Robust Reliable H^{∞} Control of Continuous/Discrete Uncertain Time Delay Systems using LMI

Jong Hae Kim and Hong Bae Park

Abstract : In this paper, we present robust reliable H^{∞} controller design methods of continuous and discrete uncertain time delay systems using LMI (linear matrix inequality) technique, respectively. Also the existence conditions of state feedback control are proposed. Using some changes of variables and Schur complements, the obtained sufficient conditions are transformed into an LMI form. The closed loop system by the obtained controller is quadratically stable with H^{∞} norm bound for all admissible uncertainties, time delay, and all actuator failures occurred within the prespecified set. We show the validity of the proposed method through numerical example.

Keywards : reliable control, robust H^{∞} control, time delay, LMI, state feedback

I. Introduction

The robust H^{∞} controller design method of parameter uncertain time delay systems has attracted the attention of many control researchers[1,2,3,4,5] because the dynamic behaviour of many physical processes contains inherent time delays and uncertainties and can be modeled by an uncertain system with time delay. However, these control designs may result in unsatisfactory performances or even unexpected instabilities in the event of control failures. In practice, failures of control components are often found in real world. Hence they should be taken into account when a practical control system is designed. Recently Seo *et al.*[6] and Veillet *et al.*[7] consider the problem of reliable H^{∞} control design. Especially, Seo *et al.*[6] considered the problem of robust and reliable H^{∞} control design for linear uncertain systems with time-varying norm-bounded parameter uncertainty in the system matrix and also with actuator failures among a prespecified subset of actuators. However they did not deal with time delay. Gu *et al.*[8] and Wang[9] treated the problem of robust H^{∞} reliable control for linear state delayed systems with parameter uncertainty through algebraic Riccati equation approach in continuous time case. However, there are some disadvantages in their works. Firstly, the results were conservative in pre-selection of some starting variables in solving algebraic Riccati equation. Secondly, their works did not consider parameter uncertainties in all system matrices. Finally, most of works treated robust reliable H^{∞} controller design algorithms in continuous time case only. Since LMI(linear matrix inequality) toolbox by convex optimization algorithms has been developed, our objective is to find static state feedback controllers in continuous time case and discrete time case through LMI technique, respectively.

In this paper, we present the design method of state feedback controller satisfying quadratic stability with H^{∞} norm bound for all admissible uncertainties, time delay, and all actuator failures occurred within the prespecified subset in continuous and discrete time case. The sufficient conditions and the controller design methods are proposed. Using LMI toolbox, the solutions can be easily obtained at the same time. Also, examples are demonstrated.

II. Main results

Consider the system described by uncertain time delay systems

$$\begin{aligned} \delta x(t) &= [A+\Delta A(t)]x(t)+[A_d+\Delta A_d(t)]x(t-d) \\ &\quad +[B_u+\Delta B_u(t)]u(t)+[B_w+\Delta B_w(t)]w(t) \\ z(t) &= [C+\Delta C(t)]x(t)+[C_d+\Delta C_d(t)]x(t-d) \\ &\quad +[D_u+\Delta D_u(t)]u(t)+[D_w+\Delta D_w(t)]w(t) \\ x(t) &= \phi(t), \quad t\in[-d,\ 0], \end{aligned} \tag{1}$$

where $x(t)\in R^n$ is the state, $u(t)\in R^m$ is the control input, $w(t)\in R^p$ is the square integrable disturbance input, $z(t)\in R^r$ is the controlled output, and $\phi(t)$ is a continuous/discrete vector valued initial function. All matrices have appropriate dimensions and we assume that all states are measurable for state feedback. In here,

$$\delta x(t) = \begin{cases} \dot{x}(t) : \text{CT} \\ x(t+1) : \text{DT} \end{cases} \tag{2}$$

and time delay is defined as

$$d = \begin{cases} \text{positive real number: CT} \\ \text{positive integer : DT} \end{cases} \tag{3}$$

where CT and DT mean continuous and discrete time, respectively. And the parameter uncertainties are defined as follows:

$$\begin{bmatrix} \Delta A(t) & \Delta B_u(t) & \Delta B_w(t) & \Delta A_d(t) \\ \Delta C(t) & \Delta D_u(t) & \Delta D_w(t) & \Delta C_d(t) \end{bmatrix} \tag{4}$$

Manuscript received : Sept. 28, 1998., Accepted : Aug. 4, 1999.
Jong Hae Kim : STRC, Kyungpook National University
Hong Bae Park : School of Electronic and Electrical Engineering Kyungpook National University

$$= \begin{bmatrix} H_x \\ H_z \end{bmatrix} F(t) [E_x \ E_u \ E_w \ E_d],$$

where H_x, H_z, E_x, E_u, E_w, E_d are known real matrices and $F(t)$ is an unknown matrix function which is bounded by

$$F(t) \in \Omega := \{ F(t) : F(t)^T F(t) \le I, \text{ the elements of } F(t) \text{ are Lebesgue measurable} \}. \tag{5}$$

Now, we classify actuators of the system (1) into two groups similar to the works[6,7]. One is a selected subset of actuators susceptible to failures, which is denoted by $\Omega \subseteq \{1, 2, \ldots, m\}$. This set of actuator is redundant in terms of the stabilization of the system, while it may contribute to and is necessary for improving control performance. The other is a set of actuators robust to failures, which is denoted by $\overline{\Omega} \subseteq \{1, 2, \ldots, m\} - \Omega$. We assume these actuators never fail and also they are required in order to stabilize a given system. Introduce a decomposition

$$B_u = B_\Omega + B_{\overline{\Omega}} \tag{6}$$

where B_Ω and $B_{\overline{\Omega}}$ are formed from B_u by zeroing out columns. In the following, we let $\alpha \subseteq \Omega$ denote a particular subset of susceptible actuators that actually fail and adopt the following notation

$$B_u = B_\alpha + B_{\overline{\alpha}} \tag{7}$$

where B_α and $B_{\overline{\alpha}}$ have meanings analogous to those of B_Ω and $B_{\overline{\Omega}}$, respectively. From definitions of B_α, $B_{\overline{\alpha}}$, B_Ω, and $B_{\overline{\Omega}}$, we can obtain the following facts

$$\begin{aligned} B_\Omega B_\Omega^T &= B_\alpha B_\alpha^T + B_{\Omega-\alpha} B_{\Omega-\alpha}^T \\ B_{\overline{\Omega}} B_{\overline{\Omega}}^T &= B_{\overline{\alpha}} B_{\overline{\alpha}}^T + B_{\Omega-\alpha} B_{\Omega-\alpha}^T. \end{aligned} \tag{8}$$

Our objective is to find a memoryless state feedback controller

$$u(t) = Kx(t) \tag{9}$$

that stabilizes the linear time-delay system (1) with a given H^∞ norm constraint on disturbance attenuation, for all admissible uncertainties, time delay, and all actuators failures occurred within the prespecified subset Ω.

Lemma 1 : For given $\gamma > 0$ and $\lambda > 0$, the system (1) is QSH^∞-AF(quadratically stabilizable with H^∞ norm bound for all admissible uncertainties, time delay, and all actuator failures occurred within the subset Ω) by state feedback control (9) if and only if the system

$$\begin{aligned} \delta x(t) &= Ax(t) + A_d x(t-d) + B_{\overline{\Omega}} u(t) \\ &\quad + [B_w \ \gamma\lambda H_x \ B_\Omega] \, \widehat{w}(t) \\ \widehat{z}(t) &= \begin{bmatrix} C \\ \frac{1}{\lambda} E_x \end{bmatrix} x(t) + \begin{bmatrix} C_d \\ \frac{1}{\lambda} E_d \end{bmatrix} x(t-d) \\ &\quad + \begin{bmatrix} D_{\overline{\Omega}} \\ \frac{1}{\lambda} E_u \end{bmatrix} u(t) + \begin{bmatrix} D_w & \gamma\lambda H_z & D_\Omega \\ \frac{1}{\lambda} E_w & 0 & 0 \end{bmatrix} \widehat{w}(t) \end{aligned} \tag{10}$$

is QSH^∞-AF for the same state feedback control (9). Therefore the original system (1) can be transformed into the system without parameter uncertainties and particular subset of the susceptible actuators using some manipulations [6,9,12,13].

For simplicity of manipulation, rewrite the system (10) as follows:

$$\begin{aligned} \delta x(t) &= Ax(t) + A_d x(t-d) + Bu(t) + \widehat{B}\widehat{w}(t) \\ \widehat{z}(t) &= \widehat{C}x(t) + \widehat{C}_d x(t-d) + D_1 u(t) + D_2 \widehat{w}(t) \end{aligned} \tag{11}$$

where

$$B = B_{\overline{\Omega}}, \quad \widehat{B} = [B_w \ \gamma\lambda H_x \ B_\Omega], \quad \widehat{C} = \begin{bmatrix} C \\ \frac{1}{\lambda} E_x \end{bmatrix},$$

$$\widehat{C_d} = \begin{bmatrix} C_d \\ \frac{1}{\lambda} E_d \end{bmatrix}, \ D_1 = \begin{bmatrix} D_{\overline{\Omega}} \\ \frac{1}{\lambda} E_u \end{bmatrix}, \ D_2 = \begin{bmatrix} D_w & \gamma\lambda H_z & D_\Omega \\ \frac{1}{\lambda} E_w & 0 & 0 \end{bmatrix} \tag{12}$$

$$\widehat{z}(t) = \begin{bmatrix} z(t) \\ \widetilde{z}(t) \end{bmatrix}, \quad \widehat{w}(t) = \begin{bmatrix} w(t) \\ \widetilde{w}(t) \\ v(t) \end{bmatrix}.$$

Here $\widetilde{w}(t)$ and $\widetilde{z}(t)$ are additional input and output, and $v(t)$ is the output of faulty actuators. When we apply the control (9) to the system (11), the closed loop system from $\widehat{w}(t)$ to $\widehat{z}(t)$ is given by

$$\begin{aligned} \delta x(t) &= A_K x(t) + A_d x(t-d) + \widehat{B}\widehat{w}(t) \\ \widehat{z}(t) &= \widehat{C_K} x(t) + \widehat{C_d} x(t-d) + D_2 \widehat{w}(t) \end{aligned} \tag{13}$$

where $A_K = A + BK$ and $\widehat{C_K} = \widehat{C} + D_1 K$.

Lemma 2 : For given $\gamma > 0$ and $\lambda > 0$, the system (1) is QSH^∞-AF with the controller (9) if there exist positive definite matrices P and R such that

i) CT case

$$\begin{bmatrix} A_K^T P + PA_K + R & PA_d & P\widehat{B} & \widehat{C_K^T} \\ * & -R & 0 & \widehat{C_d^T} \\ * & * & -\gamma^2 I & D_2^T \\ * & * & * & -I \end{bmatrix} < 0 \tag{14}$$

ii) DT case

$$\begin{bmatrix} -P^{-1} & A_K & A_d & \widehat{B} & 0 \\ * & -P+R & 0 & 0 & \widehat{C_K^T} \\ * & * & -R & 0 & \widehat{C_d^T} \\ * & * & * & -\gamma^2 I & D_2^T \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{15}$$

hold for time delay and all actuators failures occurred within the subset Ω. Here $*$ mean symmetric terms.

Proof : Firstly, we define Lyapunov functional as

$$V(x(t)) := \begin{cases} x(t)^T P x(t) + \int_{t-d}^{t} x(\tau)^T R x(\tau) d\tau : \text{ CT} \\ x(t)^T P x(t) + \sum_{i=t-d}^{t-1} x(i)^T R x(i) : \text{ DT} \end{cases} \tag{16}$$

And it is noticed that conditions (14) and (15) imply

i) CT case

$$\begin{bmatrix} A_K^TP+PA_K+R & PA_d \\ A_d^TP & -R \end{bmatrix} < 0 \qquad (17)$$

ii) DT case

$$\begin{bmatrix} A_K^TPA_K-P+R & A_K^TPA_d \\ A_d^TPA_K & -R+A_d^TPA_d \end{bmatrix} < 0, \qquad (18)$$

respectively. Taking the derivative of the Lyapunov functional (16) along the solution of (13) and the difference of the Lyapunov functional (16) yields

i) CT case

$$\begin{aligned} \dot V(x(t)) &= \dot x(t)^TPx(t)+x(t)^TP\dot x(t) \\ &\quad + x(t)^TRx(t)-x(t-d)^TRx(t-d), \end{aligned} \qquad (19)$$

ii) DT case

$$\begin{aligned} \Delta V_t &= V(x(t+1))-V(x(t)) \\ &= x(t+1)^TPx(t+1)-x(t)^T(P-R)x(t) \\ &\quad -x(t-d)^TRx(t-d). \end{aligned} \qquad (20)$$

When assuming the zero input, we have

i) CT case

$$\begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}^T \begin{bmatrix} A_K^TP+PA_K+R & PA_d \\ A_d^TP & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix} < 0 \qquad (21)$$

ii) DT case

$$\begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}^T \times \begin{bmatrix} A_K^TPA_K-P+R & A_K^TPA_d \\ A_d^TPA_K & -R+A_d^TPA_d \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix} < 0 \qquad (22)$$

which ensure the quadratic stability of the closed loop system. In the next place, assume the zero initial condition and let us introduce

$$J := \begin{cases} \int_0^\infty [\hat z(t)^T\hat z(t)-\gamma^2\hat w(t)^T\hat w(t)]dt : & \text{CT} \\ \sum_{t=0}^{\infty} [\hat z(t)^T\hat z(t)-\gamma^2\hat w(t)^T\hat w(t)] : & \text{DT} \end{cases} \qquad (23)$$

Then for any nonzero square integrable disturbance input

$$w(t) \in \begin{cases} L_2[0,\infty) : & \text{CT} \\ l_2[0,\infty) : & \text{DT} \end{cases} \qquad (24)$$

the performance measures are represented by

$$J_a := \begin{cases} \int_0^\infty [\hat z(t)^T\hat z(t)-\gamma^2\hat w(t)^T\hat w(t)+\dot V(x(t))]dt : & \text{CT} \\ \sum_{t=0}^{\infty} [\hat z(t)^T\hat z(t)-\gamma^2\hat w(t)^T\hat w(t)]+\Delta V_t : & \text{DT} \end{cases} \qquad (25)$$

and further substituting (19) and (20) into (25), respectively. And let $\delta(t)=[x(t)\ x(t-d)\ \hat w(t)]$, then

$$J_a := \begin{cases} \int_0^\infty \delta(t)^TZ_c\delta(t)dt : & \text{CT} \\ \sum_{t=0}^{\infty} \delta(t)^TZ_d\delta(t) : & \text{DT} \end{cases} \qquad (26)$$

where Z_c and Z_d are defined as

$$Z_c = \begin{bmatrix} A_K^TP+PA_K+\widehat{C}_K^T\widehat{C}_K+R & PA_d+\widehat{C}_K^T\widehat{C}_d & P\widehat{B} \\ * & -R+\widehat{C}_d^T\widehat{C}_d & \widehat{C}_d^TD_2 \\ * & * & -\gamma^2I+D_2^TD_2 \end{bmatrix} < 0, \qquad (27)$$

$$Z_d = \begin{bmatrix} A_K^TPA_K-P+R+\widehat{C}_K^T\widehat{C}_K & A_K^TPA_d+\widehat{C}_K^T\widehat{C}_d & A_K^TP\widehat{B}+\widehat{C}_K^TD_2 \\ * & A_d^TPA_d-R+\widehat{C}_d^T\widehat{C}_d & A_d^TP\widehat{B}+\widehat{C}_d^TD_2 \\ * & * & -\gamma^2I+D_2^TD_2+\widehat{B}^TP\widehat{B} \end{bmatrix} < 0. \qquad (28)$$

This $Z_c<0$ and $Z_d<0$ imply $\|z(t)\|_2 \le \gamma\|w(t)\|_2$ for any nonzero $w(t)$ of (24). Therefore when $Z_c<0$ and $Z_d<0$ are quadratically stable with an H^∞ norm bound γ by the controller (9). Using Schur complements, $Z_c<0$ and $Z_d<0$ are transformed into (14) and (15), respectively. ■

However the conditions (14) and (15) are not an LMI form in terms of each finding variable P, R, K. It is shown that the (14) and (15) are transformed into an LMI form in the following theorem.

Theorem 1': Consider closed loop system (13). For given $\gamma>0$ and $\lambda>0$, if there exist a matrix M and positive definite matrices Q, S such that

i) CT case

$$\begin{bmatrix} QA^T+AQ+M^TB^T+BM+A_dSA_d^T & \widehat{B} & M^TD_1^T+Q\widehat{C}^T+A_dS\widehat{C}_d^T & Q \\ * & -\gamma^2I & D_2^T & 0 \\ * & * & -I+\widehat{C}_dS\widehat{C}_d^T & 0 \\ * & * & * & -S \end{bmatrix} < 0 \qquad (29)$$

ii) DT case

$$\begin{bmatrix} -Q+A_dSA_d^T & AQ+BM & \widehat{B} & A_dS\widehat{C}_d^T & 0 \\ * & -Q & 0 & Q\widehat{C}^T+M^TD_1^T & Q \\ * & * & -\gamma^2I & D_2^T & 0 \\ * & * & * & -I+\widehat{C}_dS\widehat{C}_d^T & 0 \\ * & * & * & * & -S \end{bmatrix} < 0 \qquad (30)$$

holds for time delay and all actuators failures occurred within the subset Ω.

Proof : Using Schur complements and the changes of variables

$$M = KP^{-1},\ Q = P^{-1},\ S = R^{-1}, \qquad (31)$$

the obtained sufficient conditions (14) and (15) are changed to (29) and (30), respectively. ■

Remark 1 : The (29) and (30) are an LMI form in terms of changed variables. Therefore robust reliable H^∞ state feedback controller K can be calculated from the $M=KP^{-1}$ after finding the LMI solutions Q, M, and

S from the (29) and (30). Using LMI toolbox[11], the solutions can be easily obtained at the same time.

Remark 2 : In the case of continuous time-varying delay systems, the proposed method can be extended easily. If the time-varying delay is assumed as

$$0 \le d(t) < \infty, \quad \dot{d}(t) \le \beta < 1, \tag{32}$$

then the sufficient conditions are changed as

$$\begin{bmatrix} QA^T + AQ + M^TB^T + BM + A_d\tilde{S}A_d^T & \hat{B} & M^TD_1^T + Q\hat{C}^T + A_d\tilde{S}\widehat{C_d}^T & Q \\ * & -\gamma^2 I & D_2^T & 0 \\ * & * & -I + \widehat{C_d}\tilde{S}\widehat{C_d}^T & 0 \\ * & * & * & -S \end{bmatrix} < 0 \tag{33}$$

respectively. Here, $\tilde{S} = (1-\beta)^{-1}S$.

Remark 3 : Using the proposed method, the following system

$$\begin{aligned} \delta x(t) &= [A + \Delta A(t)]x(t) + [A_d + \Delta A_d(t)]x(t-d_1) \\ &\quad + [B_u + \Delta B_u(t)]u(t) + [B_d + \Delta B_d(t)]u(t-d_2) \\ &\quad + [B_w + \Delta B_w(t)]w(t) \\ z(t) &= [C + \Delta C(t)]x(t) + [C_d + \Delta C_d(t)]x(t-d_1) \\ &\quad + [D_u + \Delta D_u(t)]u(t) + [D_d + \Delta D_d(t)]u(t-d_2) \\ &\quad + [D_w + \Delta D_w(t)]w(t) \end{aligned} \tag{34}$$

can be solved. In order words, the robust reliable H^∞ static state feedback controller of parameter uncertain time delay systems in both states and control inputs can be obtained.

III. Numerical example

Consider the uncertain time delay system of the same example in [9] with

$$A = \begin{bmatrix} 4 & 0.02 & -0.1 \\ -0.3 & 3 & -0.2 \\ 0.3 & -0.1 & 2 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.2 & 0.05 & 0.01 \\ 0 & -0.3 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B_u = \begin{bmatrix} 5 & 0.2 & 0 \\ 0 & 3 & 0.1 \\ 0.1 & 0 & 0.03 \end{bmatrix}, \quad B_w = \begin{bmatrix} 0.01 & 0 \\ 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, \quad H_x = \begin{bmatrix} 0.08 & 0 & 0 \\ 0 & 0.08 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$E_x = \begin{bmatrix} 0.04 & 0 & 0 \\ 0 & 0.02 & 0 \\ 0 & 0 & 0.04 \end{bmatrix}, \quad E_d = \begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0.02 & 0 \\ 0 & 0 & 0.02 \end{bmatrix},$$

and other matrices are zero matrices with proper dimensions. For simulation, we take $\gamma = 3$, $\lambda = 1$, $F(t) = \sin t$, $d = 5$, and $\Omega = \{3\}$, we have

$$B_{\bar{\Omega}} = \begin{bmatrix} 5 & 0.2 & 0 \\ 0 & 3 & 0 \\ 0.1 & 0 & 0 \end{bmatrix}, \quad B_{\Omega} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.1 \\ 0 & 0 & 0.03 \end{bmatrix}.$$

In the case of continuous time case, all solutions and state feedback gain are

$$Q = \begin{bmatrix} 25.5851 & -0.8625 & -0.8744 \\ -0.8625 & 34.7574 & 0.7573 \\ -0.8744 & 0.7573 & 0.0723 \end{bmatrix},$$

$$S = \begin{bmatrix} 79.1138 & -0.0509 & 1.6534 \\ -0.0509 & 80.1428 & -0.7005 \\ 1.6534 & -0.7005 & 87.1236 \end{bmatrix},$$

$$M = \begin{bmatrix} -31.7756 & 4.9475 & 0.0007 \\ 0.2205 & -51.0113 & -0.3589 \\ 0 & 0 & 0 \end{bmatrix},$$

$$K = \begin{bmatrix} -2.6781 & 1.0132 & -43.0195 \\ 0.8449 & -2.0233 & 26.4649 \\ 0 & 0 & 0 \end{bmatrix}.$$

The obtained continuous time state feedback control guarantees QSH^∞-AF. The states trajectories, control inputs, disturbance inputs, and controlled outputs are shown in Fig. 1. In the (a) of Fig. 1., the states converge to zero as time goes to infinity. From this fact, the obtained controller stabilizes the system against the time delay, parameter uncertainty, and actuator failure. The control inputs are given by (b), (c), (d) of Fig. 1. The (d) of Fig. 1. shows the third actuator failure. From the relation (e), (f), and (g), the H^∞ norm bound of the closed loop system is guaranteed within the prescribed γ. The actual value of γ is 0.0719 (<3).

Similarly to the continuous time case, all solutions and discrete time state feedback gain is obtained as follows:

$$Q = \begin{bmatrix} 17.3768 & 8.7877 & -1.4222 \\ 8.7877 & 304.2086 & 13.6712 \\ -1.4222 & 13.6712 & 1.0808 \end{bmatrix},$$

$$S = \begin{bmatrix} 253.9382 & 231.4889 & 26.5253 \\ 231.4889 & 624.3931 & -4.4061 \\ 26.5253 & -4.4061 & 837.7026 \end{bmatrix},$$

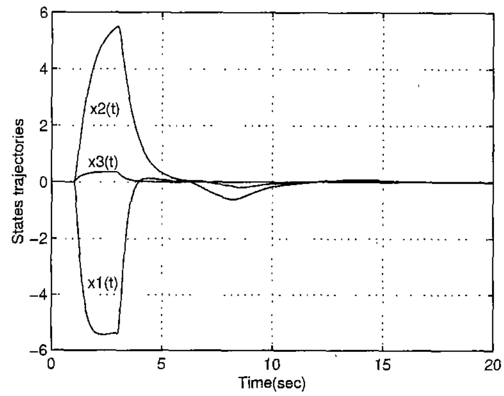
$$M = \begin{bmatrix} -13.6346 & 4.2473 & 1.4096 \\ -5.8558 & -291.9699 & -11.2058 \\ 0 & 0 & 0 \end{bmatrix},$$

$$K = \begin{bmatrix} -0.8970 & 0.0795 & -0.8819 \\ 0.9999 & -1.3483 & 8.0030 \\ 0 & 0 & 0 \end{bmatrix}.$$

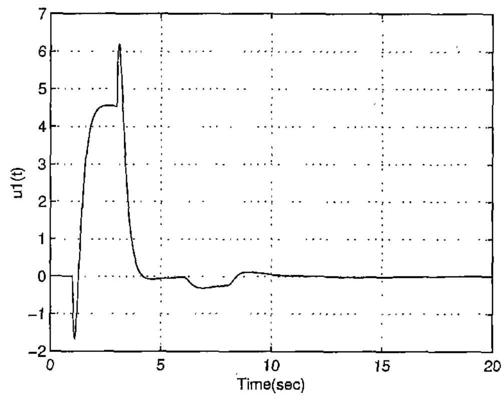
Also the obtained discrete time state feedback controller guarantees QSH^∞-AF of the closed loop system. The states trajectories, control inputs, disturbance inputs, and controlled outputs in discrete time case are shown in Fig. 2. Since the states converge to zero as time goes to infinity, the obtained controller stabilizes the system against the time delay, parameter uncertainty, and actuator failure in the (a) of Fig. 2. The control inputs are given by (b), (c), (d) of Fig. 2. The (d) of Fig. 2. shows the third actuator failure. From the relation (e), (f), and (g), the H^∞ norm bound of the closed loop system is guaranteed within the prescribed γ. The actual value of γ is 0.1232(<3).

IV. Conclusion

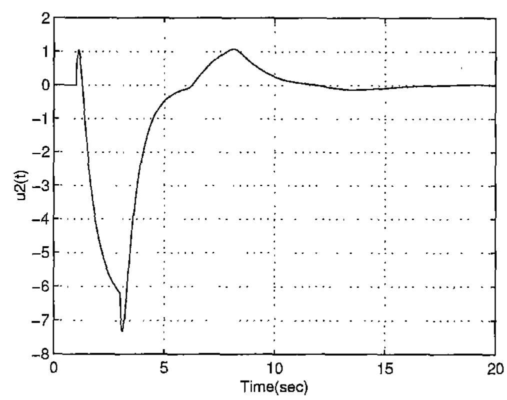
We presented controller design algorithms of continuous and discrete uncertain time delay systems through LMI approach. From the Lyapunov functions



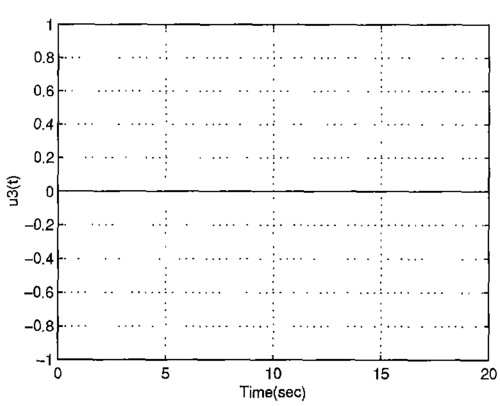
(a) $x_1(t)$, $x_2(t)$, and $x_3(t)$.



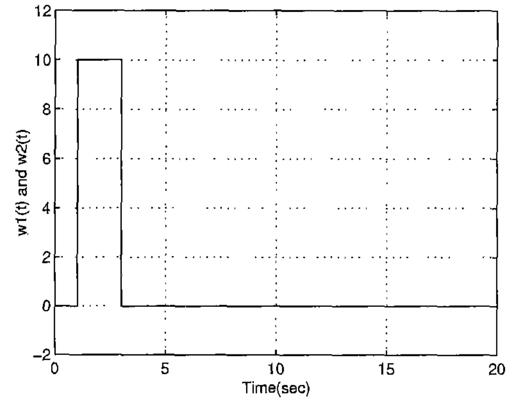
(b) $u_1(t)$.



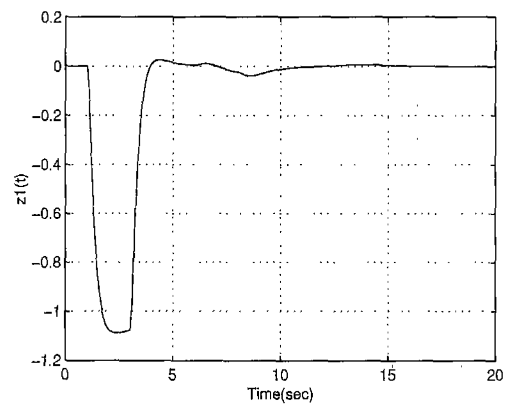
(c) $u_2(t)$.



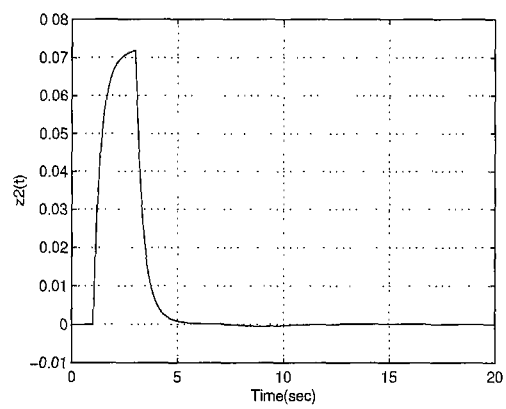
(d) $u_3(t)$.



(e) $w_1(t)$ and $w_2(t)$.



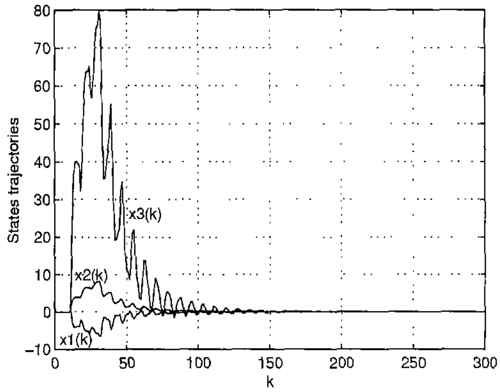
(f) $z_1(t)$.



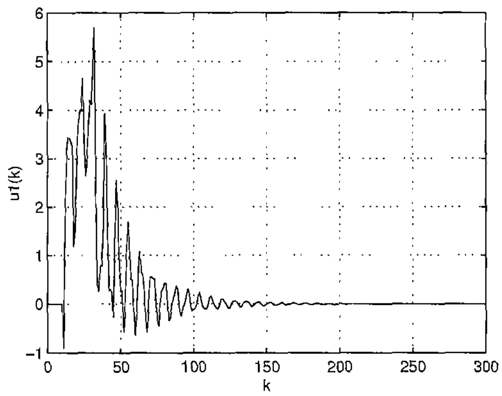
(g) $z_2(t)$.

Fig. 1. The states trajectories, control inputs, disturbance inputs, and controlled outputs in continuous time case.

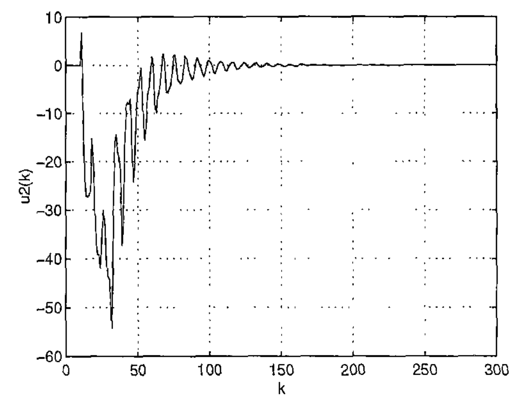
and performance measures, the existence conditions of state feedback controller were given. Also, the obtained sufficient conditions were transformed into an LMI form using some changes of variables and Schur complements. Through numerical examples, the closed loop system by the obtained state feedback controller was quadratically stable with H^{∞} norm bound for all admissible uncertainties, time delay, and all actuator failures occurred within the subset Ω.



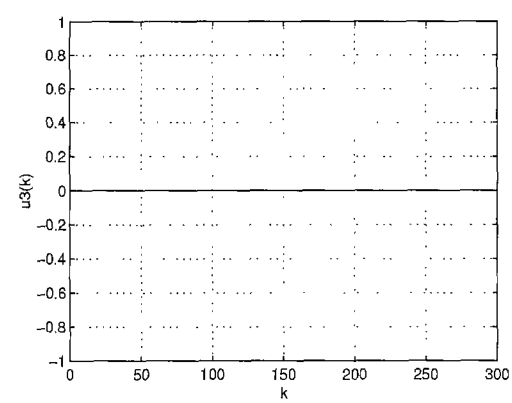
(a) $x_1(k)$, $x_2(k)$, and $x_3(k)$.



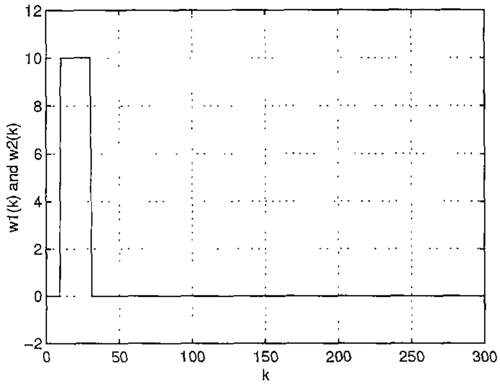
(b) $u_1(k)$.



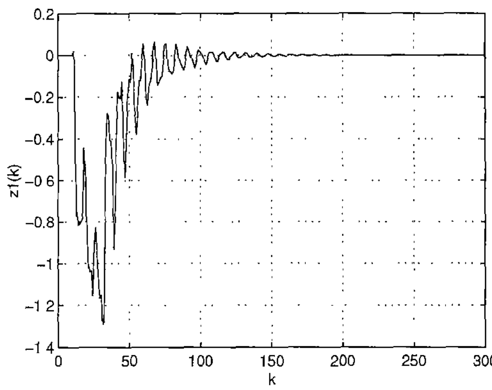
(c) $u_2(k)$.



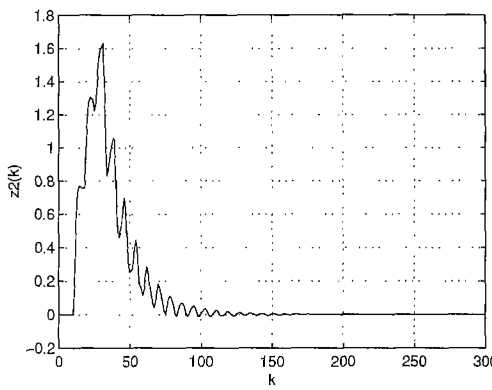
(d) $u_3(k)$.



(e) $w_1(k)$ and $w_2(k)$.



(f) $z_1(k)$.



(g) $z_2(k)$.

Fig. 2. The states trajectories, control inputs, disturbance inputs, and controlled outputs in discrete time case.

References

[1] J. H. Kim, E. T. Jeung, and H. B. Park, "Robust control for parameter uncertain delay systems in state and control input," *Automatica*, vol. 32, no. 9, pp. 1337-1339, 1996.

[2] E. T. Jeung, D. C. Oh, J. H. Kim, and H. B. Park, "Robust controller design for uncertain systems with time delays: LMI approach," *Automatica*, vol. 32, no. 8, pp. 1229-1231, 1996.

[3] J. H. Ge, P. M. Frank, and C. F. Lin, "Robust H^∞ state feedback control for linear systems with

state delay and parameter uncertainty," *Automatica*, vol. 32, no. 8, pp. 1183-1185, 1996.

[4] L. Yu, J. Chu, and H. Su, "Robust memoryless H^∞ controller design for linear time-delay systems with norm-bounded time-varying uncertainty," *Automatica*, vol. 32, no. 12, pp. 1759-1762, 1996.

[5] E. T. Jeung, J. H. Kim, and H. B. Park, "H^∞ output feedback controller design for linear systems with time-varying delayed states," *IEEE Trans. Autom. Contr.*, vol. 43, no. 7, pp. 971-974, 1998.

[6] C. J. Seo and B. K. Kim, "Robust and reliable H^∞ control for linear systems with parameter uncertainty and actuator failure," *Automatica*, vol. 32, no. 3, pp. 465-467, 1996.

[7] R. J. Veillette, J. V. Medanic, and W. R. Perkins, "Design of reliable control systems," *IEEE Trans. Autom. Contr.*, vol. 37, no. 3, pp. 290-304, 1992.

[8] Y. Gu, C. Geng, J. Qian, and L. Wang, "Robust reliable H^∞ control for uncertain time-delay systems," *American Control Conference in Philadelphia, Pennsylvania*, pp. 2415-2416, 1998.

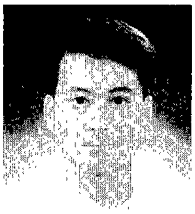
[9] Z. Wang, "Robust H^∞ reliable control for linear state delayed systems with parameter uncertainty," *American Control Conference in Philadelphia, Pennsylvania*, pp. 2415-2416, 1998.

[10] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, 1994.

[11] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *LMI Control Toolbox*, The Math Works Inc., 1995.

[12] K. Gu, "H^∞ control of systems under norm bounded uncertainties in all system matrices," *IEEE Trans. Autom. Contr.*, vol. 39, no. 6, pp. 1320-1322, 1994.

[13] L. Yuan, L. E. K. Achenie, and W. Jiang, "Robust H^∞ control for linear discrete-time systems with norm-bounded time-varying uncertainty," *Systems & Control Lett.*, vol. 27, pp. 199-208, 1996.



Jong Hae Kim

He was born in Korea, on January 10, 1970. He received the B. S., M. S., and Ph. D. degrees in electronic engineering from Kyungpook National University, Taegu, Korea, in 1993, 1995, and 1998, respectively. He is currently with STRC(Sensor Technology Research Center) at Kyungpook National University since 1998. He received 'International Scholarship Award' from SICE(Japan) in 1999 and 'Young Researcher Paper Award' from ICASE in 1999. His areas of research interest are robust control, mixed H^2/H^∞ control, nonlinear control, the stabilization of time-delay systems, non-fragile control, reliable control, and industrial application control. He is a member of IEEE, IEEK and ICASE.

Hong Bae Park

He was born in Korea, on March 6, 1951. He received the B. S. and M. S. degrees in electronic engineering from Kyungpook National University, Taegu, Korea, in 1977 and 1979, respectively, and the Ph. D. degree in electrical and computer engineering from the University of New Mexico, Albuquerque, New Mexico, in 1988. He is currently a Professor in School of Electronic and Electrical Engineering, Kyungpook National University. He received 'Hae-Dong Paper Award' from IEEK in 1998. His current research interests include robust control, optimal control to industrial applications, and guidance control. He is a member of IEEE, IEEK, and ICASE.