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Replacement Policies under Minimal Repair with Cyclic Failure Rates

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ABSTRACT

In many situations, the system failures depend on the operating environmental conditions that vary on time, usually with periodical manners. We use nonhomogeneous Poisson processes whose rate functions exhibit cyclic behavior as well as a long-term evolutionary trend to model the stochastic process of the failures when the rate of occurrence of the failures varies periodically, for example from day to day or between seasons. In this study, we compare optimal policies under the nonhomogeneous process with/without a cyclic component in the failure rate function. The analytical results for various situations are presented along with numerical examples using simulated data.

1. INTRODUCTION

The replace problems under minimal repair have been studied to minimize the expected replacement and repair costs [1~4], since Barlow and Hunter [5] introduced the idea of minimal repair. This paper investigates the problem of determining optimal replacement policies for equipment subject to failures with cyclic rates.

The arrival pattern for system failures has been modeled as a point process. In many cases, the observed failures of the system exhibit periodical variations corresponding to the environmental characteristics at a site. When the rate of occurrence of system failures varies periodically, for example from day to day or between seasons, successive system failure events are stochastically interdependent. It seems that a nonhomogeneous Poisson process (NHPP) with an appropriate rate function is the most plausible, general type of time-varying process for modeling system failure events. Cox and Lewis [6] pointed out that the continuous rate functions for such a process can be approximated to arbitrary accuracy with an exponential-polynomial function (EPF). For an NHPP whose behavior is locally cyclic with a long term evolutionary trend, Lee et al. [7] suggested to use an exponential-polynomial trigonometric function (EPTF) to model the instantaneous arrival rate. The EPTF is an exponential function whose exponent is the sum of polynomial and trigonometric components. The cyclic variation of failure rate can be represented with a trigonometric model identified by three components: frequency, amplitude and phase. The parameterization of these components provides physically interpretable values with which to characterize the cyclic behavior of system failures. Periodicity of failure rate is described by a value of the frequency. The amplitude reflects the range of variation in the number of failures, while the phase is associated with the increasing/decreasing period of failure.

We use an NHPP with an exponential rate function for the system failure model and trigonometric models to represent periodical characteristics in failure processes. In this study, we compare optimal policies under the nonhomogeneous process with/without a cyclic component in the failure rate function using Barlow and Hunter policy II [5] for periodic replacement with minimal repair at failure.

2. NHPP WITH EPTF-TYPE RATE FUNCTION

An NHPP $\{N(t): t \geq 0\}$ is a generalization of a Poisson process in which the instantaneous arrival rate $\lambda(t)$ at time t is a nonnegative integrated function of time. The mean value function (or the integrated function) of the NHPP is defined by

$$\mu(t) \equiv E[N(t)] = \int_0^t \lambda(z)dz \quad \text{for } \forall t \geq 0 .$$

In this study, An NHPP displaying cyclic behavior is assumed to have an EPTF-type rate function. An EPTF of degree m has the form

$$\lambda(t) = \exp\left\{\sum_{i=0}^{m} c_i t^i + A\sin\left(\omega t + \phi\right)\right\} \tag{1}$$

where: $\{c_0, c_1, \cdots, c_m, A, \omega, \phi\}$ is the vector of unknown parameters; the first term in the exponent of (1) is an ordinary function representing the general trend over time; and the second term is a trigonometric function representing a cyclic effect exhibited by the process.

3. BARLOW AND HUNTER POLICY II FOR PERIODIC REPLACEMENT WITH MINIMAL REPAIR AT FAILURE

The idea of minimal repair was introduced by Barlow and Hunter [3]. This idea is that if the system fails, a repair can be made which does not materially change the condition of the system from its condition immediately before failure. Under the assumption of this idea, they suggested the replacement policy with the designation "Policy II". If the replacement period is T, the Barlow and Hunter policy II selects T so as to minimize the total cost per unit time

$$C(T) = \lim_{t\to\infty} \left\{\$_f E\left[N_f(t)\right] + \$_r E\left[N_r(t)\right]\right\}/t$$

where $\$_f$ and $\$_r$ are minimal repair cost and replacement cost respectively.

In [3], it was shown that $E\left[N_f(t)\right] = \mu(T)$ which is the expected number of system failures during T and

$$C(T) = \left[\$_f \mu(T) + \$_r\right]/T\,. \tag{2}$$

4. COST BEHAVIOR FOR FAILURE PROCESSES WITH CYCLIC EFFECT

The failure rate in a system generally increases, even though it may exhibit local fluctuation, as time goes by. In this section, we investigated the cost behavior for the system failure processes with cyclic effect using the EPTF-type rate function of the first order polynomial degree:

$$\lambda(t) = \exp\{\alpha + \beta t + A\sin(\omega t + \phi)\} \tag{3}$$

where (α,β) are the coefficients associated with the initial level and increase rate of the system failure process respectively, and (A,ω,ϕ) are the trigonometric parameters corresponding to amplitude, frequency and phase of periodicity of the process respectively. In (3), the phase parameter ϕ is set to 1.5π so as to have the minimum rate at the initial point. Figure 1 shows changes in the failure rate and expected failure number of system processes related to the EPTF-type rate function when varying the trigonometric parameters of the rate function with fixed initial failure-level $\alpha = 0.0$ and increase failure-rate $\beta = 0.5$. The figure contains in the left the graphs which illustrate variation of the values related to the rate function with constant frequency $\omega = 1.0$ for different amplitudes $A =$ 0.1, 0.5, 1.0, and in the right, with constant amplitude $A = 0.5$ for different frequencies $\omega = 0.5, 1.0, 2.0$. For fixed trigonometric parameters ($A = 0.5$ and $\omega =$ 1.0), Figure 2 displays the graphs for the rate functions with initial failure-level $\alpha = 0.0$ for different increase failure-rates $\beta = 0.1, 0.5, 1.0$ in the left, and with increase failure-rates $\beta = 0.5$ for different initial failure-level $\alpha = -2..0, 0.0, 2.0$ in the right. Figure 1 and 2 also show the total costs per unit time of (2) for the system failure processes with cyclic behavior corresponding to the EPTF-type rate functions considered in these figures when using $\$_f = 1.0$ and $\$_r = 5.0$. For $\$_f =$ 1.0 and various $\$_r$'s, Table 1 and 2 contain the results which were numerically estimated for the optimal replace-time and minimal cost per unit time for the rate functions considered in Figure 1 and 2 respectively. It is clear that the optimal time for the replacement becomes shorter and the minimal cost increases as the initial failure-level and linear failure-rate are raised higher. As shown in Table 1, variations in the trigonometric components result in changing the optimal time and minimal cost. Especially, lager values of amplitude make the minimal cost increase.

Next, we compared the total cost per unit time when using the EPTF-type rate function of (3) for the failure processes with cyclic behavior to the one when using the EPF-type rate function $\lambda_0(t)$:

$$\lambda_0(t) = \exp\{\alpha_0 + \beta_0 t\}. \tag{4}$$

The coefficients (α_0,β_0) of (4) were estimated with maximum likelihood for the system failure processes associated with EPTF-type rate functions. Figure 3 shows the estimated EPF-type rate and mean value functions, and corresponding EPTF-type functions. Table 3 contains the results of maximum likelihood estimates of (α_0, β_0) for the system processes related to the EPTF-type rate functions with various parameters. For $\$_f = 1.0$ and various $\$_r$'s, the results of total costs per unit time are shown in Table 4 when applying the optimal replacement-times which

Table 1. Optimal replacement time (in parenthesis) and minimal cost per unit time for various replacement costs when using EPTF rate functions of different cyclic parameters with $\alpha = 0.0$ and $\beta = 0.5$.

$\$_r$	$\omega = 1.0$			$A = 0.5$		
	$A = 0.1$	$A = 0.5$	$A = 1.0$	$\omega = 0.5$	$\omega = 1.0$	$\omega = 2.0$
1.5	2.472	2.518	2.686	2.472	2.518	2.507
	(2.009)	(2.194)	(1.305)	(2.327)	(2.194)	(1.641)
5.0	3.992	4.062	4.312	3.900	4.062	4.047
	(2.963)	(2.329)	(2.300)	(2.554)	(2.329)	(2.638)
10.0	5.563	5.618	5.978	5.779	5.618	5.697
	(3.333)	(3.278)	(3.274)	(2.766)	(3.278)	(3.614)
20.0	8.153	8.237	8.752	8.112	8.237	8.311
	(4.226)	(4.246)	(4.257)	(4.424)	(4.246)	(4.133)

Table 2. Optimal replacement time (in parenthesis) and minimal cost per unit time for various replacement costs when using EPTF rate functions of different failure constant levels and increase ratios with $A = 0.5$ and $\omega = 1.0$.

$\$_r$	$\alpha = 0.0$			$\beta = 0.5$		
	$\beta = 0.1$	$\beta = 0.5$	$\beta = 1.0$	$\alpha = -2.0$	$\alpha = 0.0$	$\alpha = 2.0$
1.5	1.649	2.518	3.100	0.805	2.518	11.195
	(4.273)	(2.194)	(1.221)	(3.294)	(2.194)	(1.191)
5.0	2.228	4.062	5.779	1.625	4.062	14.053
	(7.274)	(2.329)	(1.383)	(5.211)	(2.329)	(1.255)
10.0	2.808	5.618	8.112	2.560	5.618	17.928
	(9.283)	(3.278)	(2.212)	(6.198)	(3.278)	(1.324)
20.0	3.697	8.237	12.520	4.148	8.237	22.616
	(12.276)	(4.246)	(2.319)	(7.193)	(4.246)	(2.248)

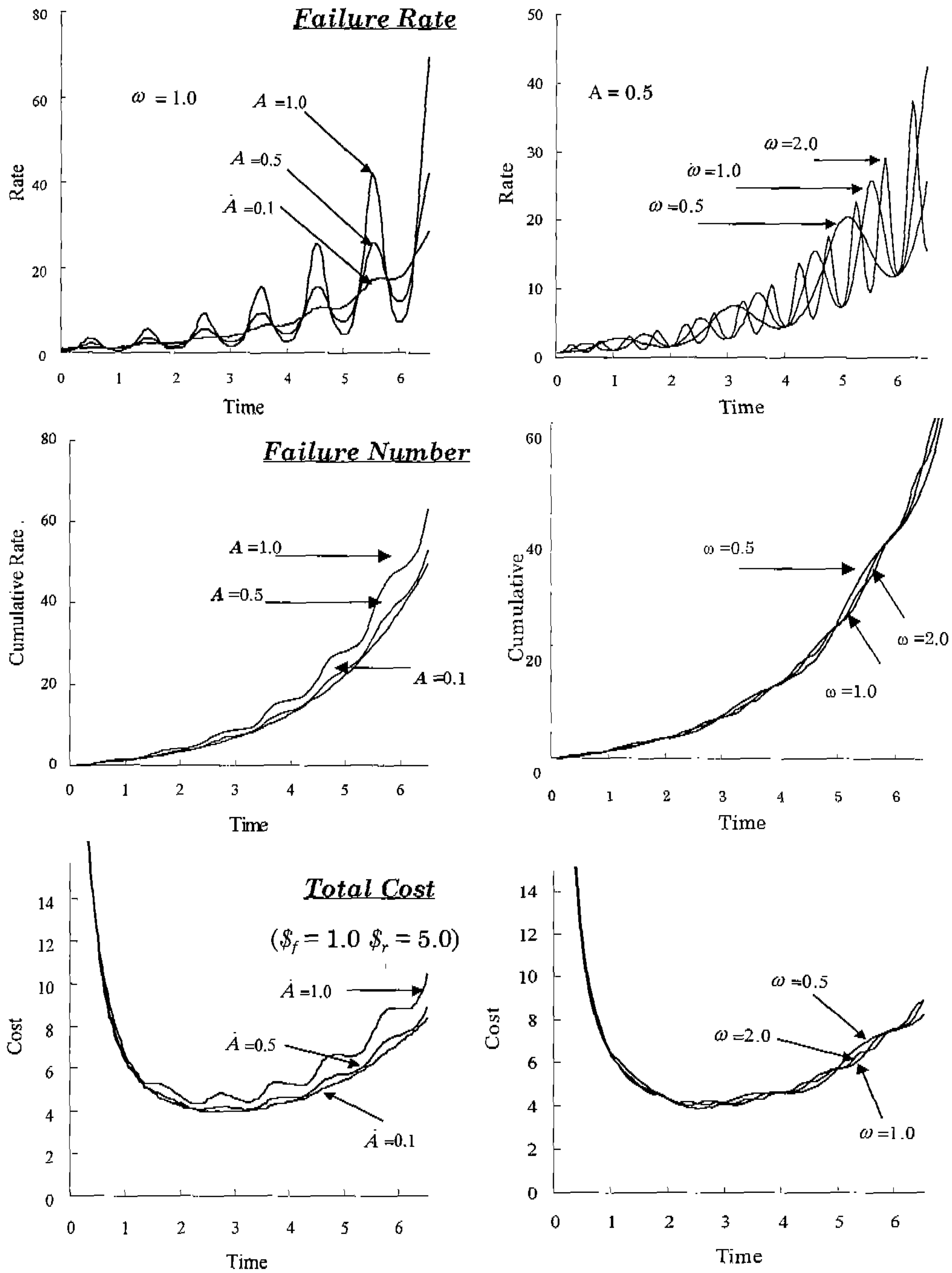

Figure 1. Failure rates, mean values and total costs corresponding to EPTF–type rate functions with $\alpha = 0.0$ and $\beta = 0.5$.

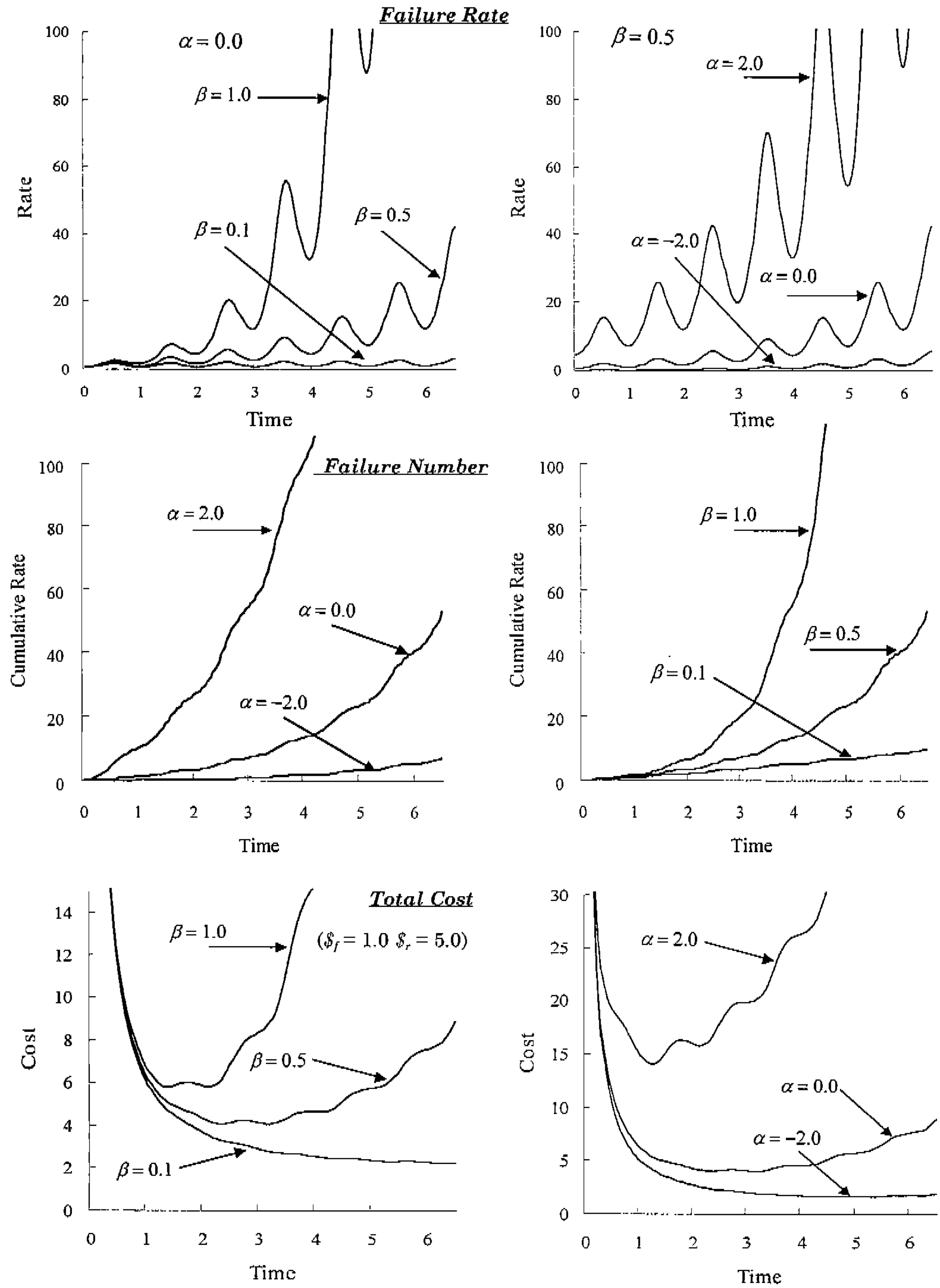

Figure 2. Failure rates, mean values and total costs corresponding to EPTF−type rate functions with A = 0.5 and ω = 1.0.

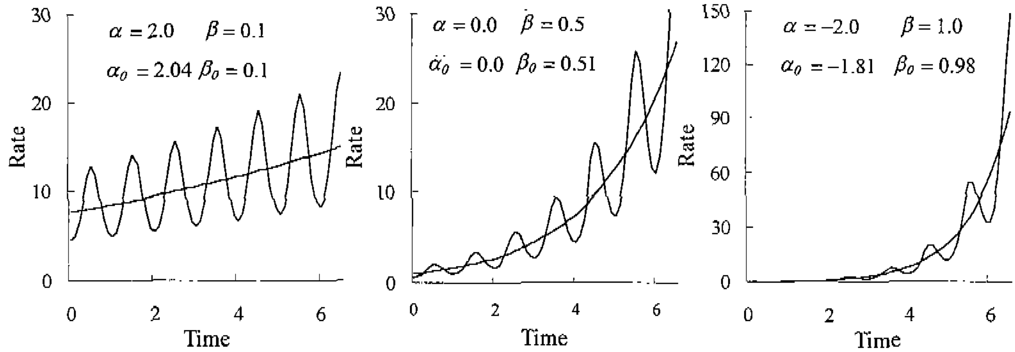

Figure 3. Estimated EPF rate functions for system failure processes with cyclic behavior corresponding to EPTF−type rate function of A = 0.5 and ω = 1.0.

Table 3. Estimated coefficient values of EPF rate functions for system failure processes with cyclic behavior corresponding to EPTF rate function.

EPTF Parameters for Failure Process			Estimated EPF Coefficients		
α	β	A	ω	α_0	β_0
−2.0	0.1	0.5	1.0	−2.200	0.112
−2.0	0.5	0.5	1.0	−2.208	0.534
−2.0	1.0	0.5	1.0	−1.814	0.978
0.0	0.1	0.5	1.0	0.000	0.107
0.0	0.5	0.1	1.0	0.000	0.500
0.0	0.5	0.5	1.0	0.000	0.506
0.0	0.5	0.5	0.5	0.000	0.502
0.0	0.5	0.5	2.0	0.000	0.512
0.0	0.5	1.0	1.0	0.000	0.543
0.0	1.0	0.5	1.0	−0.283	1.109
2.0	0.1	0.5	1.0	2.036	0.104
2.0	0.5	0.5	1.0	2.068	0.489
2.0	1.0	0.5	1.0	1.817	1.167

Table 4. Minimal cost per unit time for various replacement costs when using EPF rate functions for system failure processes with cyclic behavior corresponding to EPTF rate function.

$\$_r$	$\alpha=0.0\ \beta=0.5\ \omega=1.0$			$\alpha=0.0\ \beta=0.5\ A=0.5$		
	$A=0.1$	$A=0.5$	$A=1.0$	$\omega=0.5$	$\omega=1.0$	$\omega=2.0$
1.5	2.480	2.665	3.140	2.665	2.665	2.576
5.0	3.996	4.224	4.736	3.960	4.224	4.075
10.0	5.571	5.672	5.983	6.015	5.672	5.805
20.0	8.153	8.256	9.088	8.200	8.256	8.311
$\$_r$	$\alpha=0.0\ A=0.5\ \omega=1.0$			$\beta=0.5\ A=0.5\ \omega=1.0$		
	$\beta=0.1$	$\beta=0.5$	$\beta=1.0$	$\alpha=-2.0$	$\alpha=0.0$	$\alpha=2.0$
1.5	1.674	2.665	3.105	0.834	2.665	12.685
5.0	2.231	4.224	6.016	1.647	4.224	14.086
10.0	2.837	5.672	8.189	2.604	5.672	19.134
20.0	3.700	8.256	12.830	4.214	8.256	22.644

Table 5. Percentages of cost reduction by using EPTF rate functions as compared with using EPF-type rate functions for system failure processes with cyclic behavior.

$\$_r$	$\alpha=0.0\ \beta=0.5\ \omega=1.0$			$\alpha=0.0\ \beta=0.5\ A=0.5$		
	$A=0.1$	$A=0.5$	$A=1.0$	$\omega=0.5$	$\omega=1.0$	$\omega=2.0$
1.5	0.3%	5.5%	14.5%	7.2%	5.5%	2.7%
5.0	0.1%	3.8%	9.0%	1.5%	3.8%	0.7%
10.0	0.1%	1.0%	0.1%	3.9%	1.0%	1.9%
20.0	0.0%	0.2%	3.7%	1.1%	0.2%	0.0%
$\$_r$	$\alpha=0.0\ A=0.5\ \omega=1.0$			$\beta=0.5\ A=0.5\ \omega=1.0$		
	$\beta=0.1$	$\beta=0.5$	$\beta=1.0$	$\alpha=-2.0$	$\alpha=0.0$	$\alpha=2.0$
1.5	1.5%	5.5%	0.1%	3.5%	5.5%	11.7%
5.0	0.1%	3.8%	3.9%	1.3%	3.8%	0.2%
10.0	1.0%	1.0%	0.9%	1.7%	1.0%	6.3%
20.0	0.1%	0.2%	2.4%	1.6%	0.2%	0.1%
$\$_r$	$\alpha=-2.0\ A=0.5\ \omega=1.0$			$\alpha=2.0\ A=0.5\ \omega=1.0$		
	$\beta=0.1$	$\beta=0.5$	$\beta=1.0$	$\beta=0.1$	$\beta=0.5$	$\beta=1.0$
1.5	0.1%	3.5%	1.5%	7.8%	11.7%	14.6%
5.0	0.1%	1.3%	3.1%	0.4%	0.2%	8.8%
10.0	0.8%	1.7%	0.1%	0.0%	6.3%	2.6%
20.0	0.4%	1.6%	2.3%	2.4%	0.1%	0.7%

were estimated using the EPF-type rate functions to the cyclic processes related to the EPTF-type rate functions. Table 5 demonstrates the percentages of cost reduction by using the EPTF-type rate functions as compared with using the EPF-type rate functions for the cyclic processes. The results show that the minimal cost is generally reduced when considering cyclic components in the failure rate function. Especially the reduction amount is considerable if the replacement cost is not much greater than the repair cost.

5. CONCLUSIONS

It seems in many cases that the failure rate in a system generally increases over time and also exhibits local fluctuation. For the system failure processes with cyclic effect, we investigated the cost behavior using the periodic rate function of EPTF-type, and compared the total cost per unit time between using the periodic rate function and using the non-periodic rate function. The experimental results indicate that we can reduce the minimal cost per unit time by selecting the replacement period using the periodic failure-rate function for the system failure processes exhibited a cyclic effect.

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