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Capacity-constrained Outsourcing to Two Contract Manufacturers with Different Improvement Capabilities

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ABSTRACT

We investigate a supply chain arrangement where a manufacturing company outsources its assembly operations to two contract manufacturers. Each contract manufacturer is different in improvement capability: e.g., one is more capable than the other. This improvement capability is supposed to induce supply cost reduction that ultimately benefits the manufacturing company. Over time, the manufacturer has to decide *how much it should outsource to each contract manufacturer* and *how much processed/assembled the semi-finished units should be when they are shipped back to the manufacturing company from the contract manufacturers.* We employ the optimal control theory to answer the questions, and suggest numerical examples focused on the relationship among the optimal outsourcing amounts, contract manufacturers' improvement capabilities, and their capacity constraints.

1. INTRODUCTION

As a mode of supply chain, contract manufacturing enables a manufacturing company to outsource some of its internal manufacturing (e.g., assembly operations) to contract manufacturers [18]. For example, there is a turnkey arrangement, which allows the contract manufacturers to order the parts directly from the suppliers pre-approved by the manufacturing company. In this paper, we consider a turnkey contract context involving one manufacturing company and its two contract manufacturers (e.g., suppliers). Semi-finished units (not just finished units) can be shipped from the contract manufacturers back to the manufacturing company. Therefore, the manufacturing company must decide how much *'processed'* should be the units when they are shipped from the contract manufacturers. For instance, if the company receives only partially finished units from the contract manufacturers, it must *additionally* process the units until they become finished and ready for sale to the market.

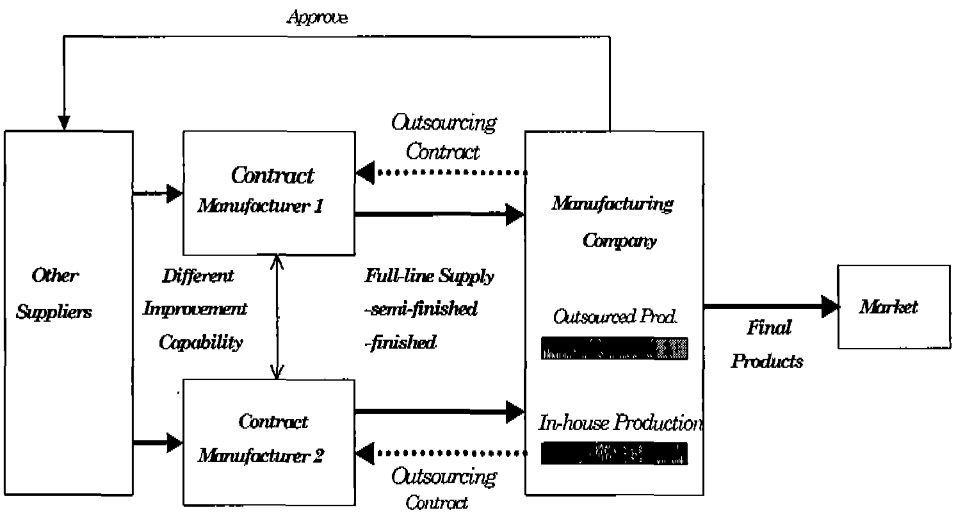

Figure 1. A Context of Contract Manufacturing Supply Chain

Sometimes, a contract manufacturer embarks on improvement activities in its production process leading to supply cost reduction that benefits the manufacturing company [13]. Because of the fundamental nature of organizational capability, however, contract manufacturers are at different levels of improvement capability [10,19]. Here we explore a manufacturing company's dynamic decision on outsourcing to contract manufacturers with varying improvement capabilities: it is assumed that contract manufacturers' improvement activities such as experiments and problem solving projects will induce supply cost reduction that enables the manufacturing company to realize higher profit [20].

In the literature, similar research has been done in the context of various uncertainties and/or global supply chain management [5]. Hodder and Jucker [11] investigated a location decision related with outsourcing alternatives, emphasizing price uncertainty [12]. Cohen and Lee [4] dealt with the outsourcing issues from a broad supply chain management perspective [9]. Kogut [15,16] viewed having multiple sourcing options as retaining operational flexibility in the global market [17]. By incorporating the contract manufacturer (CM)'s capability explicitly into analysis, we try to shed light on a more dynamic capability-centered perspective for decision making on outsourcing.

In the next section, we develop an analytical model to formulate the research problem outlined above. It is an optimal control model, adopting the distributed parameter systems approach: the analysis is two-dimensional across *time* and *processing (assembling) level*. Section 3 solves the optimal control problem and suggests economic insights. In section 4, we present numerical examples and derive some practical inferences. Finally, we close the paper with proposing managerial implications and some issues for future research.

2. RESEARCH PROBLEM MODELING

We define a manufacturing system as a network of processing/assembling centers/stages and stocking points, e.g., multi-stage processing lines [4]. Consistent with the definition, we develop an optimal control model for our research problem, employing the distributed parameter systems approach [3,7]. For analytical simplicity, we deal with a supply chain that consists of one manufacturing company and two contract manufacturers. Although it can process/assemble all of its intermediate/semi-finished products using its own production system, the manufacturing company can also outsource some of its processing operations to contract manufacturers. The company receives semi-finished units from the contract manufacturers, and further processes them until they become finished products that can be sold to the final customers. Proposing a two contract-manufacturer (i.e., two-supplier) model is not too restrictive: in fact, Dyer and Ouchi [8] suggested that utilizing two suppliers is usually enough to give the firm competitive benefits which could be derived from having more than two.

For the following analysis, we concentrate on the supply network consisting of a manufacturing company and two contract manufacturers each providing a full line of intermediate products/materials to the manufacturer, which engages itself in the entire production process, fabricating intermediate products h-processed, $1 \le h \le H$. At the same time, the manufacturing company can outsource to the contract manufacturers intermediate products/materials at any processing level h, $1 \le h \le H$. If the company receives intermediate products h-processed, $1 \le h \le H$, then it must internally process them for additional $H-h$

processing levels: provided normalizing the unit of h is done properly, the manufacturer must process the intermediate products for $H-h$ time period.

In this paper, we presume a competitive market: that is, although it can not affect the market price of the product, the manufacturing company can sell as many as it can produce. We also need to clearly state that the primary perspective we take for the research in this paper is the manufacturing company's and therefore, the objective of the analysis is to maximize the net profit of the manufacturer.

2.1 Constructs in the Mathematical Model

As mentioned already, relevant decision variables for the manufacturing company are two dimensional, i.e., (τ, h) where τ for time dimension and h for processing level. We denote the manufacturing company as M_1, and the contract manufacturers as CM_i, $i=1,2$.

2.1.1 Control and State Variables

For two full-line contract manufacturers, the model needs two control variables, $u_i(\tau,h)$'s, each of which represents the outsourced amount of h-processed products/units at τ from CM_i. The state variable, $x(\tau,h)$, stands for the stock of products (intermediate units) h-processed at τ inside M_1: $x(1,h)=x(\tau,1)=0$ is assumed as initial conditions without loss of generality. In this paper, the relevant decision horizon is $1 \le \tau \le T$: whether we choose $\tau=0$ or $\tau=1$ as the starting time does not affect the ensuing analysis. But, by using $\tau=1$ as the initial time, we can avoid a technical difficulty during the analysis: since we will formulate cost functions with a form of h^x and τ^y, where $0 \le x \le 1$ and $y \le 0$, the analysis might encounter irregularity while $0 \le h < 1$ and/or $0 \le \tau < 1$. Thus, without loss of generality, we select $\tau=1$ as the starting time for the current decision horizon.

There are two types of cost involved, outsourcing (or purchase) cost and internal processing cost. M_1 pays CM_i for the semi-finished or finished units: in the analysis, $P_i(\tau,h)$, $i=1,2$, represents the 'unit outsourcing cost' associated with $u_i(\tau,h)$. The unit cost of internal processing associated with $x(\tau,h)$ is $P_c(\tau,h)$. For the ensuing analysis, we define the specific cost structure as $P_c(\tau,h)=K_c h^\gamma$, $0 \le \gamma \le 1$ and $K_C \ge 0$ a given constant.

2.1.2 Outsourcing Capacity Constraints

Each contract manufacturer has an 'economic scale' for its internal production of intermediate products, denoted by $\bar{u}_i$ for CM_i. This economic scale compares with an economic production/order quantity, and any deviation from it causes a penalty cost to be paid by the manufacturing company. We assume the cost incurs in proportion to the quadratic deviation of the actual outsourcing amount from the economic scale. Denote r_i as a unit deviation cost imposed by CM_i. Thus, M_1 pays a total deviation cost to the CM s that amounts to $r_1(u_1 - \bar{u}_1)^2 + r_2(u_2 - \bar{u}_2)^2$: we assume $\bar{u}_i$ is constant over (τ, h), that is, the CM has a uniform economic production quantity for its entire product line. In a realistic situation, $\bar{u}_i$ would be determined by a contract between CM_i and M_1, then $\bar{u}_i$ is the economic order quantity for M_1.

Likewise, M_1 has its own capacity limit, $\bar{u}$, on the total amount of outsourcing from CM s for each (τ, h), i.e., $u_1(\tau, h) + u_2(\tau, h) < \bar{u}$. It is also assumed that $\bar{u}$ is constant.

2.1.3 Sales Revenue and Salvage Value

Fully processed, i.e., H-processed units, can sell in the market at a unit rate of $V(\tau)$ at τ. For simplicity without loss of generality, we assume $V(\tau) = V_0$, constant, and thus the manufacturer's sales revenue at τ amounts to $V_0\, x(\tau, H)$. At the end of the current decision horizon, T, a unit unfinished product generates a salvage value of $S(h)$, which is assumed to be $S_0 h$, where S_0 is constant. Thus, the total salvage value of h-processed products at T would be $S_0 hx(T, h)$.

2.1.4 Learning/Improvement Capability

CM_i's improvement capability is reflected in $P_i(\tau, h)$. In a previous section, we have argued that each contract manufacturer/supplier (or, a group of CM s/suppliers) faces a specific technological infrastructure in the industry: that is, a CM's internal improvement capability can be influenced by autonomous forces in the industry. Based on this argument, we put forth that a CM's improvement capability in a particular environment (e.g., market, industry, or country) is correlated with an autonomous innovation capacity embedded in the environment as a whole [1, 2].

In addition, it is reasonable to impose that a 'more processed' intermediate unit is more expensive than a 'less processed' since 'more processed' units have required/consumed more efforts as well as material inputs than the 'less processed.'

Taking into account these two conditions, we can establish the cost structure so that $\partial P_i(\tau,h)/\partial\tau \le 0$ and $\partial P_i(\tau,h)/\partial\tau \ge 0$.

Following a widely-applied economic modeling, we specify the cost structure as follows: $P_1(\tau,h)=K_1h^{\alpha}\tau^{n}$ for CM_1 and $P_2(\tau,h)=K_2h^{\beta}\tau^{m}$ for CM_2, where K_1 and K_2 are constant base purchase (outsourcing) costs affected by neith Twenty-two points, plus triple-word-score, plus fifty points for using all my letters. Game's over. I'm outta here.er learning nor processing level, yet: we impose $0 \le \alpha, \beta \le 1$ and $n, m < 0$. The cost structure follows the spirit of a learning curve [21] and a production function, e.g., Cobb-Douglas production function [6]. In this paper, we assume that CM_2 has higher innovation capability than CM_1, while its initial unit cost/price is more expensive. Appendix 1 summarizes the key variables and parameters defined so far.

2.2 An Optimal Control Theory Model

Based on the formulation in the previous section, an optimal control model for the manufacturing company can be set up as follows.

Maximize Z

$$= \int_1^T\int_1^H -\{r_1(u_1-\bar{u}_1)^2 + r_2(u_2-\bar{u}_2)^2 + K_1h^{\alpha}\tau^{n}u_1 + K_2h^{\beta}\tau^{m}u_2 + K_c h^{\gamma}x(\tau,h) - \kappa(\bar{u}-u_1\text{-}u_2)\}dh\,d\tau$$

$$+\ V_0\int_1^T x(\tau,H)d\tau\ + S_0\int_1^H hx(T,h)dh \qquad \text{(E1)}$$

Subject to $$\frac{\partial x(\tau,h)}{\partial\tau} = -\frac{\partial x(\tau,h)}{\partial h} + u_1(\tau,h) + u_2(\tau,h) \qquad \text{(E2)}$$

$$u_1 + u_2 \le \bar{u} \qquad \text{(E3)}$$

$$0 \le u_1 \qquad \text{(E4)}$$

$$0 \le u_2, \qquad \text{(E5)}$$

where $\kappa \ge 0$ is the market value of M_1's unit outsourcing capacity unused for the contract manufacturing.

The objective functional (E1) consists of three parts: the first integration represents the negative of total costs (including capacity deviation penalty, outsourcing, and internal production costs); the second integration calculates the total sales revenue; the third is the total salvage value at the end of the decision time horizon. (E2) can be derived in the following way. Since the unit of h is scaled so as to be equivalent to the time unit and $x(\tau,h)$ represents the stock of products h-processed at τ, $x(\tau+\Delta\tau,h)$ is comprised of three elements, $x(\tau,h-\Delta\tau)$, i.e., one due to the internal processing of products $(h-\Delta\tau)$-processed at τ, and $u_1(\tau,h)\,\Delta\tau$ and $u_2(\tau,h)\Delta\tau$, i.e., h-processed products from the two contract manufacturers at an instantaneous moment. Therefore,

$$x(\tau+\Delta\tau,h)=x(\tau,h-\Delta\tau)+u_1(\tau,h)\Delta\tau+u_2(\tau,h)\Delta\tau \tag{E6}$$

By dividing both sides of (E6) with $\Delta\tau$ and let $\Delta\tau\to 0$, we obtain (E2).

The associated Hamiltonian is, after related Lagrange multipliers are assigned,

$$\begin{aligned}H_Z=&-\left\{r_1(u_1-\bar{u}_1)^2+r_2(u_2-\bar{u}_2)^2+K_1h^{\alpha}\tau^{n}u_1+K_2h^{\beta}\tau^{m}u_2+K_ch^{\gamma}x-\kappa(\bar{u}-u_1-u_2)\right\}\\&+\lambda\left(-\frac{\partial x}{\partial h}+u_1+u_2\right)+\mu(\bar{u}-u_1-u_2)+\mu_1u_1+\mu_2u_2,\end{aligned} \tag{E7}$$

where λ is the costate variable for (E2), and μ, μ_1, and μ_2 are Lagrangian multipliers for (E3), (E4), and (E5), respectively.

We need to consider an important complication stemming from dealing with a two-dimensional (rather than one-dimensional) space, i.e., (τ,h)-space. It is due to that since each product needs to be processed for $H-1$ time period to generate a full market value and the current time horizon is T, the intermediate products processed less than $\tau-(T-H)$, i.e., $h<\tau-(T-H)$, and procured from a CM at $\tau>T-H+1$ can not be completed by T. Thus, the values generated by these intermediate products must differ from others' that contribute full market values.

To rectify the problem, we partition the (τ,h)-space into three regions, taking into account H and T. The division results in regions O_1, O_2, and O_3 with two internal boundaries R_1 and R_2, as in Figure 2. In addition, we have two sets of external boundaries, $(\{1\}\times[1,H])\cup([1,T]\times\{1\})$ represented by the thick lines on τ- and h-axis and $([1,T]\times\{H\})\cup(\{T\}\times[1,H])$ shown as dotted lines in Figure 2. $x(\tau,h)$ is affected by different initial conditions, i.e., $x(1,h)$ while in O_1, and $x(\tau,1)$ while in O_2 and O_3. That is, the optimal values of $x(\tau,h)$ depend on the initial boundary condition $(\{1\}\times[1,H])\cup([1,T]\times\{1\})$.

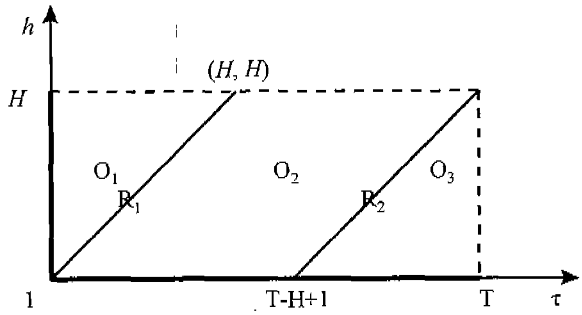

Figure 2. Partition of (τ, h)−space

As in a usual economic analysis, the costate variable $\lambda(\tau,h)$ represents the shadow or marginal value of $x(\tau,h)$. We know that there are two ways for $x(\tau,h)$ to contribute values to the company, either as finished products at τ, i.e., $x(\tau,H)$ for $1\le\tau\le T$, or as unfinished ones with salvage values at T, i.e., $x(T,h)$ for $1\le h\le H$. Therefore, we can see that the optimal $\lambda^*(\tau,h)$ hinges on the terminal boundary condition $([1,T]\times\{H\})\cup(\{T\}\times[1,H])$.

3. Optimal Control Theory Analysis

For the analysis using the optimal control theory, we consider two separate cases, $u_1^*+u_2^*<\overline{u}$ and $u_1^*+u_2^*=\overline{u}$. For the first case, we focus on calculating the value of surplus outsourcing capacity. For the case of $u_1^*+u_2^*=\overline{u}$, we conduct the comparative statics analysis.

Now, the relevant Hamiltonian is as follows,

$$H_Z=-\left\{r_1(u_1-\overline{u}_1)^2+r_2(u_2-\overline{u}_2)^2+K_1h^{\alpha}\tau^{n}u_1+K_2h^{\beta}\tau^{m}u_2+K_ch^{\gamma}x-\kappa(\overline{u}-u_1-u_2)\right\}$$
$$+\lambda(-\frac{\partial x}{\partial h}+u_1+u_2)+\mu(\overline{u}-u_1-u_2)\,. \qquad \text{(E8)}$$

Assuming that the manufacturing company can sell all of its products in the market, i.e., the market has a sufficient demand for the finished products, it is reasonable to assume that u_1 and u_2 are always positive. Therefore, for the remaining analysis, we can safely ignore the two non-negativity constraints, (E4) and (E5). One way to make sure that our assumption is valid is to obtain an optimal solution with the proposed relaxation, and then confirm that the $u_i^*(t)$'s are indeed positive for $1 \le t \le T$. If the assumption does not hold as suggested, we should reinstate the relaxed constraints.

For u_1, we set $\dfrac{\partial H_Z}{\partial u_1} = -\left\{2r_1(u_1 - \bar{u}_1) + K_1 h^{\alpha}\tau^{n}\right\} + \lambda - \kappa - \mu = 0.$

Thus, $u_1^* = \bar{u}_1 + \dfrac{1}{2r_1}(\lambda - \kappa - \mu - K_1 h^{\alpha}\tau^{n})$. (E9)

$\mu \ge 0$, $u_1 + u_2 \le \bar{u}$, and $\mu(\bar{u} - u_1 - u_2) = 0$. (E10)

To ensure we have a maximized solution, we take $\partial^2 H_Z / \partial u_i^2 = -2r_i$ which is non-positive as long as $r_i \ge 0$, as assumed throughout the analysis.

We can interpret (E9) through the following steps:

$\bar{u}_1$: the base quantity, i.e., economic order/production quantity,

$\Lambda \equiv \lambda - \kappa - \mu - K_1 h^{\alpha}\tau^{n}$: if $u_1 + u_2 < \bar{u}$, then $\mu = 0$ and M_1 would sell its outsourcing capacity to the outside market at κ. Since λ is the shadow value of $x(\tau, h)$, i.e., the value that would be generated from an intermediate unit h-processed at τ, κ the opportunity cost due to internally using one unit of outsourcing capacity, and $P_1(\tau, h)$ the outsourcing cost of an intermediate unit h-processed at τ, $\Lambda = \lambda - \kappa - P_1(\tau, h)$ represents the net contribution of unit outsourcing of the intermediate product. On the other hand, if $u_1 + u_2 = \bar{u}$, $\kappa(\bar{u} - u_1 - u_2) = 0$ and can be dropped from (E8). Thus, we can make a similar interpretation for $\Lambda = \lambda - \mu - P_1(\tau, h)$ except that now we need to take into account the shadow price of capacity, μ, rather than its opportunity cost.

$\Lambda / 2r_1$: since r_1 is the penalty cost associated with a quadratic deviation from the economic production quantity, $2r_1$ can be regarded as a linearized deviation cost; recall $(\partial / \partial u_i)\left\{r_i(u_i - \bar{u}_i)^2\right\} = 2r_i(u_i - \bar{u}_i)$. Thus, by dividing the net contribution from an intermediate product h-processed at τ with the linear deviation cost, we can express an additional quantity that is adjusted according to the deviation penalty.

Therefore, $u_1^* > \bar{u}_1$ if $\Lambda > 0$, i.e., the intermediate product has a positive net contribution, whereas $u_1^* < \bar{u}_1$ if $\Lambda < 0$. But, the actual magnitude of difference is determined by the adjusted value of Λ according to the linearized deviation penalty cost, $2r_1$. Likewise, we can obtain for u_2, $u_2^* = \bar{u}_2 + (1/2r_2)(\lambda - \kappa - \mu - K_2 h^{\beta} \tau^{m})$.

Once u_i^* is obtained, we can calculate optimal state and costate variables. As mentioned before, specific structures of x^* and λ^* depend on the (τ, h)-*space division. Detailed analytical procedures to calculate* x^* *and* λ^* can be referred to Butkovskiy [3]. A more intuitive procedure can be found in [14] and Appendix 2.

After plugging λ^* into (E9) and (E10), u_i^* can be used to attain x^*. Using the optimal values, u_i^* and x^*, we can finally determine the optimal objective value of the optimal control problem, Z^*. Although computing x^* and Z^* is also interesting, we focus more on the behavior of u_i^* since our primary research question is about the dynamics of control variables, i.e., optimal outsourcing decisions for contract manufacturing.

Before continuing our main analysis, we have to state the boundary condition: in order to have a continuous solution, we need to impose that u_i^*'s should have the same value on R_2, i.e., on R_2, $H = T - \tau + h$ and thus $HS_0 = V_0$. It is easy to prove the statement, using the fact that from (E9) and (E10), we know that u_i^*'s depend on λ^*'s, which in turn must be considered separately either in $(\tau, h) \in O_1 \cup O_2$ or in $(\tau, h) \in O_3$.

As shown in Figure 2, a feasible solution for u_i^* needs to be continuous on R_2. For u_1^*, this implies that the two u_1^*'s defined above must be equal on R_2. This condition can be always met by setting $H = T - \tau + h$ and thus $HS_0 = V_0$. We can derive the same result for u_2^*.

3.1 Case of $u_1 + u_2 < \bar{u}$

When $u_1 + u_2 < \bar{u}$, there remains a slack capacity, i.e., $\bar{u} - (u_1 + u_2) > 0$. Thus, it becomes important to assess the market value of the manufacturer's unit production (outsourcing) capacity, κ, that makes it more profitable to sell part of its capacity in the market rather than to use all of it in processing intermediate products from its two contract manufacturers. For example, such κ might be the unit price the company charges other firms for processing a unit product for them.

For the case of $u_1 + u_2 < \overline{u}$, $\mu = 0$ from (E10).

$$u_1^* = \overline{u}_1 + \frac{1}{2r_1}(\lambda - \kappa - K_1 h^{\alpha}\tau^{n}) \text{ and } u_2^* = \overline{u}_2 + \frac{1}{2r_2}(\lambda - \kappa - K_2 h^{\beta}\tau^{m}) \tag{E11}$$

Theorem 1. Value of Manufacturer's Unit Outsourcing Capacity. Define $\widetilde{\kappa}$ as the minimum value of κ that makes it more profitable for the manufacturer to sell a unit of its outsourcing capacity. Then $\widetilde{\kappa} = \{(2r_1r_2)/(r_1+r_2)\}\left\{\hat{u}_1^* + \hat{u}_2^* - \overline{u}\right\}$, where $\hat{u}_i^*$ is the optimal outsourcing amount when no capacity constraint is imposed.

Proof. We can obtain $\widetilde{\kappa}$ by evaluating κ that holds up $u_1 + u_2 < \overline{u}$, i.e., κ that satisfies (E12).

$$u_1^* + u_2^* = \overline{u}_1 + \frac{1}{2r_1}(\lambda - \kappa - K_1 h^{\alpha}\tau^{n}) + \overline{u}_2 + \frac{1}{2r_2}(\lambda - \kappa - K_2 h^{\beta}\tau^{m}) < \overline{u} \tag{E12}$$

By arranging (E14), we have

$$\frac{2r_1r_2(\overline{u}_1 + \overline{u}_2 - \overline{u})}{r_1 + r_2} + \frac{r_1r_2}{r_1 + r_2}\left[\frac{\lambda - K_1 h^{\alpha}\tau^{n}}{r_1} + \frac{\lambda - K_2 h^{\beta}\tau^{m}}{r_2}\right] < \kappa \tag{E13}$$

This can be rearranged so that

$$\frac{2r_1r_2}{r_1 + r_2}\left\{\left[\overline{u}_1 + \frac{\lambda - P_1(\tau,h)}{2r_1}\right] + \left[\overline{u}_2 + \frac{\lambda - P_2(\tau,h)}{2r_2}\right] - \overline{u}\right\} = \frac{2r_1r_2}{r_1 + r_2}\left\{\hat{u}_1^* + \hat{u}_2^* - \overline{u}\right\} < \kappa .$$

Thus, $\widetilde{\kappa} = (2r_1r_2)/(r_1+r_2)\left\{\hat{u}_1^* + \hat{u}_2^* - \overline{u}\right\}$, where $\hat{u}_i^* = \overline{u}_i + \lambda - P_i(\tau,h)/2r_i$ is the optimal outsourcing amount when the capacity constraint is not imposed: it can be easily verified from (E9). □

Therefore, the value of processing/outsourcing capacity depends on r_i's and the difference between the optimal amount of 'capacity-unconstrained' outsourcing, $\hat{u}_1^* + \hat{u}_2^*$, and the processing capacity, $\overline{u}$.

The opportunity cost of the manufacturer's capacity is determined by not only its own production/outsourcing capacity, but also its contract manufacturers' economic production quantities and related penalty costs: production decisions of the manufacturing company and its contract manufacturers become intricately connected.

3.2 Case of $u_1 + u_2 = \overline{u}$

If $u_1 + u_2 = \overline{u}$ and therefore $\mu \geq 0$, we can simplify the Hamiltonian by substituting u_1 with $\overline{u} - u_2$. The simplified Hamiltonian becomes

$$H_Z = -\left\{r_1(\overline{u} - \overline{u}_1 - u_2)^2 + r_2(u_2 - \overline{u}_2)^2 + K_1 h^\alpha \tau^n (\overline{u} - u_2) + K_2 h^\beta \tau^m u_2 + K_c h^\gamma x\right\} + \lambda\left(-\frac{\partial x}{\partial h} + \overline{u}\right) \quad \text{(E14)}$$

with only one control variable u_2. To derive an optimal solution, we take:

$$\frac{\partial H_Z}{\partial u_2} = -\left\{-2r_1(\overline{u} - \overline{u}_1 - u_2) + 2r_2(u_2 - \overline{u}_2) - K_1 h^\alpha \tau^n + K_2 h^\beta \tau^m\right\} = 0,$$

$$\therefore u_2^* = \frac{r_1(\overline{u} - \overline{u}_1) + r_2 \overline{u}_2}{r_1 + r_2} + \frac{K_1 h^\alpha \tau^n - K_2 h^\beta \tau^m}{2(r_1 + r_2)} \text{ and } u_1^* = \overline{u} - u_2^*. \quad \text{(E15)}$$

In this case, u_i^* does not involve λ, i.e., if the manufacturer's capacity is constrained and binding, the shadow value of $x(\tau, h)$ becomes irrelevant to calculating optimal solutions. Since the outsourcing capacity is constrained and fully used, the shadow value of a unit capacity becomes irrelevant.

Theorem 2. Comparative Statics Analysis. For the comparative statics analysis, we postulate the following two properties. As CM_i's economic production quantity (constraint) increases, the optimal amount of outsourcing to CM_i also increases. The optimal outsourcing amount to CM_i decreases as CM_i's improvement/innovation capability deteriorates, whereas it increases as CM_j's, $j \neq i$, improvement/innovation capability deteriorates.

Proof. Using (E15), we can show

(i) $$\frac{\partial u_2^*}{\partial \overline{u}_i} \begin{cases} \geq 0 & \text{if } i=2 \\ < 0 & \text{if } i=1 \end{cases}$$

(ii) $$\frac{\partial u_2^*}{\partial m} = \frac{1}{2(r_1 + r_2)}\left\{- ln\tau K_2 h^{\beta}\tau^{m}\right\} = \frac{-ln\tau P_2(\tau,h)}{2(r_1 + r_2)} < 0 \text{ and}$$
$$\frac{\partial u_2^*}{\partial n} = \frac{1}{2(r_1 + r_2)}\left\{K_1 h^{\alpha}\tau^{n} ln\tau\right\} = \frac{P_1(\tau,h) ln\tau}{2(r_1 + r_2)} > 0 .$$

Since $u_1^* = \bar{u} - u_2^*$, the theorem is proved. □

4. NUMERICAL EXAMPLES

In this section, we present numerical examples based on the analysis results in the previous section: we concentrate on the optimal outsourcing dynamics, u_1^* and u_2^*, as h and τ vary. The setting for numerical examples can be summarized as follows.

The current time horizon is $[1, 31]$, that is, $T = 31$. The length of a production cycle is equivalent to 2 time units because the initial processing level is $h = 1$ and the fully processed level is $H = 3$. The company's decision horizon is 15 times the production cycle. Consider the PC industry. The above setting could imply a situation where it takes 2 weeks to complete a PC and the company plans to produce the PC for 30 weeks.

The sales price of a PC is \$15 (V_0=15) and the salvage value of a semi-finished product h-processed at T is \$5$\times h$ (S_0=5). The base outsourcing cost from CM_1 is \$1 (K_1=1) whereas that from CM_2 is \$1.5 (K_2=1.5), indicating that CM_1's current price (supply cost) is cheaper than CM_2's. M_1's base internal processing cost is \$1 (K_C=1).

Since we are primarily interested in the dynamic patterns of outsourcing from two contract manufacturers with different improvement capabilities, we use the same parameter values for the two contractors such as returns to scale ($\alpha = \beta = \gamma = 0.9$), economic order/production quantity levels ($\bar{u}_1 = \bar{u}_2 = 5$), and related deviation penalty costs ($r_1 = r_2 = 1$), except for the learning rates that proxy the CM's improvement capabilities. We assume M_1's outsourcing capacity at each (τ, h) is $\bar{u} = 10$, where applicable.

As previously assumed, CM_1 is less innovative than CM_2 in terms of reducing the supply costs. Thus, throughout the numerical exercise, it is assumed that $n = 0.0$ while we change the value of m, which represents CM_2's improvement capability. Applying the concept of learning rate in the learning curve literature [21], three specific values are used for m, in the following numerical examples: $m = ln0.90/ln2$ =-0.152, $m = ln0.85/ln2$=-0.234, and $m = ln0.80/ln2$=-0.322. For instance, m=-0.234 means CM_2's learning rate is 0.15 and therefore, as τ doubles, the cost portion related with time will be reduced by 15%.

In the numerical examples, we present the optimal dynamics of u_i^* for the capacity-constrained case, assuming it is economical to fully utilize the total outsourcing capacity. Figure 3 graphs the dynamics of u_i^* with respect to τ given $h = 2$, while Figure 4 presents the dynamics with respect to h given $\tau = 15$.

Figure 5 and 6 show the relative dynamics, i.e., u_2^*/u_1^*, as the contract manufacturers' capacity ratio, $\overline{u}_2/\overline{u}_1$, varies, given $m = -0.322$. As in Figure 3 and 4, Figure 5 displays the dynamics of u_2^*/u_1^* with respect to τ given $h = 2$, while Figure 6 indicates the dynamics with respect to h given $\tau = 15$.

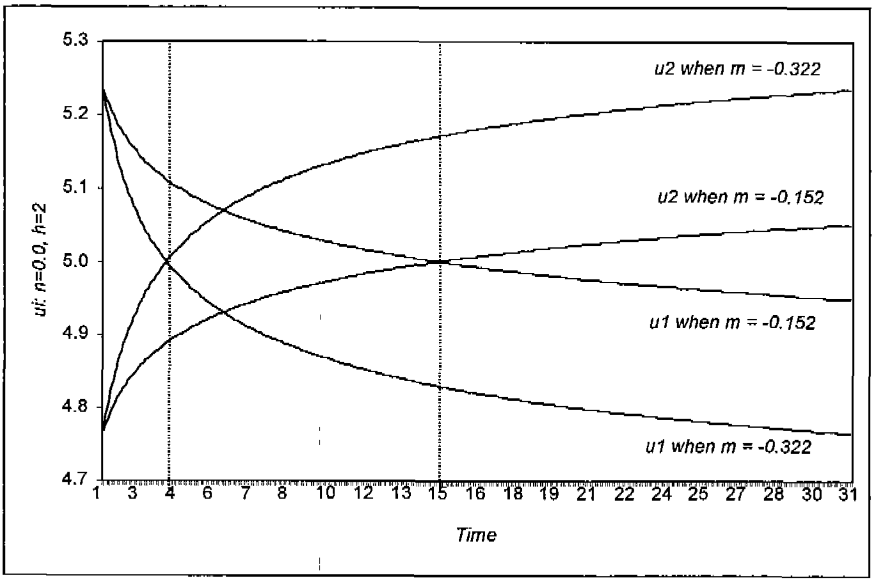

Figure 3. Dynamics of u_i^* with respect to τ, given $h = 2$

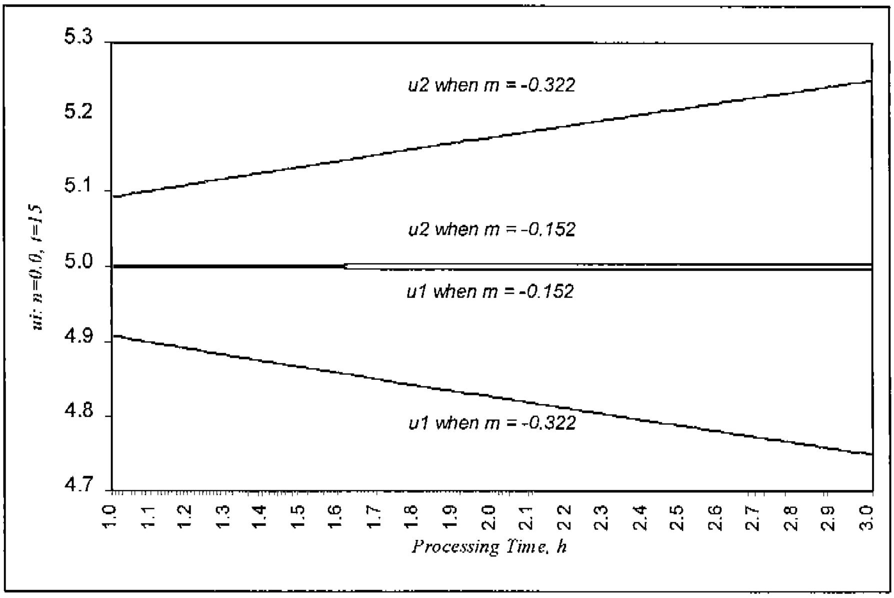

Figure 4. Dynamics of u_i^* with respect to h, given $\tau = 15$

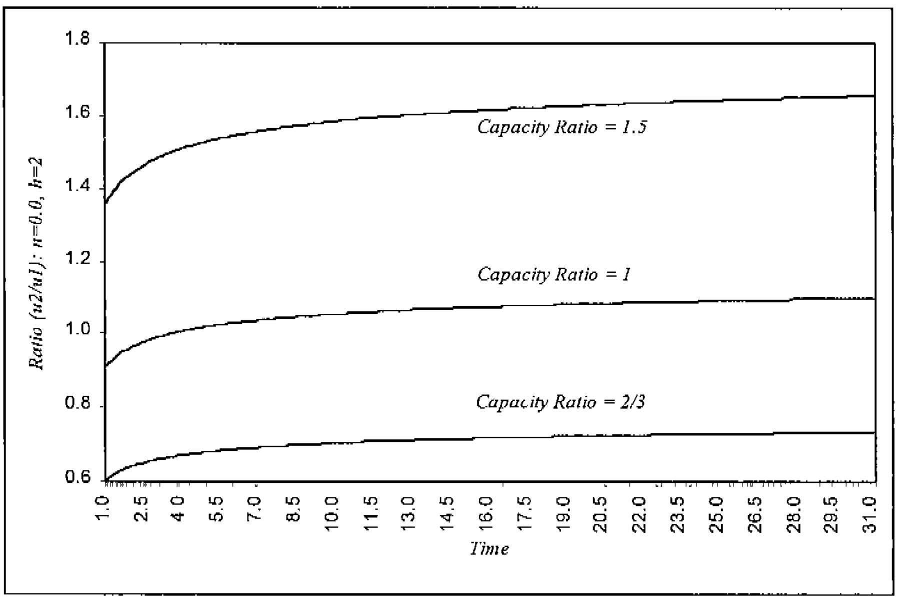

Figure 5. Dynamics of u_2^* / u_1^* with respect to τ, given $h = 2$ with $m = -0.322$

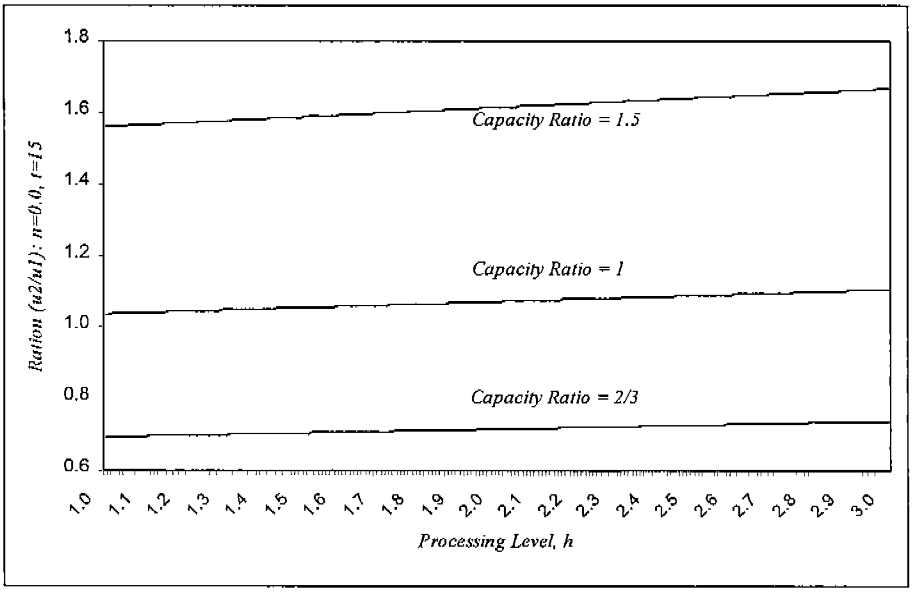

Figure 6. Dynamics of u_2^* / u_1^* with respect to h , given $\tau = 15$ with $m = -0.322$

We can derive important managerial implications from the numerical examples. Figure 3 clearly shows that $du_2^* / d\tau > 0$: since CM_2's improvement capability is higher than CM_1's, the optimal outsourcing to CM_2 increases over time, given a processing level. It also demonstrates that the higher the improvement capability, the earlier the time at which it becomes that $u_2^* > u_1^*$.

Figure 4 shows similar dynamics, i.e., $du_2^* / dh > 0$. But, the effect of improvement capability on the optimal outsourcing amount gets more pronounced as the processing level advances. In other words, the difference between u_1^* and u_2^* becomes larger as h increases, given a time point.

Figure 5 and 6 display the optimal dynamics of relative outsourcing amounts as the contract manufacturers' capacity ratio changes. These examples indicate that even though a contract manufacturer has higher improvement capability, the optimal amount of outsourcing to the contract manufacturer can not increase significantly unless its production capacity can support the increased outsourcing with little additional costs.

These specific dynamics might come from the particular parameter values we have chosen, and thus may not be extensively generalizable. The primary contribution of this numerical exercise, therefore, is to understand the intricate interplay among contract manufacturers' improvement capability, processing level of the intermediate units, and other decision parameters in making optimal outsourcing decisions.

5. CONCLUSION AND MANAGERIAL IMPLICATIONS

We have tried to develop an optimal control theory model to suggest an economic insight into an outsourcing decision making. The specific research context involves two contract manufacturers, each having a certain level of improvement capability. As alluded before, a global market can be an appropriate setting. First of all, it is more logical to assume that a group of suppliers provides a full product line (i.e., intermediate products/inputs) to the manufacturing company than to suppose a single supplier does. Secondly, suppliers in a country face a similar infra-structural environment for innovation, and therefore share a similar level of learning capability. Thus, it will be easy to label suppliers in a particular country as a supplier group with a certain (similar) level of improvement capability (which is determined by the same infra-structural environment).

For example, consider the PC (personal computer) industry. Although a PC maker (like DEC) does fabricate some of components for its final product (i.e., a PC), its major task is assembling parts supplied by outside vendors (e.g., contract manufacturers or suppliers). Let h represent the level of being processed of a PC, i.e., the processing level, where $1 \leq h \leq H$: for instance, a completed PC (that is, a final unit/product) is labeled with $h = H$, whereas an initial unit (which is not processed at all) with $h = 1$. Denote D_l as a semi-finished PC which is completed up to a level $h = l$. Suppose the company receives a D_l from a CM . In order to sell the unit in the market, the company must internally process the semi-finished D_l until its processing level reaches $h = H$. If the unit of processing level is normalized so that one unit of h is equivalent to a unit time, we can say that a completed PC requires processing for total $H - 1$ time units, either internally or externally.

In the above example, we can think of two different situations regarding the contract manufacturers. First, suppose there are two individual CM s: then each of them is presumed to supply a full range of intermediate units for producing completed PCs. The other situation involves two different groups of CM s. For instance, the PC company can outsource to a group of suppliers or CM s all of whom are either in an Asian or European country. In this latter situation, a single supplier/*CM* does not have to supply a full product line: it would suffice for the suppliers in a country as a whole to offer the full range of intermediate products to the company. From the PC maker's standpoint, it does not matter whether a group of *CM* s or an individual *CM* provides the units as long as it can obtain all the necessary intermediate inputs for assembling and/or fabricating PCs. For the second case, the suppliers/*CM* s in a country are assumed to have a similar improvement capability since they all face the same technological infrastructure in their country which is essential for autonomous innovation at an industry/society level. This context handily accommodates issues of global supply chain management where a global firm has to procure materials from suppliers in different countries. For a global firm, the primary decision making is concerned with how much to procure from suppliers in which country with a certain level of autonomous innovation capacity, and how the answer to the above question changes for each of intermediate products/materials at different processing levels, i.e., different h s. For example, the firm might need to ask "Right now, should we procure 'highly processed intermediate products' more than 'less processed ones' from 'a supplier/*CM* (or, a group of suppliers/*CM* s) with higher improvement capability' or 'the other with lower improvement capability, but offering currently lower prices'?"

In addition to the analytical side, we have tried to generate useful managerial insights for the critical role taken up by a contract manufacturer's improvement capability in influencing the manufacturing company's outsourcing decision. To supplement such economic insights, we designed numerical examples.

Based on the analysis and related numerical examples, we were able to derive a few managerial implications. First, the manufacturing company must take into account the difference in improvement capability, not just the current cost structure, when outsourcing to multiple contract manufacturers. As the numerical examples indicate, the manufacturer also has to consider the processing levels of semi-finished units when making an outsourcing decision. For instance, it must ask whether it is better to procure more processed units rather than less processed ones from the contract manufacturer with higher improvement capability, and how the outsourcing dynamics changes over time.

This analysis can help managers reach a better outsourcing decision by supporting a communication or planning mechanism for strategy formulation. With appropriate adjustments, the model could be applied to other managerial situations as well. For instance, one can explore a human resource planning problem with this model: H can now be regarded as a required education period, and the suppliers are two sources of new and/or skilled workforce.

Our research has some limitations. First, our analytical model in this paper does not deal with stochastic elements in the outsourcing decision mainly because we tried to avoid too much complication in the analysis, but this can cause our analysis to fail under an uncertain environment. We also assumed that the processing level h can be normalized so as to be equivalent to the time unit. But, sometimes it might be very restrictive to impose this assumption. A more serious limitation is concerned with assuming continuity of decision variable u_i : that is, a contract manufacturer is assumed to provide the manufacturer with intermediate units/products at any processing level. In reality, it should be more accurate to treat u_i as a discrete variable representing a finite number of discrete processing stages. We needed the continuity assumption for analytical tractability. Assuming a single value of $\overline{u}_i$ across (τ, h) might pose another difficulty in practical application: we presumed that $\overline{u}_i$ is determined by contract negotiation between the manufacturing company and its contract manufacturers. Finally, we made a presumption that the contract manufacturer's learning capability is correlated with time: it is more valid when the improvement has a nature of autonomous innovation.

Although these limitations could hamper applying the research results to a real-world situation, they suggest some helpful guidelines for improvement in future research. The analytical results themselves are offering important managerial insights into the dynamic decision making for outsourcing to contract manufacturers.

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Appendix 1: Summary of Variables and Parameters

M_1 : manufacturing company

CM_i : contract manufacturer i, $i = 1, 2$; CM_2 has more innovation/ improvement capability than CM_1

$u_i(\tau, h)$: outsourcing amount of intermediate products h-processed at τ from CM_i

$x(\tau, h)$: stock of intermediate products h-processed at τ inside M_1

$\overline{u}_i$: economic order quantity for outsourcing to CM_i

r_i : penalty cost of outsourcing deviation associated with $\overline{u}_i$

$\overline{u}$: manufacturing company's capacity limit on outsourcing

$1 \le \tau \le T$: current decision time horizon

$1 \le h \le H$: product throughput/cycle time; level of being processed/assembled

$x(1, h) = x(\tau, 1) = 0$: initial condition for $x(\tau, h)$

$P_1(\tau, h) = K_1 h^{\alpha} \tau^{n}$: unit cost of outsourcing to CM_1, associated with $u_1(\tau, h)$

$P_2(\tau, h) = K_2 h^{\beta} \tau^{m}$: unit cost of outsourcing to CM_2, associated with $u_2(\tau, h)$

$P_c(\tau, h) = K_c h^{\gamma}$: M_1's unit internal processing cost

$K_1, K_2, K_C > 0$: given constants

$K_1 \le K_2$: base outsourcing cost; cheaper for CM_1

α, β, γ : coefficients of returns to scale associated with h, $\alpha \le \beta$ assumed, $0 \le \alpha, \beta, \gamma \le 1$

n, m : improvement capability coefficients for CM_1 and CM_2, respectively

$V(\tau) = V_0$: sales price/revenue of a finished product/unit

$S(h) = S_0 h$: salvage value at T of an h-processed intermediate product/unit

Appendix 2: An Intuitive Derivation of x^* and λ^*

We provide an intuitive interpretation of x^* and λ^*, which are given as follows.

$$x^*(\tau,h)=\begin{cases}\int_1^{\tau}\{u_1(q,h-\tau+q)+u_2(q,h-\tau+q)\}\,dq & (\tau,h)\in O_1\\ \int_1^{h}\{u_1(\tau-h+q,q)+u_2(\tau-h+q,q)\}\,dq & (\tau,h)\in O_2\cup O_3\end{cases}\tag{A1}$$

$$\lambda^*(\tau,h)=\begin{cases}V_0-\int_h^{H}K_c\rho^{\gamma}d\rho & (\tau,h)\in O_1\cup O_2\\ (T-\tau+h)S_0-\int_h^{T-\tau+h}K_c\rho^{\gamma}d & (\tau,h)\in O_3\end{cases}\tag{A2}$$

For $x(\tau,h)$ when $(\tau,h)\in O_1$ in (A1), if q=1, the right hand side (RHS) becomes $u_1(1,h-\tau+1)+u_2(1,h-\tau+1)$, which will constitute $x(\tau,h)$ after $\tau-1$ period passes. Similarly, when $q=\hat{\tau}<\tau$, the RHS becomes $u_1(\hat{\tau},h-(\tau-\hat{\tau}))+u_2(\hat{\tau},h-(\tau-\hat{\tau}))$, which will also be part of $x(\tau,h)$ after $\tau-\hat{\tau}$ period. Finally, when $q=\tau$, the RHS becomes $u_1(\tau,h)+u_2(\tau,h)$ which is part of $x(\tau,h)$. The reason we need to take an integration over $q\in[1,\tau]$ is that if $(\tau,h)\in O_1$, $\tau\le h$ is always true, and therefore taking integration with regard to h would have caused $x(\tau,h)$ to be outside of $(\tau,h)\in O_1$. Since $x(\tau,h)$ while $(\tau,h)\in O_1$ is affected by the boundary of {1}×[1, H] but the initial condition $x(1,h)=0$ is assumed, we have the solution in (A1).

Using the similar logic, we can obtain $x(\tau,h)$ when $(\tau,h)\in O_2\cup O_3$ as in (A1): here we take an integration over $q\in[1,h]$ since $\tau\ge h$ while $(\tau,h)\in O_2\cup O_3$, $x(\tau,h)$ is affected by the boundary of [1,T]×{1} and also the initial condition $x(\tau,1)=0$ is assumed.

As mentioned before, $\lambda^*(\tau,h)$ is the net value of the contribution made by $x(\tau,h)$. Since $x(\tau,h)$ while $(\tau,h)\in O_1\cup O_2$ can generate a full market value, V_0, $\lambda^*(\tau,h)$ is V_0 subtracted by $\int_h^H K_c\,\rho^{\gamma}d$ which is the sum of the internal processing cost during the remaining processing time, $[h,H]$.

Now $x(\tau,h)$ when $(\tau,h)\in O_3$ will not be finished by T, the end of current decision horizon. Such $x(\tau,h)$ will be $(T-\tau+h)$-processed by T since the remaining time until T is T-τ. Therefore, $\lambda^{*}(\tau,h)$ while $(\tau,h) \in O_3$ is the salvage value, $S(T-\tau+h) = S_0(T-\tau+h)$, subtracted by $\int_{h}^{T-\tau+h} K_c \rho^{\gamma} d$ which is the sum of the internal processing cost during the remaining processing time, $[h, T-\tau+h]$.