Design of T-S(Takagi-Sugeno) Fuzzy Control Systems Under the Bound on the Output Energy

Kwang-Tae Kim, Joog-Seon Joh, and Woo-Hyen Kwon

Abstract : This paper presents a new T-S(Takagi-Sugeno) fuzzy controller design method satisfying the output energy bound. Maximum output energy via a quadratic Lyapunov function to obtain the bound on output energy is derived. LMI(Linear Matrix Inequality) problems which satisfy an output energy bound for both of the continuous-time and discrete-time T-S fuzzy control system are also derived. Solving these LMIs simultaneously, we find a common symmetric positive definite matrix P which guarantees the global asymptotic stability of the system and stable feedback gains K_i's satisfying the output energy bound. A simple example demonstrates validity of the proposed design method.

Keywords : Takagi-Sugeno fuzzy model, PDC, LMI, stability LMI, output energy bound

I. Introduction

The most important issue concerning the properties of nonlinear control systems may be stability analysis. Stability analysis for fuzzy control systems has been considered as difficult issue because fuzzy systems are essentially nonlinear systems.

Recently, some useful stability techniques for the so-called T-S fuzzy control systems, which are based on nonlinear stability theory, have been reported. Tanaka and Sugeno[1] proposed a theorem on the stability analysis of the T-S fuzzy model. Wang *et al.* [2] proposed the so-called PDC(Parallel Distributed Compensation) as a design framework and also modified the Tanaka's stability theorem to include control. It is, however, not a design method since it needs pre determined feedback gains. Wang *et al.*'s method can be considered as a stability checking method for pre-designed system. Joh *et al.* Proposed a systematic design method to overcome such drawback[3][4]. It applies the Schur complements[5, page 7] to the previous stability criterion to treat the feedback gains as unknown and derives LMIs for desired regions for closed-loop poles to obtain the desired control performances[6][7].

This paper proposes a new design method for T-S fuzzy controllers which guarantees global asymptotic stability under the maximum output energy constraint of the closed-loop system. It is accomplished by considering LMI for the maximum output energy constraint of the closed-loop system. Output energy constraint means that response speed of the system output can be controlled. Convex optimization technique involving LMIs are utilized to find a common symmetric positive definite matrix P and stable state-feedback gains satisfying maximum output energy constraint.

This paper is organized into five sections. Section 1 is an introduction. Some background materials are summarized in Section 2. The proposed T-S fuzzy control system design method satisfying the output energy bound is explained in Section 3. A T-S fuzzy controller for an inverted-pendulum with a cart is designed using the proposed method in Section 4. Finally, some concluding remarks are given in Section 5.

Manuscript received : Sep. 10, 1998., Accepted : June. 24, 1999.
Kwang Tae Kim : Depatment of Electronic Engineering, Doowon Technical College.
Joog Seon Joh : Department of Control and Instrumentation Engineering, Changwon National University.
Woo Hyen Kwon : School of Electronic and Electrical Engineering, Kyungpook National University.
※ This work was supported in part by the Korea Science and Engineering Foundation(KOSEF) through the Machine Tool Research Center at Changwon National University.

II. Background Materials

1. T-S Fuzzy Model of Nonlinear Dynamic Systems and Its Stability

Takagi and Sugeno[8] proposed an effective way to represent fuzzy model of nonlinear dynamic systems. It uses input-output relationship to represent each plant rule. A T-S fuzzy model is composed of r plant rules that can be represented as

Plant rule i : if $x_1(t)$ is $M_1^i \cdots$and$\cdots x_n(t)$ is M_n^i

$$\text{then } \delta x(t) = A_i x(t) + B_i u(t),\ i = 1,2,\cdots,r \quad (1)$$

where x_j is j-th state variable, M_j is fuzzy term set of x_j, M_j^i is a fuzzy term of M_j selected for plant rule i, $x(t)\in\mathbf{R}^n$ is the state vector, $u(t)\in\mathbf{R}^m$ is the input vector, $A_i\in\mathbf{R}^{n\times n}$ and $B_i\in\mathbf{R}^{n\times m}$. It should be noted that $\delta x(t)=\dot{x}(t)$ for CFS(Continuous-time Fuzzy Systems) and $\delta x(t)=x(t+1)$ for DFS(Discrete-time Fuzzy Systems).

For any current state vector $x(t)$ and input vector $u(t)$, the T-S fuzzy model infers $\delta x(t)$ as the output of the fuzzy model as follows:

$$\delta x(t) = \frac{\sum_{i=1}^{r}\omega_i[A_i x(t) + B_i u(t)]}{\sum_{i=1}^{r}\omega_i} \quad (2)$$

where

$$\omega_i = \prod_{k=1}^{r} M_k^i(x_k(t)). \tag{3}$$

For a free system(i.e., $u(t) \equiv 0$), (2) can be written as

$$\delta x(t) = \frac{\sum_{i=1}^{r} \omega_i A_i x(t)}{\sum_{i=1}^{r} \omega_i} \tag{4}$$

From now on, it is assumed that a proper continuous or discrete T-S fuzzy model is available.

Tanaka and Sugeno[1] suggested an important criterion for the stability of the discrete T-S fuzzy model.

Theorem 1. [Stability Criterion for T-S Fuzzy Model][CFS] The equilibrium of the continuous-time T-S fuzzy model (4) (namely, $x = 0$) is globally asymptotically stable if there exists a common symmetric positive definite matrix P such that

$$A_i^T P + PA_i < 0 \text{ for all } i = 1,2,\cdots,r \tag{5}$$

proof : See [1].

Theorem 2. [Stability Criterion for T-S Fuzzy Model][DFS] The equilibrium of the discrete-time T-S fuzzy model (4) (namely, $x = 0$) is globally asymptotically stable if there exists a common symmetric positive definite matrix P such that

$$A_i^T PA_i - P < 0 \text{ for all } i = 1,2,\cdots,r \tag{6}$$

proof : See [1].

Remark 1. It is well-known that the total system may not be stable even though every subsystems are stable[1].

Remark 2. It should be noted that (5) and (6) are *sufficient conditions* for stability but not necessary condition. Therefore, better criteria may exist for stability of T-S fuzzy model.

2. PDC-type T-S Fuzzy Control System and Its Stability

Wang *et al.*[2] proposed a framework which can be used as a guideline to design a T-S controller using existing T-S fuzzy model. In this case, we can use a proper linear control method for each pair of control rule and plant rule. Wang *et al.*[2] named it PDC.

A PDC-type T-S fuzzy controller which uses full state feedback is composed of r control rules that can be represented as

Control rule i : if $x_1(t)$ is $M_1^i \cdots$and$\cdots x_n(t)$ is M_n^i
then $u(t) = K_i x(t)$ $\quad$ i = 1,2,$\cdots$,r (7)

For any current state vector $x(t)$, the T-S fuzzy controller infers $u(t)$ as the output of the fuzzy controller as follows:

$$u(t) = \frac{\sum_{j=1}^{r} \omega_j K_j x(t)}{\sum_{j=1}^{r} \omega_j}. \tag{8}$$

It has very important advantage because it makes easy (or manageable) to apply (8) to (2). Therefore, the closed-loop behavior of the T-S fuzzy model (1) with the T-S fuzzy controller (7) using PDC can be obtained by substituting (8) into (2) as follows:

$$\delta x(t) = \frac{\sum_{i=1}^{r}\sum_{j=1}^{r} \omega_i\omega_j (A_i + B_i K_j) x(t)}{\sum_{i=1}^{r}\sum_{j=1}^{r} \omega_i\omega_j} \tag{9}$$

The corresponding sufficient condition for the stability of (9) can be obtained in terms of Lyapunov's direct method and resulting LMI formulations can be resuced by the application of [2].

Corollary 1. [Stability Condition for PDC-type T-S Fuzzy Control System using State Feedback][CFS] The equilibrium of the continuous-time PDC-type T-S fuzzy control system (namely, $x = 0$) is globally asymptotically stable if there exists a common symmetric positive definite matrix P such that

$$\begin{aligned} (A_i + B_i K_i)^T P + P(A_i + B_i K_i) < 0, &\quad i = 1,2,\cdots,r \\ G_{ij}^T P + PG_{ij} < 0, &\quad i < j \le r, \end{aligned} \tag{10}$$

where

$$G_{ij} = \frac{(A_i + B_i K_j) + (A_j + B_j K_i)}{2}, \quad i < j \le r. \tag{11}$$

Proof : See [2].

The sufficient condition for stability for discrete-time PDC-type T-S fuzzy control systems is summarized in the Corollary 2.

Corollary 2. [Stability Condition for PDC-type T-S Fuzzy Control System using State Feedback][DFS] The equilibrium of the discrete-time PDC-type T-S fuzzy control system (namely, $x = 0$) is globally asymptotically stable if there exists a common symmetric positive definite matrix P such that

$$\begin{aligned} (A_i + B_i K_i)^T P(A_i + B_i K_i) - P < 0, &\quad i = 1,2,\cdots,r \\ G_{ij}^T PG_{ij} - P < 0, &\quad i < j \le r. \end{aligned} \tag{12}$$

Proof : See [2].

Remark 3. The sufficient condition for stability (10) and (12) can be used only for the purpose of checking of the stability of the T-S fuzzy control system in which the feedback gains K_i's, $i = 1,2,\cdots,r$ are predetermined by a proper linear controller design method.

III. Derivation of LMIs under the Bound on Output Energy

In this paper, we first show how to find a state feedback gain K_i in each subsystem such that the output energy of the closed-loop system in less than a specified value.

1. The case of CFS

We assume that the initial state condition $x(0)$ is given. To obtain the bound on output energy, we seek first the maximum output energy for the CFS given a certain initial state condition as follows:

$$\max\left\{\int_0^{\infty} y(t)^T y(t)dt \mid \delta x(t) = \frac{\sum_{i=1}^{r}\omega_i\left(A_i x(t) + B_i u(t)\right)}{\sum_{i=1}^{r}\omega_i}, y(t) = Cx(t)\right\} \quad (13)$$

Suppose there exists a quadratic Lyapunov function $V(x(t)) = x(t)^T Px(t)$ such that

$$\delta V(x) \le -y(t)^T y(t), \quad (14)$$

for every x(t) and $y(t)$. Then, integrating (14) from t=0 to T, wc obtain

$$V(x(T)) - V(x(0)) \le -\int_0^T y(t)^T y(t)dt, \quad (15)$$

for every $T \ge 0$.

Since $V(x(T)) \ge 0$, we conclude that $V(x(0))=x(0)^T Px(0)$ is an upper bound on the maximum energy of the output $y(t)$ given the initial state condition $x(0)$.

We now derive MIs(Matrix Inequalities) that satisfy upper bound constraint on the output energy for the T-S fuzzy control system.

Theorem 3. [Stability Criterion for PDC-type T-S Fuzzy Control System under the Output Energy Bound for CFS] The equilibrium of the continuous-time PDC-type T-S fuzzy control system (namely, $x = 0$) under the output energy (i.e., $x(0)^T Px(0) \le \gamma$, γ is positive constant which is specified by designer) is globally asymptotically stable if there exists a common symmetric positive definite matrix P such that

(16)

is posit

$$(A_i + B_i K_i)^T P + P(A_i + B_i K_i) + C^T C \le 0, i, 2, \cdots, \gamma$$
$$G_{ij}^T P + PG_{ij} + C^T C \le 0, i < j \le \gamma$$ ive
$$x(0)^T Px(0) \le \gamma, \gamma$$

constant

Proof : The third equation in (16) is already an LMI. Therefore, the first two equations in (16) will be derived to prove the theorem. (14) becomes as follows

$$\frac{x^T(t)\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i\omega_j\left(A_i + B_i K_j\right)^T Px(t)}{\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i\omega_j} + \frac{x^T(t)\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i\omega_j P\left(A_i + B_i K_j\right)x(t)}{\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i\omega_j} \le -x(t)^T C^T Cx(t). \quad (17)$$

Therefore, following equation is satisfied.

$$\frac{\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i\omega_j\left(A_i + B_i K_j\right)^T P}{\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i\omega_j} + \frac{\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i\omega_j P\left(A_i + B_i K_j\right)}{\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i\omega_j} \le -C^T C. \quad (18)$$

In (18), $C^T C$ term can be rewritten as (19).

$$C^T C = \frac{\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i\omega_j C^T C}{\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i\omega_j} = \frac{\sum_{i=1}^{r}\omega_i\omega_i C^T C + 2\sum_{i<j}^{r}\omega_i\omega_i C^T C}{\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i\omega_j} \quad (19)$$

Substituting of (19) into (17) and using Corollary 1 yields

$$\left(A_i + B_i K_i\right)^T P + P\left(A_i + B_i K_i\right) + C^T C \le 0, \quad i = 1,2,\cdots,r$$
$$G_{ij}^T P + PG_{ij} + C^T C \le 0, \quad i < j \le r. \quad (20)$$

(End of Proof)

Therefore, PDC-type T-S fuzzy control system under the bound on the output energy is designed by solving following LMIs.

Theorem 4. [Design of PDC-type T-S Fuzzy Control System under the Output Energy Bound for CFS] A continuous-time T-S fuzzy controller which guarantees globally asymptotic stability and an output energy less than specified maximum output energy $\gamma > 0$ can be solving following LMIs

$$\begin{bmatrix} -\gamma & x^T(0) \\ x(0) & -Q \end{bmatrix} \le 0,$$

$$\begin{bmatrix} A_i Q + QA_i^T + B_i V_i + V_i^T B_i^T & (CQ)^T \\ CQ & -I \end{bmatrix} \le 0,$$
$$i = 1,2,\cdots,r$$

$$\begin{bmatrix} A_i Q + QA_i^T + A_j Q + QA_j^T + B_i V_j + V_j^T B_i^T + B_j V_i + V_i^T B_j^T & (CQ)^T \\ CQ & -I \end{bmatrix} \le 0, \quad (21)$$
$$i < j \le r.$$

Proof : If we let $Q=P^{-1} > 0$ and pre- and post-multiply (20) by Q, then (20) becomes

$$A_i Q + QA^T + B_i K_i Q + QK_i^T B_i^T + QC^T CQ \le 0,$$
$$i = 1,2,\cdots,r$$
$$A_i Q + QA_i^T + A_j Q + QA_j^T + B_i K_j Q + QK_j^T B_i^T + B_j K_i Q + QK_i^T B_j^T + QC^T CQ \le 0, \quad (22)$$
$$i < j \le r.$$

By letting $K_i Q=V_i$ and using Schur complements then (21) is obtained.

(End of Proof)

2. The Case of DFS

Maximum output energy for DFS is obtained as the same form as CFS using summation notation instead of integral in (13) and (15). And, in discrete-time case, $\delta V(x)$ in (14) becomes $V(x(t+1)) - V(x(t))$. The following LMIs for stability criterion under the bound on the ouput energy for DFS can be made by using the similar concept in the CFS.

Theorem 5. [Stability Criterion for PDC-type T-S Fuzzy Control System under the Output Energy Bound for DFS] The equilibrium of the discrete-time PDC-type T-S fuzzy control system (namely, $x = 0$) under the output energy bound (i.e., $x(0)^T Px(0) \le \gamma$, γ is positive constant which is specified by designer) is globally asymptotically stable if there exists a common symmetric positive definite matrix P such that

$$(A_i + B_i K_i)^T P(A_i + B_i K_i) - P + C^T C \le 0, i = 1, 2, \ldots, \gamma$$
$$G_{ij}^T PG_{ij} - P + C^T C \le 0, i < j \le \gamma \quad (23)$$
$$x(0)^T Px(0) \le \gamma, \gamma$$
is positive constant

Proof : The third equation in (23) is already an LMI.

Therefore, the first two equations in (23) will be derived to prove the theorem. (14) becomes as follows

$$\frac{x^T(t)\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i\omega_j(A_i+B_iK_j)^T P(A_i+B_iK_j)x(t)}{\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i\omega_j}-\underbrace{\frac{x^T(t)\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i\omega_j Px(t)}{\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i\omega_j}}_{\le -x(t)^T C^T Cx(t)} \quad (24)$$

Substituting of (19) into (24) and using Corollary 2 yields

$$(A_i+B_iK_i)^T P(A_i+B_iK_i)-P+C^TC\le 0,\ \ i=1,2,\cdots,r$$
$$G_{ij}^T PG_{ij}-P+C^TC\le 0,\ \ i<j\le r \quad (25)$$

(End of Proof)

Therefore, PDC-type T-S fuzzy control system under the bound on the output energy is designed by solving following LMIs.

Theorem 6. [Design of PDC-type T-S Fuzzy Control System under the Output Energy Bound for DFS] A discrete-time T-S fuzzy controller which guarantees globally asymptotic stability and an output energy less than specified maximum output energy $\gamma>0$ can be designed by solving following LMIs

$$\begin{bmatrix} -\gamma & x^T(0) \\ x(0) & -Q \end{bmatrix}\le 0,$$

$$\begin{bmatrix} -Q & QA_i^T+V_i^TB_i^T & 0 \\ A_iQ+B_iV_i & -Q & (CQ)^T \\ 0 & CQ & -I \end{bmatrix}\le 0,$$
$$i=1,2,\cdots,r$$

$$\begin{bmatrix} -Q & \frac{1}{2}(QA_i^T+V_j^TB_i^T+QA_j^T+V_i^TB_j^T) & 0 \\ \frac{1}{2}(A_iQ+B_iV_j+A_jQ+B_jV_i) & -Q & (CQ)^T \\ 0 & CQ & -I \end{bmatrix}\le 0, \quad (26)$$
$$i<j\le r.$$

where $V_i=K_iQ$ and $Q=P^{-1}$.

Proof : If we let $Q=P^{-1}$ and pre- and post-multiply (25) by Q then (25) becomes

$$P^{-1}(A_i+B_iK_i)^T P(A_i+B_iK_i)P^{-1}-P^{-1}+P^{-1}C^TCP^{-1}\le 0,\ \ i=1,2,\cdots,r$$
$$\frac{1}{4}P^{-1}\{(A_i+B_iK_j)+(A_j+B_jK_i)\}^T P\{(A_i+B_iK_j)+(A_j+B_jK_i)\}P^{-1}$$
$$-P^{-1}+P^{-1}C^TCP^{-1}\le 0,\ \ i<j\le r. \quad (27)$$

By Schur complements, (27) becomes

$$\begin{bmatrix} -P^{-1} & \{(A_i+B_iK_i)P^{-1}\}^T \\ (A_i+B_iK_i)P^{-1} & P^{-1}C^TCP^{-1} \end{bmatrix}\le 0,\ \ i=1,2,\cdots,r$$

$$\begin{bmatrix} -P^{-1} & \frac{1}{2}P^{-1}\{(A_i+B_iK_j)+(A_j+B_jK_i)\}^T \\ \frac{1}{2}\{(A_i+B_iK_j)+(A_j+B_jK_i)\}P^{-1} & P^{-1}C^TCP^{-1}-P^{-1} \end{bmatrix}\le 0,\ \ i<j\le r \quad (28)$$

Therefore, (28) is equivalent to the second and third LMI in (26).

(End of Proof)

IV. Simulation

The Proposed design method is verified by designing a controller for an inverted pendulum on a cart which is adopted from Wang *et al.*[2]. The equation of motion for the pendulum are

$$\delta x_1(t)=x_2(t)$$
$$\delta x_2(t)=\frac{g\sin(x_1(t))-amlx_2^2(t)\sin(2x_1(t))/2-a\cos(x_1(t))u(t)}{4l/3-aml\cos^2(x_1(t))} \quad (29)$$

where $x_1(t)$ is the angle(in radian) of the pendulum

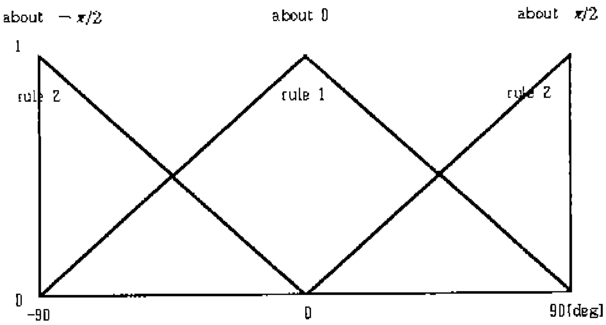

Fig. 1. Membership functions of "about 0" and "about $\pm\frac{\pi}{2}$".

from the vertical, $x_2(t)$ is the angular velocity, and $u(t)$ is the control force(in Newton) applied to the cart. The other parameters are as follows:

g : the gravity constant($9.8m/s^2$),
m : mass of the pendulum($2.0Kg$),
M: mass of the cart($8.0Kg$),
$2l$: length of the pendulum($1.0m$),

$$a=\frac{1}{m+M}$$

The T-S fuzzy model in Wang *et al.*[2] is adopted in this paper. It is composed of two plant rules.

Plant rule 1 : if x_1 is about 0 then $\delta x=A_1x+B_1u$

Plant rule 2 : if x_1 is about $\pm\frac{\pi}{2}\left(|x_1|<\frac{\pi}{2}\right)$ then $\delta x=A_2x+B_2u$ (30)

where

$$A_1=\begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3-aml} & 0 \end{bmatrix},\ B_1=\begin{bmatrix} 0 \\ -\frac{a}{4l/3-aml} \end{bmatrix}$$
$$A_2=\begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi(4l/3-aml\beta^2)} & 0 \end{bmatrix},\ B_2=\begin{bmatrix} 0 \\ -\frac{a\beta}{4l/3-aml\beta^2} \end{bmatrix}$$

and $\beta=\cos(88°)$. Refer to Wang *et al.*[2] for detailed descrip-tion. Membership functions of "about 0" and "about $\pm\frac{\pi}{2}$" are shown in Fig. 1.

A T-S controller has the following structure accor-ding to PDC framework.

Control rule 1 : if x_1 is about 0 then $u=K_1x$

Control rule 2 : if x_1 is about $\pm\frac{\pi}{2}\left(|x_1|<\frac{\pi}{2}\right)$ then $u=K_2x$ (31)

and the resulting output of the controller is

$$u=\omega_1K_1x+\omega_2K_2x \quad (32)$$

since $\omega_1+\omega_2=1$ from Fig 1. Here, (1 and (2 are membership grades of antecedent parts of control rules 1 and 2 respectively.

As solving LMIs in (21), we can obtain the solution as follows

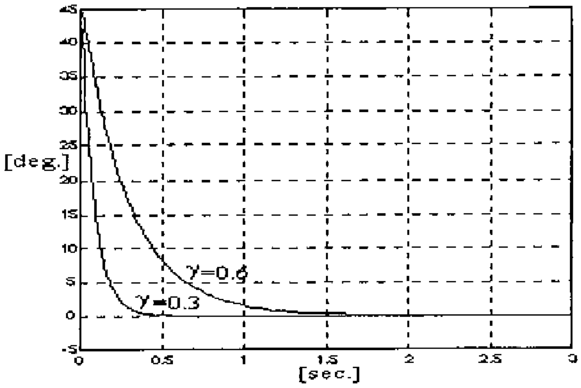

Fig. 2. The resulting output responses.

$$Q=\begin{bmatrix} 9.825 & -9.805\times10^{1} \\ -9.805\times10^{1} & 1.678\times10^{3} \end{bmatrix}_{\gamma=0.3}$$
$$K_1=\begin{bmatrix}5.776\times10^{3} & 5.377\times10^{2}\end{bmatrix}_{\gamma=0.3} K_2=\begin{bmatrix}3.165\times10^{4} & 2.682\times10^{3}\end{bmatrix}_{\gamma=0.3}$$
$$Q=\begin{bmatrix} 5.322 & -1.652\times10^{1} \\ -1.652\times10^{1} & 6.366\times10^{1} \end{bmatrix}_{\gamma=0.6}$$
$$K_1=\begin{bmatrix}6.648\times10^{2} & 1.875\times10^{2}\end{bmatrix}_{\gamma=0.6} K_2=\begin{bmatrix}3.041\times10^{3} & 8.526\times10^{2}\end{bmatrix}_{\gamma=0.6} \tag{33}$$

Since all of the eigen-values of matrix Q in accordance with given γ are positive, Q and P are obviously symmetric positive definite matrices. Therefore, the PDC-type T-S fuzzy controller under the output energy bound is globally asymptotically stable. It verifies the stability of the proposed design method.

The responses of $y(t)$ according to γ for given initial condition($x_1=45°$ and $x_2=0$) are shown in Fig. 2.

From the simulation, we can see that the designed controller satisfies the output energy constraint. That is,

$$\int_0^\infty y^T(t)y(t)dt = 2.86\times10^{-2} < \gamma(=0.3)$$
$$= 9.86\times10^{-2} < \gamma(=0.6) \tag{34}$$

V. Concluding Remarks

A new design method for PDC-type T-S fuzzy controller under the output energy bound is proposed. The method uses LMI approach to find a common symmetric positive definite matrix P and state feedback gains numerically. The stability of the closed-loop system is represented as LMIs where the output energy of the closed-loop system is to be bounded within given maximum output energy γ. Therefore, the state-feedbacks gains guarantees globally asymptotic stability under the output energy constraint.

The validity of the proposed design method is verified by designing a PDC-type T-S fuzzy controller for an inverted pendulum with a cart.

References

[1] K. Tanaka and M. Sugeno, "Stability analysis and design of fuzzy control systems," *Fuzzy Sets and Systems*, vol. 45, pp. 135-156, 1992.

[2] H. O. Wang, K. Tanaka, and M. Griffin, "An analytical framework of fuzzy modeling and control of nonlinear systems : stability and design issues," *Proc. American Control Conference, Seattle*, Washington, pp. 2272-2276, 1995.

[3] J. Joh, R. Langari, and W. J. Chung, "A new design method for continuous takagi-sugeno fuzzy controller with pole placement constraints : an LMI approach," *Proc. 1997 IEEE Int. Conf. on Systems*, Man, and Cybernetics, Orlando, Florida, pp. 2969-2974, 1997.

[4] J. Joh, S. K. Hong, Y. Nam, and W. J. Chung, "On the systematic design of takagi-sugeno fuzzy control systems," *Proc. Int. Symp. on Engineering of Intelligent Systems/EIS'98*, Tenerife, Spain, pp. 113-119, 1998.

[5] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, "Linear matrix inequalities in system and control theory," *SIAM Studies in Applied Mathematics*, 1994.

[6] M. Chilali and P. Gahinet, "H_∞ design with pole place-ment constraints : an LMI approach," *IEEE Trans. Automatic Control*, vol. 41, no. 3, March, 1996.

[7] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, "LMI control toolbox : for use with MATLAB," *The MATH WORKS Inc.*, 1995.

[8] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Systems, Man, and Cybernetics*, vol. SMC-15, no. 1, pp. 116-132, 1985.

Kwang-Tae Kim

He was born in 1961, in Wonju Korea. He received the B.S., and the M.S. degrees in Electronic Engineering from Kyungpook National University, Korea, in 1984 and 1987, respectively. From 1987 to 1996, he was with the Agency for Defense Development. Since 1996, he has been with the Depatment of Electronic Engineering, Doowon Technical College, Korea, where he is now an Assistant Professor. His research interests include fuzzy control, adaptive control, and robotics.

Woo-Hyen Kwon

He was born in 1953, Korea. He received the B.S. degree in Electronic Engineering from the Sogang University, Korea, in 1977 and the M.S. and Ph. D. degrees in Electrical Engineering from the KAIST, Korea, in 1979 and 1993, respectively. Since 1979, he has been with the School of Electronic and Electrical Engineering, Kyungpook National University, Korea, where he is now a Professor. His research interests include motor control, power electronics, and factory automation.

Joong-Seon Joh

He was born in 1958, in Hong-seong, Korea. He received the B.S. degree in Mechanical Engineering from the Inha University, Korea, in 1981, the M.S. degree in Mechanical Design and Production Engineering from the Seoul National University, Korea, in 1983, and the Ph. D. degree in Mechanical Engineering from the Georgia Institute of Technology in 1991. From 1983 to 1986, he was with the central research center of Daewoo Heavy Industries. He was also with the Agency for Defense Development from 1991 to 1993. Since 1993, he has been with the Department of Control and Instrumentation Engineering, Changwon National University, Korea, where he is now an Associate Professor. His research interests include fuzzy control, neuro-fuzzy control, automatic control, and robotics.