray scale image $I(i,j)$ is an image function. B is a binary matrix which is obtained by enhancing and threshing edge pixels, can be generated using gradient edge detector such as Sobel operator. Let's $h_x(i,j)$ and $h_y(i,j)$ for x and y direction operators respectively for any available edge detector. Then the gradient magnitude matrix G_m, the gradient direction matrix G_θ, and binary edge map B can be obtained as follows:

$$G_\theta(i,j) = \arctan(\ G_x(i,j) /\ G_y(i,j)),$$

$$G_m(i,j) = \sqrt{\ G_x(i,j)^2 + G_y(i,j)^2},$$

$$B(i,j) = \begin{cases} 1, \text{if } G_m(i,j) \geq \tau \\ 0, \textit{otherwise}, \end{cases}$$

$$G_x(i,j) = \sum_{u=1}^{k} \sum_{v=1}^{l} h_x(u,v) I(i+u, j+v),$$

$$G_y(i,j) = \sum_{u=1}^{k} \sum_{v=1}^{l} h_y(u,v) I(i+u, j+v)$$

where k and l are the x and y components of selected edge detector respectively, and $i=1..N$, $j=1..M$ if an image $I(i,j)$ is stored as $N \times M$ matrix. Threshold τ may be determined by the equation $\tau = \mu + c\sigma$, where μ is the mean of $I(i,j)$ and σ is the standard deviation of $I(i,j)$ and c the arbitrary constant weight to be selected heuristically.

If the normal parameterization is used, a line can be uniquely represented as $\rho = j \sin\ \theta\ +\ i \cos\ \theta$, $\theta \in [0, \pi)$. We define a cell in parameter space centered at (ρ_k, θ_l) as a rectangle $\Omega_{k,l}$ of size $\Delta\rho \times \Delta\theta$. Each pixel, a voter, is to vote for the cells in the parameter space, which may reflect the instances of true linear features in an image matrix. In SHT[10][11], each voter (i_o, j_o) is qualified for voting for multiple candidates provided that it has a sufficient gradient magnitude. The geographic position of a pixel and the parameter equation fixed by its position confines the scope of the candidates in parameter space for voting. That is,

$$\{(\theta, \rho) \mid |\rho - j_0 \sin\ \theta - i_0 \cos\ \theta| \leq (\Delta\rho/2), \forall\rho, \theta\}.$$

The geographic location and the shape of a candidate $\Omega_{k,l}$ determines its electoral district in an image space(refer to [12] for details). That is, it represents a particular constraint that is a collection of particular instances of lines, and this constraint can be mapped out in feature(image) space by evaluating

$$\{(i,j) \mid (|\rho_k - j \sin\theta_l - i \cos\theta_l| \leq (\Delta\rho/2)) \cap ((\rho_k, \theta_l) \in \Omega_{k,l}), \forall i,j\}.$$

From the observation above, the SHT has a dual interpretation, which might cause two different implementations (1) and (2) as follows:

$$H(k,l) = \sum_{i=1}^{N} \sum_{j=1}^{M} B(i,j)\ \delta_{\Delta}(\rho_k - j\sin\theta_l - i\cos\theta_l),$$
$$\forall l,k,\ l=1\ldots L,\ k=1\ldots K \qquad (1)$$

where

$\Delta\theta = \pi/L,\ \ \theta_l = \Delta\theta(l-1),\ \ \Delta\rho = 2\sqrt{N^2 + M^2}/K,$

$\rho_k = (k - K/2)\cdot\Delta\rho,$

$$\delta_{\Delta}(x) = \begin{cases} 1, & -\Delta\rho/2 < x \le \Delta\rho/2 \\ 0, & otherwise. \end{cases}$$

The values of K and L define the resolution of parameter space H.

procedure SHT(n,m,N,M,O,K,L var H); (2)
  $\Delta\theta = 2\pi/L$; $\Delta\theta = 2\pi/L$; $\Delta\rho = 2\sqrt{N^2 + M^2}/K$;
   for i=1 to N do
    for j= 1 to M do
     if $B(i,j)$=1 then
      for l=1 to L do
        $\theta_l = \Delta\theta(l-1)$; k=round(O-(j-m) $\sin\theta_l$-(n-i))$\cos\theta_l$); $H(k,l)$=$H(k,l)$+1;
end; end; end;

The O is the center of the ρ-axis and (n,m) is the center of $I(i,j)$.

3. The Modified Fuzzy Hough Transform (MFHT)

The simple edge operators such as Sobel and Prewitt are well established and commonly used to extract edge features, chiefly because of their computational simplicity. It was shown that, even for a straight step edge of uniform contrast, the gradient magnitudes and directions computed by these edge operators may vary considerably because of their inherent orientation bias or distance bias[14]. This poses difficulties for Hough schemes to utilize the gradient magnitudes and directions as voting parameters. In order to avoid this problems, several modified Hough schemes were suggested[1,13]. However, those algorithms may generate the other issues, such as the selection of additional parameters, which are usually to be determined heuristically. From this observation SHT is modified by using the concepts from fuzzy set theory so that the local variation in the observed gradient direction at a pixel should be properly utilized. The observed gradient direction and its local variation are considered as a fuzzy variable and its fuzzy membership function respectively. Such a fuzzy model is estimated by using image model and computational methodologies suggested by Malin[14]. In the MFHT algorithms, each pixel can vote for all candidates whose gradient directions are similar to the voter. Hence the voting mechanism may be more refined by considering inherent propensity at a pixel position as well as the geometric location and the shape of candidate.

3.1 IMAGE AND EDGE MODEL

Assume that image function is given as follows (the origin is assumed to be the center of image for convenience):

$$f(x,y) = \begin{cases} 1, & x\cos\theta + y\sin\theta \ge r \\ 0, & otherwise. \end{cases}$$

The image plane is tessellated with a square grid of unit length. A pixel is identified with each of these squares and taken as its receptive field. The edge model is a step edge of unit contrast; zero intensity on one side, unit intensity on the other. Hence the intensity of a pixel is the integration of the light intensity over its associated square with uniform weighting over the square. The intensity for a pixel is computed as follows: first, because of the symmetry of the pixel grid and the edge operator, we need consider only orientations in the range 0° to 45°. The computation can be easily extended to other angles with a little algebra. Take the center of a pixel as its origin. Then for an edge with orientation θ and offset ρ, the integrated intensity $f(\rho,\theta)$ for the pixel on that edge is the area of the intersection of the pixel with the bright side of the edge as follows:

$$
f(\rho,\theta) = \begin{cases}
1 - f(-\rho,\theta), & \text{for } \rho < 0, \\
0, & \text{for } \theta = 0^{\circ}, \rho \geq 1/2, \\
1/2 - \rho, & \text{for } \theta = 0^{\circ}, 0 \leq \rho < 1/2, \\
0, & \text{for } \theta > 0^{\circ}, 1/2 \leq \eta, \\
\tan/8 + 1/4 - \rho/2\cos\theta + 1/8\tan\theta - \rho/2\sin\theta + \rho^2/\sin 2\theta, & \text{for } \theta > 0^{\circ}, -1/2 \leq \eta < 1/2, \\
1/2 - \rho/\cos\theta, & \text{for } \theta > 0^{\circ}, \eta < -1/2,
\end{cases}
$$

where $\eta = \rho\csc\theta - 1/2\cot\theta$. The gradient magnitudes and directions are obtained by taking the Euclidean norm and this gradient vector. That is,
$G_m(i,j) = \sqrt{G_x(i,j)^2 + G_y(i,j)^2}$,
$\tan\theta = G_y(i,j) / G_x(i,j)$, where $G_x(i, j)$ and $G_y(i, j)$ are x and y components of intensity gradient respectively. From this, all gradient directions are computed given the edge model whose angle parameters happen to be the gradient direction, which can be considered as a fuzzy variable.

3.2 Membership Function and Algorithm

Here the major concern is the distributions of gradient magnitudes and directions when an image function is defined on the properly defined image matrix with given edge operator. We can assume that those distributions may reflect fuzzy behaviors of the observed gradient direction of a pixel, even though this analytical computation is done under ideal image condition. Assume that the image lines are infinitely extended and the boundary of 256 × 256 image plane is extended by one pixel outwards for computational convenience. The normal distance ρ in image function is limited in the range [0, 1]. Since we are interested in the behavior of edge operator in terms of the distributions of gradients, the outside of this range is not important for general statistics. Also because of the symmetry of the pixel grid and edge operators, the angles in the range 0° to 45° are sufficient for analysis. The Fig. 1 shows the distribution space of given

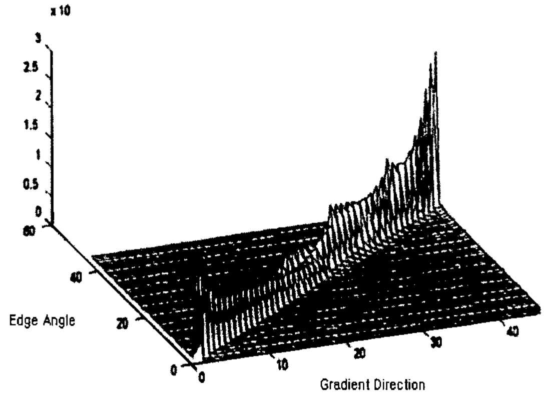


Fig. 1. The distribution space of edge angles which can be considered as an ideal gradient directions.

edge direction from 0° to 45° , which are ideal gradient directions to be observed at the pixels along the edge. The y-axis represents the edge directions. The x-axis shows the distributions of the gradient directions actually computed using Sobel edge operator. To generate the membership function of each gradient direction each column tagged by each element of x-axis is projected onto the y-axis and normalized to set the maximum value of each membership function to unity. In this example the incremental angle step is set to unity. The size of this step could be adjusted according to the requirements imposed on the applications. The Fig. 2 shows the resultant membership functions from 0° to 12° . The membership functions of other angles can be easily derived based on this base functions. The following is the MFHT algorithm.

procedure MFTH;

$\triangle\theta = 2\pi/L$; $\triangle\rho = 2\sqrt{N^2 + M^2}/K$;

for i=1 to N do

for j=1 to M do

$l = \lfloor L \cdot G_\theta(i,j)/2\pi \rfloor$; $\theta_l = \triangle\theta \cdot (l-1)$;

$k = round(O - (j-m)\cos\theta_l - (n-i)\sin\theta_l)$;

$H(k,u) = H(k,u) + G_m(i,j) \cdot \mu_{\theta_l}(u)$

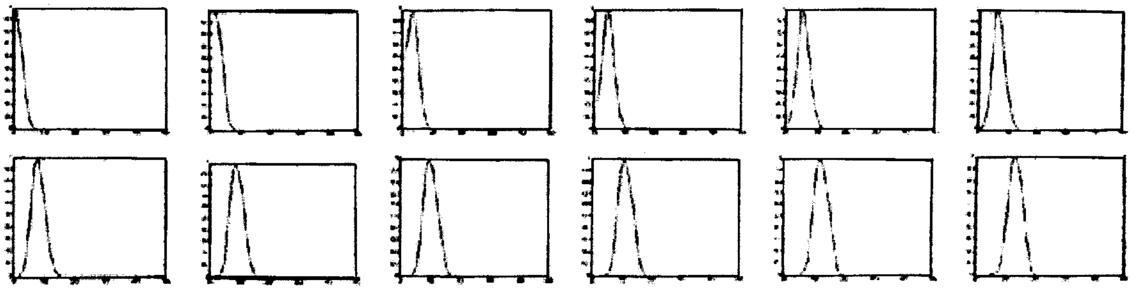


Fig. 2. The membership function $\mu_A(u)$, $u \in [0°, 12°]$. The fuzzy subsets A are of gradient directions from 0° to 12° (the order is from left to right and from top to bottom).

end; end;

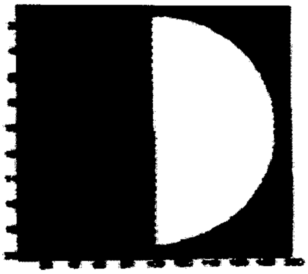
Here $\mu_{\theta_i}(u)$ is the fuzzy membership function of θ_i.

4. Experiments

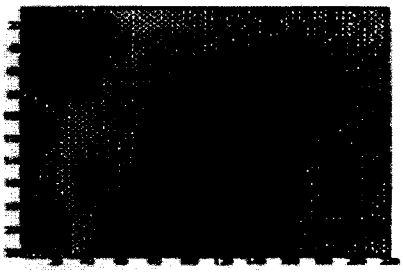
Two experiments are conducted to evaluate the performance of MFHT. First method uses the synthetic image as shown in Fig. 3(a), while in second one, wooden block image as shown in Fig. 3(b) is captured by using Panasonic WV 1600 TV camera with 50mm(f/3.5) lens and Data translation DDT2803 frame grabber is used to digitize this image(6 bits/pixel, 240×256 resolution).

A method commonly used to judge the usefulness of an image analysis algorithm is to measure its performance under varying signal-to-noise ratios. Signal-to-noise(SNR) is defined as $SNR = 10log_{10}(\Delta h^2 / \sigma^2)\ dB$, where Δh is the average step between the regions of the image, where a step is the difference in gray level between the regions. The σ is the standard deviation of the noise and σ^2 its variance. SNR is undefined if the variance of noise is zero. To simplify the computation of Δh and the measurement process, the simple half circle will be contained in a test image I_t as shown in Fig. 3(a)

The center of half circle image is centered at the image plane and rotated from 0° to 45° . Also



(a)



(b)

Fig. 3. (a) the synthetic image, (b) the wooden block image

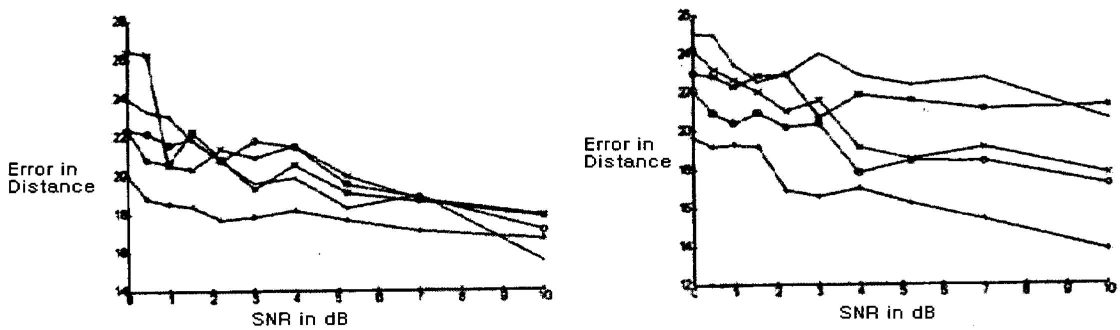
three different lengths of the diameter of circle were tested. A noise matrix I_n , consisting of normally distributed random numbers, was generated with mean at 0 and standard deviation of 1.0. The degree of noise is controlled by adjusting the variance parameters. The final image matrix is I which is $I_t + I_n$. A root-mean-square error is used to determine the closeness of the elected candidate to the expected total ballots for the expected candidates in Hough space. For example, assume that the expected locations of candidates are $\{(\rho_1, \theta_1), (\rho_2, \theta_2), (\rho_3, \theta_3), ..., (\rho_n, \theta_n)\}$. Also assume that the elected candidates are $\{(\rho_1^*, \theta_1^*), (\rho_2^*, \theta_2^*), (\rho_3^*, \theta_3^*), ..., (\rho_n^*, \theta_n^*)\}$ Now the Euclidean distances between the detected peaks and the expected peaks in parameter space is given by

$$E=(1/n)\sum_{i=1}^{n}\sqrt{(\rho_i-\rho_i^*)^2+(\theta_i-\theta_i^*)^2}.$$

The performance of MFHT was measured, and compared to the algorithms proposed by others such as SHT, FHT[15], WPHT[1][13] and PRO [16]. In FHT, each pixel votes for a single candidate if it has sufficient gradient magnitude. The orientation of its gradient and the parameter equation fixed by its position is used to select the candidate. In WPHT, every pixel votes uniformly for two candidates whose gradient orientations are closer than other candidates. The PRO can be considered as a generalized voting system. Every pixel votes for multiple candidates in Hough space. The position of each pixel and the parameter equation fixed by its position confines the scope of candidates to be voted for. Each pixel has a single ballot whose weight is determined by its gray intensity. The acquired ballots of each candidate is differentiated by comparing the voted ballots for competitor locally. After obtaining $H(k,l)$, the peak detection should be performed in parameter space in order to elect the true candidates who represent linear features. In this paper a modification of the NR scheme[13] is used. The NR scheme is a global peak detection algorithm, that is, it detects only a single peak in whole parameter space. It superimposes the local window centered at a candidate in parameter space and selects the candidate with a local maximum vote in a window, where the summing up the votes in a window results in global maximum sum. That is,

$$r, c = m, n: \max\left(\sum_{p=0}^{n_\rho-1} H(m-p, n)\right),\ where\ n_\rho-1 \le m \le \rho_{size}-1,\ 0 \le n \le \theta_{size}-1.\ (\rho_{peak}, \theta_{peak}) = k, c:\quad H(k, c) = \max(H(j, c)),\ where\ r-n_\rho+1 \le j \le r,$$

where $\rho_{size}=2\sqrt{N^2+M^2}/\Delta\rho$, $\theta_{size}=2\pi/\Delta\theta$. Fig. 4(a) and Fig. 4(b) show the results of error measurements with two different resolutions of Hough space. As expected, the suggested MFHT performs well compared with the other schemes. Fig. 5 shows the Hough spaces generated by SHT, FHT, WPHT and WHT. Those spaces are normalized by the maximum accumulation count for visual comparisons. The advantages of the suggested method are shown. The decrease of irrelevant spread in Hough space is obvious and



(a) (b)

Fig. 4. The plotted results: (a) $\Delta\theta$ = 1, (b)$\Delta\theta$ =5. o: WPHT, x: SHT, +: MFHT, .: FHT, and*: PRO.

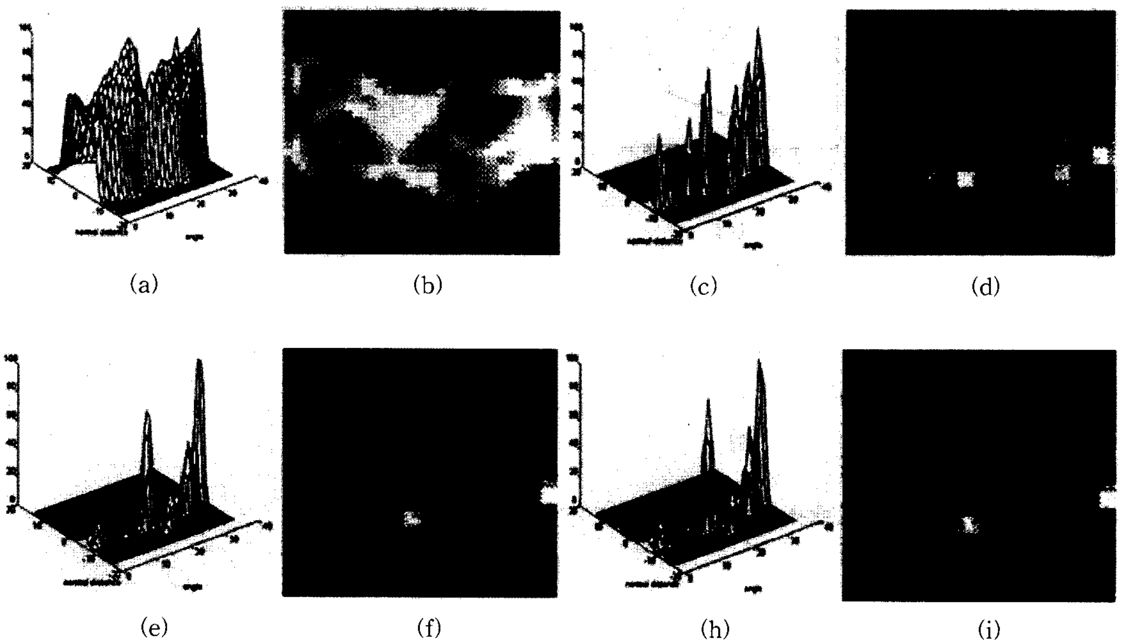


Fig. 5. The resultant Hough spaces: (a)(b)SHT, (c)(d)FHT, (e)(f) WPHT (h)(i) MFHT

the peaks are more isolated, which makes the task of peak detection easier.

5. Conclusion

To extract the linear features in gray scale images, the SHT is modified to accommodate the variations of gradient directions which are considered as fuzzy variables, The membership functions are computed using the predefined image and edge models by collecting the observed distributions of gradient directions imbedded in the synthetic image along the edge positions. The experiments demonstrate that the proposed method shows better performance than the conventional ones.

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