# 함수의 정의역 변형에 의한 신호간의 거리 측정 방법

論 文
48A-3-23

# A Modified Domain Deformation Theory for Signal Classification

金 聖 洙
(Sung-Soo Kim)

**Abstract** - The metric defined on the domain deformation space better measures the similarity between bounded and continuous signals for the purpose of classification via the metric distances between signals. In this paper, a modified domain deformation theory is introduced for one-dimensional signal classification. A new metric defined on a modified domain deformation for measuring the distance between signals is employed. By introducing a newly defined metric space via the newly defined Integra-Normalizer, the assumption that domain deformation is applicable only to continuous signals is removed such that any kind of integrable signal can be classified. The metric on the modified domain deformation has an advantage over the $L^2$ metric as well as the previously introduced domain deformation does.

**Key Words** : Domain deformation, Similarity measurement, Signal classification, Homeomorphism.

## 1. Introduction

When we measure the similarity between the waveforms, we can use many different methods for each different area of application. In most cases, we use the metric in $L^2$ which measures the least square error (LSE) between two signals as a distance. A function is a mapping between two spaces called a domain and a range. The domain deformation method defines a metric in order to measure the distance between functions using the relation that a domain can be mapped to another domain so that the values of range of one function matches the values of another range as shown in Fig. 1. Simply, the domain deformation is a mapping from one domain to another through a homeomorphic relation.

The motivation to study the Domain Deformation Theory (DDT)[1] in the metric spaces involved in the compared functions is that the DDT performs better in measuring the shape similarity between two functions. There are some cases in which the measurement via the metric $L^2$ does not agree with an intuitive understanding of it. The DDT results in a more intuitive description of the relation between two signals than the traditional metrics in $L^2$.

正 會 員 : 韓國電子通信硏究院·工博

接受日子 : 1998年 12月 3日
最終完了 : 1999年 1月 28日

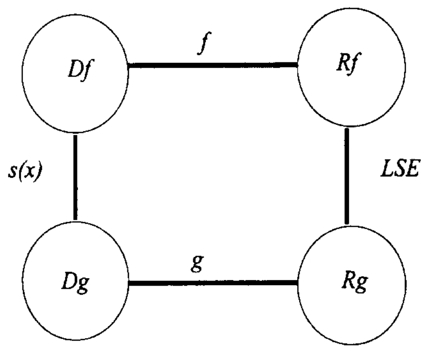


Fig. 1 Relation Between Spaces of Two Functions

Suppose that there are two 1-D signals whose shapes are to be compared in a metric space $X$, which is the space of real-valued functions of bounded variation. A case is studied and a metric is proposed for the method of shape similarity measurement between signals in [1,2] with a more intuitive understanding.

Let signals $f_1(t)$, $f_2(t)$ and $g(t)$ be the functions of bounded variation on the unit interval [0, 1] for purposes of comparison. The functions are the elements of $X$ which are shown in Fig. 2 such as:

$$g(t) = \begin{cases} 0.125, & \text{if } t \in [\alpha, \beta], \\ 0.0, & \text{otherwise} \end{cases}$$

$$f_1(t) = \begin{cases} 1.0, & \text{if } t \in [0.5, 0.51], \\ 0.0, & \text{otherwise} \end{cases}$$

$$f_2(t) = \begin{cases} 1.0, & \text{if } t \in [0.52, 0.53], \\ 0.0, & \text{otherwise} \end{cases}$$

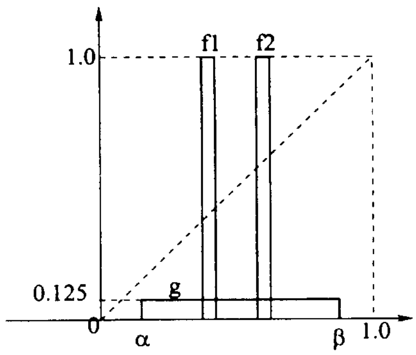


Fig. 2 Counter Example Showing Weakness of $L^2$ Metric

For example, let $\Xi(f_1(t), f_2(t)) \stackrel{\text{def}}{=} \int (f_1(t) - f_2(t))^2$ for $t \in [0, 1]$ and the functions $f_1$, $f_2$ in $X$. For the functions $f_1$, $f_2$, $g$, the metric distance in $L^2$-sense becomes $\Xi(f_1(t), g(t)) < \Xi(f_2(t), g(t))$ when $\alpha = 0$ and $\beta = 0.8$; $\Xi(f_1(t), g(t)) = 0.01859$ and $\Xi(f_1(t), f_2(t)) = 0.02$. This means that $f_1$ is closer to $g$ than to $f_2$ in the $L^2$ sense. This result does not agree with our intuitive understanding that $f_1$ is closer to $f_2$ than to $g$. However, when $\alpha = 0.1$ and $\beta \geq 0.91$ in Fig. 2, $\Xi(f_1(t), f_2(t))$ is less than $\Xi(g(t), f_2(t))$ which agrees with our intuitive understanding where $\Xi(f_1(t), g(t)) = 0.020150$ and $\Xi(f_1(t), f_2(t)) = 0.02$. This DDT is introduced in [2] so that it removes the defect of $L^2$ metric in respect to our intuitive understanding.

However, there is a constraint to the functions in the DDT. The compared functions must be continuous in order to hold the homeomorphism between their domains which yields a limit to the applicability of the DDT. The DDT employs the homeomorphism by removing the "redundant" $s$'s from $S$ the domain deformation space. In this research, the Modified Domain Deformation Theory (MDDT) is developed by removing the constraint and the "redundant" elements from the spaces involved in the DDT, so that the mapping becomes bijection. In the MDDT, deformation is applied to the domain of the operator defined as the Integra-Normalizer, not to the domain of function to be compared initially. The first advantage of the MDDT is the relaxation of the constraints, so that it becomes possible to measure distance between all integrable functions. The second advantage is that bijection between domain spaces can be obtained without excluding the "redundant." Thus, the homeomorphism in the domain deformation function is concisely defined. In addition to these advantages, a metric is introduced in the MDDT through which the distance measurement becomes more compact than the metric defined in [1].

## 2. Domain Deformation Theory (DDT)

The domain deformation theory (DDT) in [2] is defined based on the assumption that $X$ is the space of real-valued functions of bounded variation, defined in the unit interval $I$, satisfying the following:

1. At every point $t$ in the interval $[0, 1]$, the left- and right-hand limits exist.
2. $\forall t \in X$, $f(t) = f(t + 0^+)$ $or$ $f(t) = f(t + 0^-)$.
3. The values taken by $f$ at 0 and 1 are arbitrary.

For better understanding, the definition of the bounded variation is given below:

**Definition 1** (*Bounded Variation*)
A function in $I$ is said to be of bounded variation on $I$ if its total variation $Var(w)$ on $I$ is finite, where $Var(w) = sup \sum_{j=1}^{n} |w(t_j) - w(t_{j-1})|$, the supremum being taken over all partitions $0 = t_0 < t_1 < \cdots < t_n = 1$ of $I$, $n \in N$ is arbitrary.

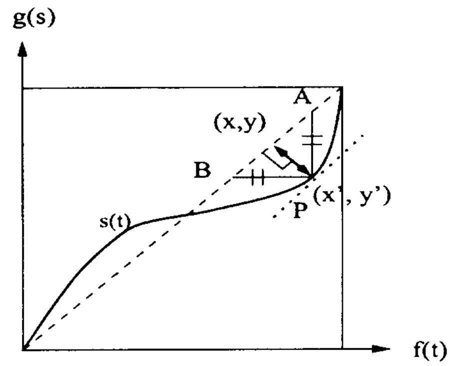


Fig. 3 Domain Deformation Shown in the Product Domain of $f_i$ and $g$

For the functions $f_1$, $f_2$, and $g$ in $X$, the function $s$ is defined as the domain deformation by $g(t) \stackrel{\text{def}}{=} f_1(s(t)) \stackrel{\text{def}}{=} (f_1 \circ s)(t)$, where $s(t)$ is an order-preserving homeomorphism of the unit interval $I$ onto itself. Fig. 3 describes pictorially the structure of deformation. The functions $g$ and $f_i$ are to be visualized normal to the plane of Fig. 3. The horizontal axis ( $t$

-axis) in Fig. 3 is the domain of $f_i$ and the vertical axis (the $x$-axis) is the domain of $g$. In order to present the domain deformation method, the spaces $S$ and $X$ are defined in [2].

**Definition 2** Let $S$ be the space of all order-preserving homeomorphisms of the unit interval $I$ to itself.

We define $S$ as the space of all order-preserving homeomorphism in the unit interval $I$ onto itself. The function $s \in S$ is a continuous, strictly increasing function in the product domain $I \times I$. The pair $(S, \circ)$ is a group and the space $S$ is convex as a set [1, 2].

**Definition 3** Let $X$ be the space of all real functions of bounded variations defined in the unit interval $I$. Let $f \in X$. A set of functions can be defined $X_f$ which is obtained from $f$ through order preserving, homeomorphic, domain deformation in $X$:

$X_f = \{ g{:}f = g \circ s, \text{ for } some\, s \in S \}$.

As is mentioned in [2], the mapping from $S$ to $X_f$ is onto, the mapping $s$ to $f \circ s^{-1}$, but not one-to-one in general. When $f(t)$ equals to 1, $X_f$ is the singleton $\{ f \}$ since $1 \circ s^{-1} = 1$ for all $s$ in $S$ as shown in [2]. This singleton occurs since the function $f$ is not a strictly increasing function. In the DDT, by removing the "redundant" $s$'s from $S$, except the identity domain deformation $s = i$, the mapping becomes one-to-one. The $f(t) = 1$ is a special case of constant value functions. In [2], for each of the corresponding constant subintervals in $f$ and $g$, by connecting left bottom and right upper corner of the kernel, the uniqueness of $s$ is kept. Then, $f$ in $X$ can be compared with $g$'s in $X_f$ only through the domain deformation $s$. An interesting question is whether the DDT can be used for measuring distance between two functions when the $s$ is not a strictly increasing function. A good example illustrating that the $s$ is not a strictly increasing function is when $f$ is an absolutely increasing function and $g$ has a constant magnitude subinterval in $I$. Obviously, the domain deformation function does not hold the homeomorphism. Another case in which the DDT does not hold homeomorphism is when $f$ in $X$ is a discontinuous function which is strictly monotonic except for the discontinuous point. Suppose that $f$ is a discontinuous function at a point $p$ in $I$ with $f(p+0^-)$ which is not equal to $f(p+0^+)$. Can the $s$ which is an order-preserving, homeomorphic, domain deformation for any possible function in $X$ be found? Only if $s$ is a straight line connecting $(0,0)$ and $(1,1)$ can $s$ be found. In this case, $s$ maps $f$ to itself, not to any others. There exists one-to-one and onto between $X_f$ and $S$ by mapping the $s$ and $f = g$ such that $X_f = \{ f \}$. There is no way to find any functions that belong to $X_f$ through this $s$, except $f$ itself. Conversely, if a function $g$ for the $f$ in $X$ is chosen first, there is no $s$ that holds the properties. Using the definition above, the distance between $f$ and the functions in $X_f$ can be measured through the domain deformation, $s$.

The question is whether the domain deformation $s$ with the properties of order-preserving homeomorphism for any function that is not an element of $X$ can be found. Since $X_f$ is a subset of $X$, there are some cases where the domain deformation function cannot be found. Thus, some modification of the theory is unavoidable.

## 3. Constraints of the DDT

In the Lemma 2.5, 2.6, and the Theorem 2.7, Corollary 2.8, and Theorem 2.10 in [1], the domain deformation is built to be a strictly increasing function with homeomorphism in $I \times I$. In this procedure in [1], all possible cases of creating singletons are eliminated, so that not all the functions in $X$, the space of real-valued function with bounded variation, can be compared to each other. Thus, when two functions in $X$ are compared, the domain deformation $s$ does not hold the homeomorphism.

For instance, when $f$ has a number of constant-valued subintervals and $g(t) = t$ is a strictly monotonic function, there are singletons in domain deformation. In this case, all the subintervals of $f$ are mapped to singletons so that the function $s$ is not an absolutely increasing function. If we remove the singletons in the deformation function, it is no longer homeomorphic. The general definition of homeomorphism is as follows:

**Definition 4** If $f$ is a $1-1$ function from a metric space $X_1$ onto a metric space $X_2$ and if $f$ and $f^{-1}$ are continuous, $f$ is a homeomorphism from $X_1$ onto $X_2$ where $X_1$ and $X_2$ are the subsets of $X$.

A simple example where homeomorphism of the domain deformation for the functions in $X$ does not hold is shown in Fig. 4. The example is an element of the $X$ space, a real-valued bounded function with bounded variation over the interval $I$ onto itself. If the example function is noted as $f$ and the function $g \in X$ $g(t) = t$, where $t \in I$ then the deformation function becomes the function as shown in Fig. 5. The jumping element of $s$ has a singleton in the domain $f$ which matches to several elements of the domain of the function $g(t)$. This clearly contradicts the homeomorphism of $s$, the domain deformation function. If the element of the domain of $f$ is excluded, then $s$ is no longer a continuous function and this exclusion will not satisfy the homeomorphism. Therefore, the Domain Deformation Theory as presented in [2], needs modification to satisfy the order-preserving homeomorphism of the deformation function.

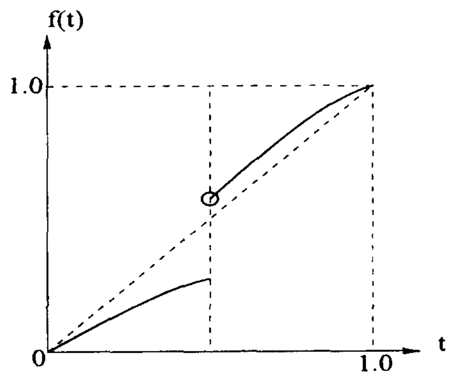

Fig. 4 A Counter Example of the DDT

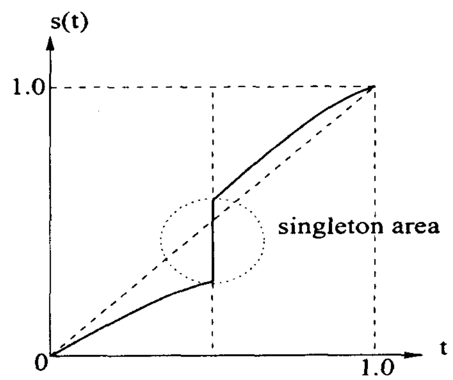

Fig. 5 Domain Deformation of the Counter Example

#### 4. Modified Domain Deformation Theory

This section defines the operator Integra-Normalizer (IN) and a transformed space through the IN, so that the order-preserving homeomorphism is satisfied. The procedures and theory involved in the IN are defined as the Modified Domain Deformation Theory (MDDT) with $S$ the same space presented in the previous section and with the integrable function defined in [4].

**Theorem 1** [Integrable Function]
A function is integrable if and only if it can be expanded into a series of step functions. Moreover, if $f \simeq f_1 + f_2 + \cdots$, where the $f_n$ are step functions and $\simeq$ means an approximation, then $\int f = \int f_1 + \int f_2 + \cdots$.

From Theorem 1, a slightly modified function is defined in Definition 5.

**Definition 5** [Approximation of Function into a Positive-Valued Function] Suppose that there are a number of intervals on which values of $f, g$ are zero. Then, without loss of generality, a value $\varepsilon$ can be defined as $\varepsilon = \frac{\delta}{N}$ where $\delta$ = the infimum of the $f_n$ at which non-zero value begins, and $N \in R$ is a large enough value. This creates a slightly increasing valued function over all the intervals whose magnitudes are all zeros, except the origin of domain. Therefore, if $\varepsilon$ is added to the values of zero in the range, the functions that have all positive values except the origin are obtained.

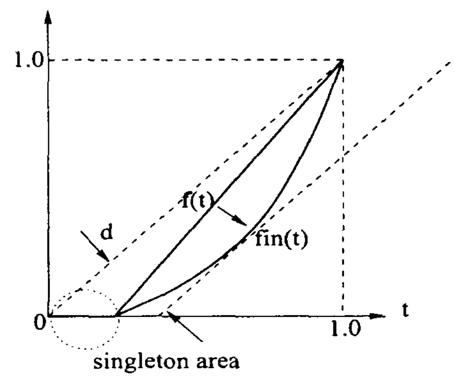

Fig. 6 Singleton in the Domain Deformation Function Obtained from the IN

Mathematically, the above definition can be described as:

$$h(t) = \begin{cases} f(t), & \text{if } f(t) \neq 0, \\ t * \frac{\varepsilon}{N}, & \text{if } f(t) = 0 \end{cases}$$

where $N$ is large enough, $\frac{\varepsilon}{N} * t$ is smaller than $\delta$ for all $t \in I$. Thus, if the function $h \in X$ becomes a positive integrable function except at the $t = 0.0$ where $t \in I$, then using the proposition, the $n$th Degree

Integra-Normalizer is defined. Without employing Definition 5, the $s$ domain deformation function can not always be obtained as shown in the following Fig. 6.

**Definition 6** [The $n$th Degree Integra-Normalizer]
With $f$ as an integrable function over the unit interval $I$, an increasing function can be generated by an $n$-times integration over $I$ with $0 \le f(t) \le 1$. Let $f$ and $g$ be integrable functions with variables $t$ in the unit interval $I$. Let

$$\Phi^n \underline{\underline{\text{def}}} \ \overbrace{\int \cdots \int}^{n}, \quad \Gamma^n \underline{\underline{\text{def}}} \ \frac{\Phi^n}{Max(\Phi^n)}$$

so that, for $n=k$, $f^k=\Gamma^k(f)$ and $f^k=\Gamma^k(g)$ become continuous strictly increasing functions over the $I$ with $f^k(1)=1$. Then, there exists a unique domain deformation function that is an absolutely increasing function as the $I \times I$ mapping connecting the origin to $(1,1)$.

**Theorem 2**
Let $H$ be the space of all real integrable positive functions except at zero defined in the unit interval $I$. Then the functions $f^k, g^k \in H$ obtained through the kth Degree Integra-Normalizer are absolutely increasing functions, and there exists a unique solution $s$ which satisfies the following: $h^k = f^k \circ s$, $s \in S$ where $S$ is the space of all order-preserving homeomorphisms of the unit interval $I \times I$.

Proof:
Let the functions $f^k, g^k \in H$ where $H$ is the space of all real integrable positive functions except at zero as defined in the unit interval $I$. Then, without loss of generality, the following can be obtained:

$$\int_0^t f(t)\,dt < \int_0^{t+\varepsilon} f(t)\,dt,$$
$$\int_0^t g(t)\,dt < \int_0^{t+\varepsilon} g(t)\,dt$$

for $t \in I$ and a small real value $\varepsilon > 0$. Then, according to the Theorem in [2], if $f$ and $g$ are strictly increasing (decreasing) functions, the solution of $g = f \circ s$ has a unique solution in $S$ and the unique solution $s \in S$ can be obtained. Thus, by induction, it is true for all $k \in N$. From Theorem 2, we can obtain the following corollary:

**Corollary 1** [Removed Constraint: Continuity]
All of the functions that are continuous, discontinuous, and the combined signal can measure their relative distance in the metric space with $N=2$, the $2nd$ degree Integra-Normalizer, if they are integrable. [Proof is trivial.]

Therefore, two issues are resolved in this section. From Definition 6, the constraints contained in the DDT are removed. An integrable function is approximated in such a way that the function becomes all positive except the origin so that the domain deformation function $s$ becomes a strictly increasing function. The domain deformation function $s \in S$ is the order-preserved homeomorphic over $I$ to itself. The output of an integrable function through the Integra-Normalizer is a continuous function. If $n=2$, we can handle most cases of continuous, discontinuous, or combinations of continuous and discontinuous functions in real applications. In the following section, the choice of a metric for the proper measurement of closeness (similarity) is discussed.

## 5. A Metric Measuring Similarity Between Signals

Choosing a proper metric for measuring distance in information space is critical. In [1, 2], the supremum metric for measuring distance is introduced, while various metric spaces are defined in many applications in order to measure distance between the several pieces of information. Finding the most appropriate metric for the problem under consideration is very important. The rule of thumb in this procedure is to define a metric that carries out the measurement so that the mathematical distance matches with our intuitive understanding. The distance between $f_i$ and $g$ is measured by the *supremum* metric defined as,

$$*d(f, g) = *d(f, f \circ s) = sup_{t \in I} |t - s(t)|$$

However, there is a possibility that the supremum metric produces the same metric distance for two different domain deformations from two different signals with respect to a commonly compared function $f(t)$ as shown in Fig. 7. The two different domain deformation functions $S$ and $S'$ classifies two different signals as the same when $d = d'$. In this research, in order to remove this drawback, a new metric is defined as a tool for measuring distance between two signals by including the effect of the least square error method.

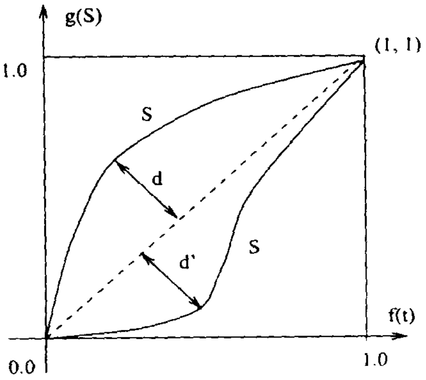


Fig. 7 Two different deformations with the same distance

**Definition 7** Let $S$ be the space of all order-preserving homeomorphisms of the unit interval $I$ to $I$. Let $f, g \in X$, the space of integrable real-valued functions. Then a metric $*d(f, g)$ can be defined as:

$*d(f, g) = *d(f, f \circ x) = \alpha sup_{t \in I}|t - s(t)| + \beta(1 - e^{-LMS(f, g)})$

where $\alpha + \beta = 1.0$ and $\alpha, \beta \geq 0.0$.

{Proof}:

1) (Positive) follows from $\alpha, \beta \geq 0$, $sup|t - s(t)| \geq 0$, and $(1 - e^{-LMS(f,g)}) \geq 0$, thus $*d(f,g)$ is real-valued, finite and nonnegative.

2) (Identity) If $\alpha = 0$, $\beta = 1$ from the condition $\alpha + \beta = 1$, such that $(1 - e^{-LMS(f,g)}) = 0$. From this, $e^{LMS(f,g)} = 1$ is obtained, so that $LMS(f, g) = 0$ which yields $f = g$.
If $\alpha \neq 0$, then $sup|t - s(t)| = 0$, so that $t = s(t)$, which implies that $f = g$. The second term $\beta(1 - e^{-LMS(f,g)})$ becomes 0 if $\beta = 0$. If $\beta \neq 0$, $f = g$. Conversely, if $f = g$, $\alpha sup_{t \in I}|t - s(t)| = 0$, $\beta(1 - e^{-LMS(f,g)}) = 0$. Therefore, $*d(f, g) = 0 \Rightarrow f = g$.

3) (Symmetry) From $s^{-1}(t)$ is a reflection of $s$ around the diagonal of $I \times I$ and $LMS(f, g) = LMS(g, f)$. $*d(f, g) = \alpha sup_{t \in I}|t - s(t)| + \beta(1 - e^{-LMS(f,g)}) = \alpha sup_{t \in I}|t - s^{-1}(t)| = + \beta(1 - e^{-LMS(g,f)}) = *d(g, f)$.

4) (Triangle Inequality)
For $f, g, h \in X$, $*d(f, g) + *d(g, h) = \alpha sup_{t \in I}|t - s(t)| + \beta(1 - e^{-LMS(f,g)}) + \alpha sup_{t \in I}|t - s'(t)| + \beta(1 - e^{-LMS(g,h)})$ where $g = f \circ s$, $h = f \circ s'$, $\alpha + \beta = 1$. Then by the following relations, we can obtain the relation, so that $*d(f, g) + *d(g, h) \leq *d(f, h)$.

$$sup|t - s(t)| + sup|t - s'(t)| = sup\{|t - s(t)| + |t - s'(t)|\} \geq sup\{|t - s(t) + t - s'(t)|\} = sup\{|s(t) - s'(t)|\}$$

Suppose that

$(1 - e^{-LMS(f,h)}) \leq (1 - e^{-LMS(f,g)}) + (1 - e^{-LMS(f,g)})$.

Then, if we apply both sides log, we get

$$1 + LMS(f, h) \leq (1 + LMS(f, g)) + (1 + LMS(g, h)) \Rightarrow LMS(f, h) \leq LMS(f, g) + LMS(g, h)$$

Thus, the assumption holds. Therefore, the statement holds.

## 6. Experimental Results: 1-D Signal Metric Distance Using the Modified Domain Deformation

A supremum metric measures the closeness between signals as shown in Fig. 3. Here $f$ and $g$ are two functions obtained using the defined linear operator $\Gamma$ onto the signals to be compared. The $s$ is a function that maps the domain of $f$ to the domain of $g$, so that $s$ becomes a bijection between $f$ and $g$. The parameter $t$ is a variable defined in the domain of $f$, and the parameter $s$ is a variable defined in the domain of $g$. In Fig. 3, the supremum metric is the length between $(x, y)$ and $(x', y')$.

As an example:

$$f_1 \stackrel{\text{def}}{=} u(t - \tau) = \begin{cases} 1.0, \text{if } t \in [\tau, \tau + 0.1], \\ 0.0, \textit{otherwise}. \end{cases}$$

Fig. 8 shows the function when $\tau = 0.4$, such as $f_1 = u(t - 0.4)$. Fig. 9 shows a signal with an impulse at $t = 0.6$ such that:

$$f_2 \stackrel{\text{def}}{=} u(t - 0.4) + 0.5\delta(t - 0.6)$$

where $\delta$ is a unit impulse function. Fig. 10 is the function $f_3$ defined as:

$$f_3 \stackrel{\text{def}}{=} \begin{cases} 1.0, \text{if } t \in [0.8, 0.9] \\ \sin(\pi t), \text{if } t \in [0.3, 0.5] 0.0, \textit{ elsewhere}. \end{cases}$$

Fig. 11 and 12 show the results obtained from applying the previously defined linear operator $\Gamma^2$ onto $f_2$ and $f_3$ respectively. The domain deformation functions for $f_2$ and $f_3$ are shown in Fig. 14 and 15. The function used in the domain of Fig. 13 is the function $u(t - 0.8)$ while the functions in Fig. 13, 14, and 15 are used in the range of the domain deformation function as shown in Table 1. Fig. 13 shows the $s$ functions for several different $\Gamma^2$'s for $f_1$ where $\tau$ varies from 0 to 0.8. In Fig. 13 the function corresponding to $\tau = 0.8$ is labeled $a$. The function corresponding to $\tau = 0.7$ is labeled $b$. In a similar manner, $i$ in Fig. 13 corresponds to the case when $\tau=0.0$.

As $\tau$ in Fig. 13 becomes closer to 0.0, the domain deformation function becomes closer to $i$ in Fig. 13. It is obvious that the functions obtained through the double integral linear operator $\Gamma^2$ are all strictly increasing functions as are the domain deformation functions. As Table 1 and Fig. 13 show, the signals defined by varying $\tau$ in $f_1$ can be classified using the metric defined above, while the LSE cannot be used for classifying the signals since they yield the same metric distance for the different $\tau$'s of $f_1$.

The similarity between signals is measured by *Similarity*$= 1.0 - *d$ where $d$ is a metric distance defined by the MDDT and LSE.

**Table 1 Similarity Measurement**

| Test Signal 1 | Test Signal 2 | Similarity (MDDT) | Similarity (LSE) |
|---|---|---|---|
| $\tau$ =0.8 | $\tau$ =0.0 | 0.44 | 0.1414 |
| $\tau$ =0.8 | $\tau$ =0.1 | 0.51 | 0.1414 |
| $\tau$ =0.8 | $\tau$ =0.2 | 0.58 | 0.1414 |
| $\tau$ =0.8 | $\tau$ =0.3 | 0.65 | 0.1414 |
| $\tau$ =0.8 | $\tau$ =0.4 | 0.72 | 0.1414 |
| $\tau$ =0.8 | $\tau$ =0.5 | 0.79 | 0.1414 |
| $\tau$ =0.8 | $\tau$ =0.6 | 0.86 | 0.1414 |
| $\tau$ =0.8 | $\tau$ =0.7 | 0.93 | 0.1414 |
| $\tau$ =0.8 | $\tau$ =0.8 | 1.0 | 0.0 |
| $\tau$ =0.8 | with impulse | 0.93 | 1.0 |
| $\tau$ =0.8 | with continuos signal | 0.52 | 0.0459 |

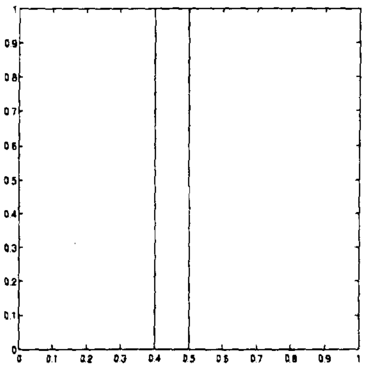

Fig. 8 A pulse function: f1

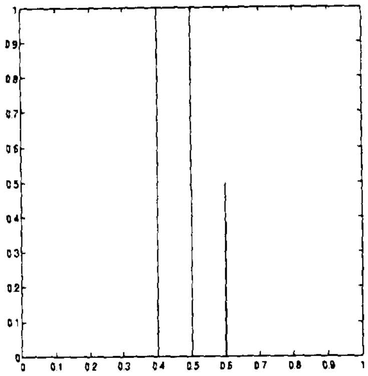

Fig. 9 A pulse function with impulse

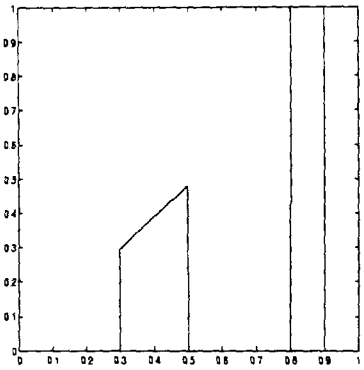

Fig 10: Pulse function with sinusoidal

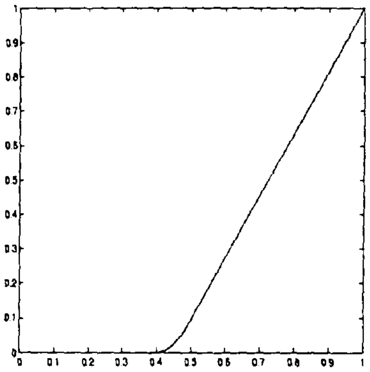

Fig. 11 $\Gamma^2(f_2)$ of $f_2$

### 7. Conclusion

This paper presents a new method of one-dimensional signal classification using the metric defined on a modified domain deformation theory. The domain deformation theory is modified by employing the newly defined Integra-Normalizer.

Thus the constraint in [1, 2] is removed by employing the double linear integration operator while the advantage of the domain deformation theory is kept. The advantage of this new method is that we can classify not only the bounded continuous signal over the unit interval $I$ but also the discontinuous signals over the unit interval $I$. Therefore any one-dimensional signal, whether continuous or discontinuous, can be classified, more intuitively than $L^2$ metric, using the modified domain deformation theory and the metric defined on it.

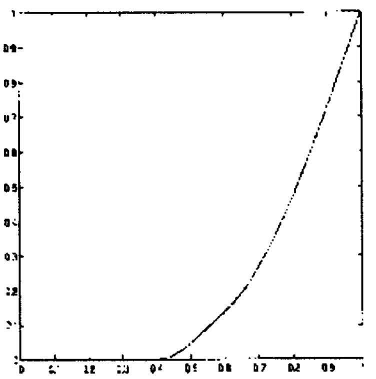

Fig. 12 $\Gamma^{(2)}(f_3)$ of $f_3$

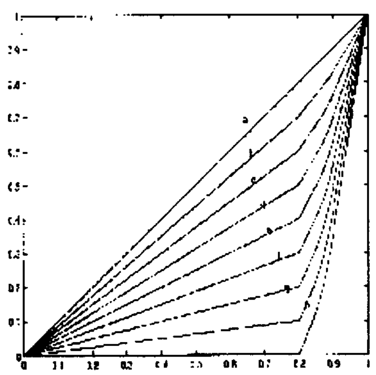

Fig. 13 modified Domain Deformation of $f_1$

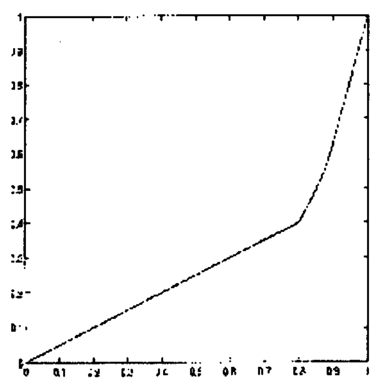

Fig. 14 Modified Domain Deformation of $f_2$

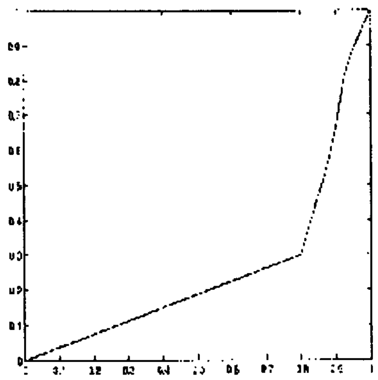

Fig. 15 Modified Domain Deformation of $f_3$

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## 저 자 소 개


**김 성 수(金 聖 洙)**

1959년 5월 17일생. 1983년 충북대학교 전기공학과 졸업. 1989년 알칸소주립대 전기공학과 졸업(석사). 1997년 중앙플로리다 주립대학 졸업(공박). 현재 전자통신연구원 Post. Doc.