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A Mechanism to Derive Optimal Contractor-type & Action Combinations of a Single-source Procurement Contract

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■ Abstract ■

In sole-source procurement contracting for government goods and services, the buyer (government) needs to derive the optimal actions[1] from the contractor so the buyer can obtain the maximum utility and the contractor, or single-source supplier, is guaranteed the equivalent of a minimum level of profit. Under the assumption of risk-neutrality for both the buyer and the contractor and the buyer's unobservability of the contractor's action, it is necessary for the buyer to design a (mathematical) model to achieve the above objective. This paper considers the mathematical formulation in which two problems - moral hazard and adverse selection - are present simultaneously; furthermore, from the formulation, a GAMS (General Algebraic Modeling System) program is used for a possible buyer to obtain the optimal actions.

1. Introduction

Among many procurement contracts, a buyer (principal, firm, military, government, etc.) frequently needs only one supplier (agent, military industry, private or public enterprises, etc.) because of the characteristics of the goods being procured. This type of contract is called a *sole-source procurement contract*. Such sole-source procurement contract situations are typically found in such areas as military or government where special skills and secrecy are required from the contractor in private industry.

The procurement of military and government goods is often characterized by negotiations between the military or government and the sole supplier. This kind of procurement is complicated by three factors : first, because items procured for the military or government often involve a unique design and require high technology, the procurement contract may involve a considerable cost risk ; second, the pro-

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1) Actions (or efforts) used in this paper refer to monetary efforts or investments made by the contractor to perform a given contract. For more information, see *ASSUMPTIONS*.

duction process for military and government items is sometimes quite complex. Therefore, the military and government have difficulties in monitoring the action (or effort) taken by the supplier to reduce costs after contracting. In military or government procurement contracting where the military or government cannot perfectly monitor the complex production processes, the sole contractor may take unmonitored action that could decrease the government's or military's expected utility ; and third, in many procurement contracts, it is often difficult to differentiate direct costs from overhead costs. Therefore, a procurement contract must be based on imperfect monitors of the production costs.

The purpose of this paper is to develop a general mathematical formulation that guarantees the buyer the maximum utility and the contractor a reasonable level of profit by deriving the optimal contractor-type and action combinations under specific assumptions that will be explained later.

2. Problem Statement

In many procurement contracts, especially in sole-source situations whose characteristics were mentioned earlier, a buyer frequently encounters moral hazard and adverse selection problems. A moral hazard problem occurs when the contractor undertakes certain actions that affect the buyer's valuation of the transaction but that the buyer cannot monitor perfectly. Therefore, incentive schemes are often used to reduce the problem of moral hazard. This means the contract must be structured so the contractor will, in its own best interests, take actions the buyer would prefer. In other words, the buyer would like to structure the contract so the contractor bears fully the consequences of its actions. A typical example of the moral hazard problem is fire insurance, where the insuree may or may not exhibit sufficient care in storing flammable materials.

Other than the problem of moral hazard, an adverse selection problem occurs when the contractor knows information pertaining to the transaction that is relevant but unknown to the buyer. Market signaling can be the solution to this problem, because the contractor who possesses superior information signals what they know through their actions. A classic example of an adverse selection problem is life insurance, where the insuree may know things about the state of their health that are unknown to the insurer (Kreps, 1990).

In addition to the above two specific situations that constrain both the buyer's and the contractor's behavior, there is another factor that affects the buyer's utility and the contractor's profits : contractor-type. According to Brown (1986), the contractor-type may consist of two factors : the contractor's risk-preference and the probability assessment of the cost of the project. In this paper, it is assumed both the buyer and the contractor are risk-neutral; therefore, the contractor-type involves only the contractor's minimum expected costs of the project.

In summary, the focus of this paper is on one of the buyer's problems : to determine what level or what amount of action in cost reduction the buyer should attempt to derive from the contractor in order to obtain maximum utility as well as guarantee the contractor a reasonable level of profit, also called *reservation price*, given that the buyer is unable to observe the contractor's efforts. The buyer's uncertainty about the contractor's action is a result of the uncertainty about the contractor-type; that is, action is a function of contractor-type. Action levels depend on the contractor's minimum expected cost for the project. This means that the likelihood of observing certain cost outcomes depends on the contractor's minimum expected cost of the project.

The paper most related to this paper is Laffont and Tirole's (1986). They emphasize using accounting data in procurement contracts when the contractor has superior information about the cost of the project. In their paper, the contractor announces an expected cost and is given an incentive contract that is linear in cost overruns.

3. Assumptions

When the government procurement contracting problem occurs in a sole-source situation, the possible final cost of the project is easily estimated by both the government and the contractor because of each party's previous and present experience with similar contracts. In most cases, however, the contractor has superior information about the cost of the project. It is not difficult for the buyer to observe the cost of the project after the project has been supplied by the contractor, but it is more difficult for the buyer to observe the contractor's action and the cost disturbance, both of which influence the final observed cost. This is because the contractor usually chooses the action it will undertake after contracting has already taken place. The interpretation of the contractor's action depends on the contract situation and its mechanisms. It may be regarded as the contractor's R & D cost for performing the project in some contracts, and sometimes it may be interpreted as an effort to reduce the project's final costs. In other cases, the contractor's action can be a kind of special labor cost used to boost the employees' morale towards the project. Examples can be found in some of the Just-In-Time (JIT) concepts practiced in Japanese auto-making industries. The company, especially at high management levels, encourages its quality circles employees who meet on a scheduled basis to discuss their function and the problems they are encountering in order to improve its productivity and quality. For this purpose, the company pays for all of the necessary expenditures, although a quality circle is made up of volunteers. To initiate this kind of quality circle, the company must provide a trained person who can lead the group. Statistically, actual evidence has been impressive. In addition to quality circles, most major Japanese companies give profit-based bonuses to all employees (Chase and Aquilano, 1989). Such bonuses may be good incentives for high productivity and good quality of their products. Therefore, the buyer needs to induce a reasonable level of action from the contractor, regardless of the interpretation of the contractor's action.

To reflect the real life contracting situation, the following general assumptions are necessary in order to formulate the contract problem explained thus far :

- Let I represent the number of possible final costs of the project in the sole-source procurement contract whose elements are $c_1, c_2, ..., c_i, ..., c_I$. As mentioned before, the number and values of possible final costs of the project can be easily assessed by both the buyer and the contractor because of their past and present experiences with similar contracts ;

- It is assumed there are J possible contractor-types in this case, and each type is denoted by t_j, where $j \in \{1,2,...,J\}$. Through the selection of a particular contract, the contractor is indicating its type. The contractor may not choose the one the buyer had hoped a contractor of its type would choose. This is why the buyer wants to design the contract to be "self-selective." *Self-selective* means the contractor of a given type finds a particular contract to be most desirable for its type and this choice has been anticipated by the buyer ;
- It is natural to think there will be a certain number of contractor actions, K, for each type. The possible action for each type is denoted as $A \in \{a_1, a_2, ..., a_K\}$. The buyer can assess a set of possible actions and feel that the likelihood of actions beyond this set is so small that it can approximate the likelihood of these occurring to be zero. It is possible the buyer can assess the possible range of actions or all of the possible discrete values of the actions (in case of this paper) for each type. Here, A indicates all of the possible actions for each contractor-type ;
- As a result of assuming J contractor-types and K contractor's actions for each type, we can think of the following J optimal contractor-type and action combinations ;

〈Table 1〉 Optimal contractor-type and action combinations

Type	1	......	j	......	J
Action	$a^*(t_1)$	......	$a^*(t_j)$	......	$a^*(t_J)$

$a^*(t_j)$: optimal action for contractor-type j.

Table 1 indicates there should be an optimal action that best suits each contractor-type. The problem of this paper is to find out the optimal contractor-type and action combinations - $(t_j, a^*(t_j))$ - from all possible combinations that will produce the buyer's maximum utility and the contractor's reservation price.

4. Formulation

As mentioned earlier, it is assumed both the buyer's and the contractor's risk-preferences are known and assumed to be risk-neutral. Therefore, the contractor-type (t_j) here means the contractor's minimum expected costs for the project. It is necessary to clarify the differences between the meaning of the contractor-true type (t_j, expected cost of the project) and that of the contractor-reported type, T_r. The contractor-true type (t_j) is the contractor's a priori expected cost for the project when the contractor's productivity is most efficient and denoted by minimum expected cost, where $j \in \{1,...,J\}$. On the contrary, the contractor-reported type T_r, $r \in \{1,...,J\}$, may be different from the contractor-true type t_j. The contractor is supposed to report its type to the buyer in the early stages of the contract, e.g., in the contract proposal. Therefore, the contractor-reported type is known to both sides before the contractor devotes its action to the project ; whereas the contractor-true type is not told to the buyer even though it is sometimes equal to the reported type. The buyer can, however, optimally design the contract by accounting for the possible types and their incentives. As a result, the expectation is that the reported type will be equal to the contractor's actual type.

When the contractor chooses its profit schedule, $P(c_i, T_r)$, it prefers to report T_r, given its type t_j, that will bring the optimal profit to itself. Hereafter, the contractor-true type is denoted by t_j and represents the contractor's believed minimum expected cost of the project when most efficient, whereas T_r represents the contractor-reported type, based on maximizing its expected profit, which is a function of the offer $P(c_i, T_r)$ over all T_r. $P(c_i, T_r)$ represents the profit awarded to the contractor reporting type T_r on the realization of the project's cost (c_i). After delivering the project, the buyer and the contractor are aware of the final cost; therefore, the profit awarded to the contractor depends on the final cost and contractor-reported type.

The buyer's uncertainty on the value of t_j, $g(t_j)$, is assumed to be assessed by the buyer with the help of experts or consultants familiar with the various assessment procedures. The buyer's and contractor's utility functions are represented by $U^0[*]$ and $U[*]$ respectively and are defined over monetary wealth. The probability distribution on cost is represented by $f(c_i \mid t_j,a_k)$, where a_k represents the contractor's action as a function of contractor-type. The final cost of the project c_i is dependent upon the contractor-true type and action. This is why the probability distribution on cost is denoted by the conditional probability $f(c_i \mid t_j,a_k)$. Once the contractor-true type is given, action is taken by the contractor, then the final cost of the project is determined according to the above conditional probability assessment. It is assumed $f(c_i \mid t_j,a_k) > 0$, for every $j \in \{1,...,J\}$, and that $\Sigma f(c_i \mid t_j,a_k) = 1$, for all $j \in \{1,...,J\}$ and $k \in \{1,...,K\}$. The buyer's uncertainty about the contractor's ultimate action is a result of the uncertainty about the contractor-true type; that is, action is a function of t_j. There exist K possible actions once the contractor-true type t_j is given. Among many possible actions, there must be the optimal action or actions ($a^*(t_j)$) for each type. The action has a direct influence on the final cost c_i.

The expected utility of the contractor, which is determined by its reported type T_r, true type t_j, and action a_k is :

$$\Sigma_{i=1..I} U[P(c_i, T_r)] f(c_i \mid t_j,a_k) - d(a_k) \quad (1)$$

where $d(a_k)$ is the contractor's disutility of action. All random variables are assumed to be discrete in this paper. Concerning the contractor's disutility of action, the definition and its application are not as complicated in such cases as R & D costs and special labor costs for decreasing the final cost, in which the contractor's action is easily measured in monetary factors. But in other cases, e.g., a JIT environment as mentioned earlier, the interpretation of the contractor's action and its application to a model are much more complicated. This is a managerial problem because the contractor's action in the JIT concept consists of less obvious sources of efficiency than those of previous cases.

The first constraint, which is denoted by *incentive-compatibility constraint*, is written as (2). K*J incentive-compatibility constraints exist as a result of the contractor's being type j and possibly reporting its type r.

$$\Sigma_{i=1..I} U[P(c_i, t_j)] f(c_i \mid t_j, a^*(t_j)) - d(a^*(t_j)) \geq \Sigma_{i=1..I} U[P(c_i, T_r)] f(c_i \mid t_j, a_k) - d(a_k) \quad (2)$$

for each j, $j \in \{1,...,J\}$, and for all r and k, $k \in \{1,...,K\}$ and $r \in \{1,...,J\}$.

The formulation seeks to determine the optimal profit schedule that induces the contractor to report its true type : the contractor must have the correct incentives to go along with the buyer's ideas of what kind of contractor-type should be used and how much action should be provided. The contractor should be cautious in taking the action if the buyer's contract mechanism includes disutilities as shown in the above constraint.

The contractor is willing to undertake the project as long as its expected net utility from performing the project is at least as large as the net utility for its next best opportunity. This is the so-called the contractor's reservation price, and it is determined by market forces or by negotiation processes. The next *participation constraint* guarantees the contractor's reservation price.

$$\Sigma_{i=1..I} U[P(c_i, t_j)] f(c_i \mid t_j, a^*(t_j)) - d(a^*(t_j)) \geq U[R] \quad (3)$$

for each j, $j \in \{1,...,J\}$

where R denotes the contractor's reservation price. The reservation price is usually a reasonably fixed value that is assumed to be known to the buyer.

Then the buyer's problem is a problem of maximization of its expected utility

$$\Sigma_{j=1..J} \Sigma_{i=1..I} U^0[V - c_i - P(c_i, t_j)] f(c_i \mid t_j, a^*(t_j)) g(t_j) \quad (4)$$

subject to (2) and (3),

where V is the buyer's valuation of the project. It is assumed the buyer's expected utility $U^0[*]$ and the contractor's expected utility $U[*]$ are continuously differentiable, strictly increasing, and concave. From the mathematical model, both the buyer and the contractor maximize their expected utilities, and the contractor is guaranteed the reservation price. However, it is sometimes impossible to obtain the optimal contractor-type and action combinations without performing a numerical analysis. The following summarizes the notations and formulation that have been explained up until now.

4.1 Notations

① c_i	: possible final costs of the project, $i \in \{1,...,I\}$;
② t_j	: contractor-true type, which means minimum expected cost of the project, $j \in \{1,...,J\}$;
③ T_r	: contractor-reported type that is given to the buyer before the contractor's action, $r \in \{1,...,J\}$;
④ a_k	: action for each type taken by contractor, k $\{1,...,K\}$;
⑤ $P(c_i, T_r)$	: profit schedule awarded to the contractor as a function of final cost c_i and contractor-reported type T_r;
⑥ R	: contractor's reservation price;
⑦ $f(c_i \mid t_j, a_k)$	: probability distribution on cost that is conditional on contractor-true type and following action;
⑧ $d(a_k)$	: contractor's disutility of action;
⑨ $g(t_j)$	: probability distribution representing the buyer's uncertainty on the value of t_j;
⑩ $U^0[*]$	: buyer's utility;
⑪ $U[*]$	: contractor's utility; and
⑫ V	: buyer's valuation of the project.[2]

2) Valuation (or Value) is different from utility. Value is the worth that a person attaches to a good or service whereas utility is the power to satisfy human wants. Put another way, value is an appraisal of utility in terms of a medium of exchange (Fabrycky and Blanchard, 1991).

4.2 Formulation

Maximize
$P(c_i, t_j)$ and $a^*(t_j)$

$$\sum_{j=1..J} \sum_{i=1..I} U^0[V-c_i-P(c_i, t_j)]\ f(c_i \mid t_j, a^*(t_j))\ g(t_j)$$

subject to

$$\sum_{i=1..I} U[P(c_i, t_j)]\ f(c_i \mid t_j, a^*(t_j)) - d(a^*(t_j)) \geq \sum_{i=1..I} U[P(c_i, T_r)]\ f(c_i \mid t_j, a_k) - d(a_k)$$

for all r and k, $r \in \{1,...,J\}$, $k \in \{1,...,K\}$;

$$\sum_{i=1..I} U[P(c_i, t_j)]\ f(c_i \mid t_j, a^*(t_j)) - d(a^*(t_j)) \geq U[R]$$

for each j, $j \in \{1,...,J\}$

5. Numerical Example

In testing the mathematical model, the GAMS[3] (General Algebraic Modeling System) is used to run a set of numerical data as follows and also to find optimal contractor-type and action combinations :

5.1 Data Examined

- Possible final costs, c_i: c_1=8,000, c_2=9,000, c_3=10,000
- Contractor-type (minimum expected cost of the project), t_j: t_L=8,500, t_H=9,500
- Possible actions for each type, a_k: a_1=1,000, a_2=1,500, a_3=2,000
- Buyer's valuation of the project, V: V=16,000
- The buyer's assessment of the contractor-type, $(g(t_L),g(t_H))$: (0.3, 0.7), (0.5, 0.5), (0.7, 0.3)
- Reservation prices, R : 0 and 500
- Probability distribution on cost, $f(c_i \mid t_j,a(t_j))$:

	t_L,a_1	t_L,a_2	t_L,a_3	t_H,a_1	t_H,a_2	t_H,a_3
c_1	.10	.30	.40	.13	.37	.39
c_2	.60	.55	.50	.30	.33	.48
c_3	.30	.15	.10	.57	.30	.13

5.2 Functions Used[4]

- Buyer's expected utility :

$$U^0[w] = U^0[V-c_i-P(c_i, t_j)]$$
$$= 2.5 * (1-\exp(-0.000001 * (V-c_i-P(c_i, t_j)))) / (1-\exp(-0.000001))$$

- Contractor's expected utility :

$$U[w] = U[P(c_i, T_r)]$$

3) The GAMS (General Algebraic Modeling System) is designed to make the construction of and solution to large and complex mathematical programming models more straightforward for programmers and more comprehensible to users of models from other disciplines, e.g., managers and economists. There are three solvers available - GAMS/BDMLP, GAMS/ZOOM, GAMS/MINOS - in the personal computer version of GAMS. GAMS/BDMLP works well for most models. However, numerical stability is not as good in GAMS/BDMLP as in GAMS/MINOS and GAMS/ZOOM. The zero-one program GAMS/ZOOM is able to solve continuous linear problems and will in general solve these problems more reliably and efficiently than GAMS/BDMLP. GAMS/MINOS is available for solving linear or nonlinear programming problems. GAMS/MINOS has been adapted from MINOS 5.2 and is also more reliable and efficient than GAMS/BDMLP (*GAMS User's Guide*, The Scientific Press [1988]). The GAMS/MINOS solver may be useful for solving the GAMS program of sole-source procurement contract that is presented in this paper because the mathematical functions that can be selected to express the buyer's and contractor's utilities, the contractor's disutility, and the buyer's assessment of the final cost of the project may be nonlinear as well as linear.

4) Although both the buyer and the contractor in this research are considered risk-neutral, risk-aversion utility functions with 0.000001 risk-aversion coefficient and 2.5 scaling constant are used to express two parties' risk-neutral utility. A reasons for this is : absolute-risk-neutral cases are rare in the real-world and both parties' risk-neutral is a special case of risk-aversion case; therefore, risk-aversion utility that can be also used for risk-neutral situation is selected for more generalizability or applicability to other similar situations. However, in strict-or absolute-risk-neutral cases, pure risk-neutral utility should be used.

$= 2.5 * (1-\exp(-0.000001 * P(c_i, T_r))) / (1-\exp(-0.000001))$

- Disutility of action :

  $DISU[a] = 10000 * \ln((a/10000) + 1)$

The data - possible final costs, possible actions for each contractor-type, and probability distribution on cost - are assumed to be discrete values that are presented in the *Data examined* section. The data have been chosen for this particular example; therefore, if other data were selected, the numerical results would be different from those presented here.

The buyer's and the contractor's utility functions used are exponential utilities with a risk-aversion coefficient of 0.000001. The exponential utilities are mainly used for risk-aversion cases, but in this paper, the same utility functions with a 0.000001 risk-aversion coefficient are used to express the buyer's and the contractor's utilities. By suppressing the risk-aversion coefficient to be very close to 0, the exponential utility can also be used for risk-neutral cases. The constant 2.5 of the utility function is a kind of scaling constant that yields utility values over a reasonable range. If the data used in this numerical example are different, this scaling constant should be replaced by another one that yields utility values over reasonable ranges. The buyer's and contractor's utilities are defined over wealth that are included in brackets. As seen from the utility functions, the contractor tries to increase its profit in order to obtain the maximum utility. At the same time, the buyer tries to decrease the payment to the contractor, or the contractor's profit, in order to get the maximum utility. The disutility function for action here is a natural logarithmic function with a scaling constant of 10,000. By using the scaling constant of 10,000, the disutility of action is assumed to be very close to linear in this example. In other situations, other disutility functions may be used to best express the disutility of the contractor's action.

6. Results and Analyses

The following are the analyses of the numerical results. The buyer's valuation of the project, V, is assumed to be 16,000, and 3 different pairs of the buyer's assessment of the contractor-type - $(g(t_1),g(t_2))$: (0.3, 0.7), (0.5, 0.5), (0.7, 0.3) - and 2 different reservation prices - 0 and 500 - have been examined for this particular numerical example. This makes six different sets of results shown in Table 2 : buyer's optimal utility, optimal actions for two contractor-type (low and high), and payment to the contractor. In other words, the results shown in Tables 2 are solutions of the buyer's problem of maximizing its utility in the presence of 14 constraints : a set of 3 different actions for each of 2 contractor-types makes 12 incentive-compatibility constraints, and the 2 contractor-types cause 2 participation constraints for each contractor-type. In addition, Table 3 shows all possible actions for each contractor-type and corresponding buyer's utility and payment to the contractor of an example case of $g(t_L)=.7$, $g(t_H)=.3$, R=0 from which the buyer's optimal utility, desirable actions for each contractor-type, and payment to the contractor are derived. (The other five sets of results in Table 2 were also extracted from the same table as Table 3 using different values of

$g(t_L)$, $g(t_H)$, and R.)

Table 2 shows the second action of 1,500 is optimal for a low contractor-type and the second action of 1,500 is also optimal for a high contractor-type. From Tables 2 and 3, we can see the buyer's payments decrease as the final costs of the project increase. This outcome is desirable because it drives the contractor to try to reduce the final costs of the project in order to avoid penalties (negative profit). It is also observed that the buyer's payments increase as the contractor's action increases. But this does not mean the contractor always tries to take high action, because high action does not always guarantee the contractor high profits. High action can affect the contractor in a negative way, as captured by the disutility function. There is also a certain pattern in the buyer's payment schedules (see Table 3). When the same action is taken for any type by the contractor, the buyer's payments are almost the same with the exception of action cases (1000,

〈Table 2〉 Summary Results of All Possible Cases

All possible cases	Buyers optimal utility	Optimal Actions for Contractor type (low, high)	Buyers optimal payment					
			Payment for low contractor-type			Payment for high contractor-type		
			$P(c_1,t_L)$	$P(c_2,t_L)$	$P(c_3,t_L)$	$P(c_1,t_H)$	$P(c_2,t_H)$	$P(c_3,t_H)$
$g(t_L)=.3$ $g(t_H)=.7$ $R=0$	16288.717	1500 & 2000	1089.343	627.696	-495.895	1620.093	342.052	-509.069
$g(t_L)=.3$ $g(t_H)=.7$ $R=500$	15791.520	1500 & 2000	1275.953	857.340	-376.800	1820.437	542.140	-309.151
$g(t_L)=.5$ $g(t_H)=.5$ $R=0$	16323.125	1500 & 1500	991.085	483.232	-25.626	1066.695	407.665	100.267
$g(t_L)=.5$ $g(t_H)=.5$ $R=500$	15826.012	1500 & 1500	1191.216	683.435	174.244	1266.929	607.767	300.307
$g(t_L)=.7$ $g(t_H)=.3$ $R=0$	**16362.896**	**1500 & 1500**	**991.004**	**483.313**	**-25.761**	**1066.695**	**407.665**	**100.267**
$g(t_L)=.7$ $g(t_H)=.3$ $R=500$	15865.792	1500 & 1500	1191.187	683.464	174.196	1266.929	607.767	300.307

Action : $a_1=1000$, $a_2=1500$, $a_3=2000$
Contractor-type : $t_L=8500$ (low), $t_H=9500$ (high)
Final cost : $c_1=8000$, $c_2=9000$, $c_1=10000$
Buyers valuation of the project : V = 16000

〈Table 3〉 Buyers Optimal Utility and Payment Schedule for the Case of $g(t_L)=.7$, $g(t_H)=.3$, $R=0$

Actions for (low type, high type)	Buyers payment to the contractor						Buyers utility
	Payment for low contractor-type			Payment for high contractor-type			
	$P(c_1,t_L)$	$P(c_2,t_L)$	$P(c_3,t_L)$	$P(c_1,t_H)$	$P(c_2,t_H)$	$P(c_3,t_H)$	
(a_1,a_1)	981.319	481.363	-18.638	996.378	337.394	264.201	15816.187
(a_1,a_2)	981.359	481.358	-18.642	1066.695	407.665	100.267	16063.687
(a_1,a_3)	1019.668	519.677	19.681	1620.093	342.052	-509.069	16011.606
(a_2,a_1)	990.973	483.344	-25.816	996.378	337.394	264.201	16115.396
(a_2,a_2)	**991.004**	**483.313**	**-25.761**	**1066.695**	**407.665**	**100.267**	**16362.896**
(a_2,a_3)	1076.083	588.583	-326.155	1620.093	342.052	-509.069	16310.798
(a_3,a_1)	1513.034	406.655	-787.251	996.378	337.394	264.201	16079.741
(a_3,a_2)	1513.034	406.655	-787.251	1066.695	407.665	100.267	16327.241
(a_3,a_3)	1551.394	444.975	-748.989	1620.093	342.052	-509.069	16275.153

Action : $a_1=1000$, $a_2=1500$, $a_3=2000$
Contractor-type : $t_L=8500$ (low), $t_H=9500$ (high)
Final cost : $c_1=8000$, $c_2=9000$, $c_1=10000$
Buyers valuation of the project : V = 16000

2000).

Table 2 shows the effects the changes in the buyer's probability assessment of the contractor-type and contractor's reservation price have on the buyer's utilities and payments. When the same reservation price (0) is used (2nd, 4th, and 6th rows of Table 2), the buyer's payments are almost the same with a little variation. As the buyer's probability assessment of a low contractor-type increases (as the buyer's probability assessment of a high contractor-type decreases), an important change in the buyer's utility happens even though there is little change in the buyer's payment to the contractor.

Here, it should be noted that the buyer's assessment of the contractor-type has an important impact on its utility and more importantly on what action the buyer would like the contractor to take. When a contractor's reservation price of 500 is used (3rd, 5th, and 7th rows of Table 2), the effects that the changes in the buyer's probability assessment of the contractor-type have on the buyer's optimal payment schedule and utility are almost identical to those in previous cases when the reservation price is 0. As the reservation price changes from 0 to 500, the buyer's optimal payment schedule increases by about 200 in all action cases. Instead of changing the buyer's payment, the change in the contractor's reservation price affects the buyer's utility in a different way. That is, as the contractor's reservation price increases from 0 to 500, the buyer's utility decreases by about 500 in all action cases.

In this paper, the sole-source procurement contracting problem in the presence of moral hazard and adverse selection problems is considered. But under the assumption of the buyer's observability of the contractor's action level as well as the output level, the problem of this paper would be changed to the buyer's problem of maximizing its utility with only participation constraints that guarantee the contractor a certain level of reservation price. Finally, all through the numerical example and its analysis, the optimal action and contractor-type combinations are obtained. By writing the program in GAMS, the numerical results and analysis can be changed easily if other conditions and assumptions need to be made. In the next section, conclusion and implications of the importance of obtaining the optimal contractor-type and action combination are discussed.

7. Conclusion and Discussion

With actual numerical data, it's been shown that the optimal actions for each contractor-type can be obtained using the GAMS program. However, it should be noted that all of the results and analyses can vary according to the different assumptions included and the different data prepared in a particular contract problem. Also, in many cases, theoretical analyses do not provide closed-form solutions, whereas numerical analyses do, as shown in this paper.

In addition to the appropriate application of the GAMS program to similar contract problems, another buyer's problem is raised immediately. ***How*** does the buyer possibly induce the optimal contractor-type and action combinations it wants from the contractor? One of the answers to this problem assumes the buyer can access the contractor's disutility function. The knowledge of this function permits the buyer to derive an incentive scheme for each potentially desirable action from the contractor. Of course, the buyer's ability to observe the contractor's action is assumed to be impossible, but it may be possible for the buyer to influence the contractor's action to some degree by using appropriate payment schedules that are contingent on observable outcomes and types announced.

The alternative way of inducing the optimal action from the contractor is through a penalty or a tax for certain action which is so unreasonable when it is compared to the outcome that the buyer does not prefer. In a contracting environment characterized by both moral hazard and private information, there are circumstances where the contractor may provide more action than the buyer prefers, so the buyer employs a penalty or a tax for the action. This phenomenon happens because increases in action may increase the buyer's payment to the contractor.

Until now, the objectives of this paper have been to study the economics of sole-source procurement contracts in a model in which the problems of moral hazard and adverse selection are present and to develop a mathematical model that can be used by a buyer of sole-source procurement contracts. But in order to include more real life situations in a model, it is necessary to think not only about economic factors such as costs, reservation prices, etc., but also of the engineering factors. Although this paper provides the buyer with a suggestion on the economic side, it doesn't give the buyer a total solution that also includes the engineering factors such as delivery, quality, reliability, etc. This is necessary because real life procurement contracting situations are more complicated than the one shown here. It may be a more difficult job to reflect these engineering factors in the model presented in this paper. Nevertheless, it is necessary to look for a model that includes engineering factors as well as economic factors in order to make the mathematical formulation more real and useful. This is an important direction for future research in the procurement contract areas related to this paper.

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