초기하분포 소프트웨어 신뢰성 성장 모델 : 일반화, 추정과 예측

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요 약

최근에 개발되어 성공적으로 적용되고 있는 초기하분포 소프트웨어 신뢰성 성장 모델은 이 모델에서 중요한 역할을 하는 반응계수(sensitivity factor)를 추정 대상인 모수로 가정하고 있다. 본 논문은 먼저 디버깅과정의 무작위성을 반영하기 위해 반응계수를 이항분포를 하는 확률변수로 가정하여 초기하분포 신뢰성 성장 모델을 일반화한다. 이러한 일반화는 초기하분포 소프트웨어 신뢰성 성장 모델의 통계적 특성을 쉽게 파악할 수 있게 한다. 특히 일반화된 모델의 모수를 최소자승법으로 추정하면 기존 모델에 최소자승법을 적용한 것과 같은 결과를 얻을 수 있음을 보이고, 더불어 최우추정치를 최소자승법으로 구하는 방법과 예측방법도 제시한다.

Hyper-Geometric Distribution Software Reliability Growth Model : Generalization, Estimation and Prediction

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ABSTRACT

The hyper-geometric distribution software reliability growth model (HGDM) was recently developed and successfully applied to real data sets. The HGDM considers the sensitivity factor as a parameter to be estimated. In order to reflect the random behavior of the test-and-debug process, this paper generalizes the HGDM by assuming that the sensitivity factor is a binomial random variable. Such a generalization enables us to easily understand the statistical characteristics of the HGDM. It is shown that the least squares method produces the identical results for both the HGDM and the generalized HGDM. Methods for computing the maximum likelihood estimates and predicting the future outcomes are also presented.

1. Introduction

In recent years software systems have been widely applied to many complex and critical systems. Since failure of a software system may result in serious damage, software systems are required to be very reliable. Therefore software reliability has become one of major issues in the software system development. In order to quantitatively assess the reliability of a software system during the testing and operational phases, many software reliability growth models (SRGMs) have been proposed in the literature. See the review papers such as Goel[1], Ramamoorthy and

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논문접수 : 1998년 6월 1일, 심사완료 : 1999년 1월 16일

Bastani[12] and Shanthikumar[13]. The SRGMs are usually used to estimate the number of remaining faults, software reliability and other software quality assessment measures. Some of currently available SRGMs enable us to predict probabilistically the time to next occurrence of failure in the operational phase. Another class of SRGMs let us to estimate the number of software faults still remaining after the debugging process. The HGDM advocated by Tohma et al.[14] belongs to the latter class of SRGMs. A series of studies on the HGDM has been made recently by Hou, Kuo and Chang[2], [3], Jacoby and Tohma [6], Minohara and Tohma [9] and Tohma et al.[15]. Hou, Kuo and Chang[4], [5] developed optimal software release policies based on the HGDM.

This paper first generalizes the HGDM to make it more realistic. The generalization is concernerd with the sensitivity factor which is the key factor of the HGDM. Then parameter estimation and prediction problems are considered. Section 2 briefly reviews the basic concept and formulation of the HGDM. Assumptions on the sensitivity factor are generalized in Section 3 to reflect the random behavior of the test-and-debug process. Then the generalized HGDM is derived in Section 4. Section 5 considers the parameter estimation problem for the HGDM and the generalized HGDM. The least squares and maximum likelihood methods are dealt with. It is shown that the least squares estimates for both models are identical and that the maximum likelihood estimates can be computed by the least squares method. Section 6 suggests a method for predicting the number of faults newly discovered by future test operations. The method is based on the expected value of the number of newly discovered faults on condition that the cumulative number of faults is known.

2. Review of the HGDM

In this section we briefly review the HGDM. At the beginning of the test-and-debug process a software system is assumed to have m initial faults. Test operations performed in a day or a week may be called a test instance. Test instances are denoted by t_i, $i=1,2,\cdots$ in accordance with the order of applying them. The sensitivity factor, w_i, represents the number of faults discovered by the application of test instance t_i. Some of the faults detected by t_i may have been detected previously by the application of test instances t_j, $j=1,\cdots,i-1$. The number of faults newly discovered by t_i is not necessarily equal to w_i. That is, each detected fault can be classified into the two categories, newly discovered faults and rediscovered faults. Let N_i denote the number of faults newly discovered by t_i and $C_i=\sum_{j=1}^{i} N_j$. The following assumptions are made on the HGDM.

- No new faults are introduced into the software system during the debugging process.
- Sensitivity factor w_i, the number of faults discovered by t_i, is the faults taken randomly out of m initial faults.
- Sensitivity factor w_i is represented as a function of m and the progress in test-and-debugging p_i, i.e., $w_i = m\,p_i$.

p_i is usually referred to as the learning factor. The probability that x_i faults are newly discovered by t_i on condition that C_{i-1} faults has been discovered up to t_{i-1} is then formulated as

$$P(N_i=x_i \mid C_{i-1}) = \frac{\binom{m-C_{i-1}}{x_i}\binom{C_{i-1}}{w_i-x_i}}{\binom{m}{w_i}}, \qquad (1)$$

where $\max(0,\, w_i - C_{i-1}) \le x_i \le \min(w_i,\, m - C_{i-1})$ for $i=1,2,\cdots$, $C_0=0$ and $x_0=0$. Thus the conditional expected value of N_i is

$$E(N_i \mid C_{i-1}) = (m - C_{i-1})\,p_i.$$

The expected value of C_i was obtained by Jacoby and Tohma[7] as

$$E(C_i) = m\left[1 - \prod_{j=1}^{i}(1-p_j)\right]. \tag{2}$$

The sensitivity factor is the key factor in the HGDM. Various functions for w_i have been devised and successfully applied to real data sets. Functional forms of the sensitivity factor are presented in Table 1 of Jacoby and Tohma[7] and Minohara and Tohma[9]. Recently Hou, Kou and Chang[2] introduced two types of sensitivity factor based on the learning curve. They are respectively referred to as the exponential sensitivity factor and the logistic sensitivity factor. Next section discusses further on the sensitivity factor.

3. Assumptions on Sensitivity Factor

As mentioned in the previous section, the HGDM assumes that sensitivity factor w_i is the w_i faults randomly chosen from m initial faults. Denoting by F_i the set of faults detected by t_i, this assumption can be divided into two statements below.

- The size of F_i is w_i, an unknown constant.
- The elements of F_i are randomly chosen from m initial faults.

We now argue that the first statement does not reflect enough the testing process. Test items' for a test instance are usually selected randomly from the input domain. Different sets of test items for a test instance would discover different number of faults. It is therefore more realistic to postulate that the number of faults detected by each test instance is a random variable. Next consider the learning factor p_i, which represents the degree of test workers' skill at the application of test instance t_i. Assuming that all m faults are detectable with equal probability, the learning factor can be practically regarded as the probability that test instance t_i discovers a fault. The sensitivity factor is thus assumed to be a binomial random variable with parameters m and p_i, i.e., for $w_i = 0, 1, \cdots, m$

$$P(W_i = w_i) = \binom{m}{w_i} p_i^{w_i}(1-p_i)^{m-w_i}. \tag{3}$$

4. A Generalized HGDM

This section generalizes the HGDM based on the following assumptions on the sensitivity factor. Other assumptions remain unchanged.

- Sensitivity factor W_i is distributed as Expression (3).
- Given that C_{i-1} faults have been detected up to test instance t_{i-1} and w_i faults are detected by test instance t_i, the number of faults newly discovered by t_i is distributed as Expression (1).

We first derive the conditional distribution of N_i given that C_{i-1} faults have been discovered up to test instance t_{i-1}. Multiplying Expressions (1) and (3),

$$\begin{aligned}
&P(N_i = x_i, W_i = w_i | C_{i-1}) \\
&\quad = \binom{m-C_{i-1}}{x_i}\binom{C_{i-1}}{w_i - x_i} p_i^{w_i}(1-p_i)^{m-w_i} \\
&\quad = \binom{m-C_{i-1}}{x_i} p_i^{x_i}(1-p_i)^{m-C_{i-1}-x_i} \cdot \\
&\qquad \binom{C_{i-1}}{w_i - x_i} p_i^{w_i - x_i}(1-p_i)^{C_{i-1}-w_i-x_i}.
\end{aligned}$$

Therefore

$$\begin{aligned}
&P(N_i = x_i | C_{i-1}) \\
&\quad = \sum_{w_i} \Pr(N_i = x_i, W_i = w_i | C_{i-1}) \\
&\quad = \binom{m-C_{i-1}}{x_i} p_i^{x_i}(1-p_i)^{m-C_{i-1}-x_i}.
\end{aligned} \tag{4}$$

This is a binomial distribution with parameters $m-C_{i-1}$ and p_i. The joint distribution of N_i, $i=1,2,\cdots,n$ is then obtained as

$$\begin{aligned}
&P(N_i = x_i, i=1,2,\cdots,n) \\
&\quad = \prod_{i=1}^{n} P(N_i = x_i | N_j = x_j, j=1,2,\cdots,i-1) \\
&\quad = \prod_{i=1}^{n} P(N_i = x_i | C_{i-1})
\end{aligned}$$

$$= \prod_{i=1}^{n} \binom{m-\sum_{j=1}^{i-1}x_j}{x_i} p_i^{x_i}(1-p_i)^{m-\sum_{j=1}^{i}x_j} \tag{5}$$

$$= \binom{m}{x_1, \cdots, x_n} \prod_{i=1}^{n} \left[p_i \prod_{j=1}^{i-1}(1-p_j) \right]^{x_i} \cdot \left[\prod_{i=1}^{n}(1-p_i) \right]^{m-\sum_{i=1}^{n}x_i} \tag{6}$$

where

$$\binom{m}{x_1, \ldots, x_n} = \frac{m!}{x_1! \cdots x_n!(m-\sum_{i=1}^{n}x_i)!}.$$

Since $\sum_{i=1}^{n}\left[p_i \prod_{j=1}^{i-1}(1-p_j)\right] = 1-\prod_{i=1}^{n}(1-p_i)$, the joint distribution of N_i, $i=1,2,\cdots,n$ is a multinomial distribution with parameters m and $p_i \prod_{j=1}^{i-1}(1-p_j)$, $i=1, \cdots, n$. Consequently C_i is binomially distributed, i.e.,

$$P(C_i = c_i) = \binom{m}{c_i}\left[1-\prod_{j=1}^{i}(1-p_j)\right]^{c_i}\left[\prod_{j=1}^{i}(1-p_j)\right]^{m-c_i}. \tag{7}$$

5. Parameter Estimation for the Generalized HGDM

Let c_i and x_i be the observed values of C_i and N_i. Suppose that the software system is tested up to test instance t_n. In order to estimate the current number of residual faults and to predict the number of residual faults after applying t_{n+d} for $d \geq 1$, we first need to estimate the parameters in the model. Due to the mathematical difficulty of the maximum likelihood method, the least squares method has been used for the HGDM. Tohma et al.[14] obtained the least squares estimates by minimizing

$$\sum_{i=1}^{n}[c_i - E(C_i)]^2. \tag{8}$$

This criterion was also employed in Hou, Kuo and Chang[2], [3] and Jacoby and Tohma[7]. However, Tohma et al.[15] minimized

$$\sum_{i=1}^{n}[x_i - E(N_i \mid C_{i-1})]^2. \tag{9}$$

The minimization of Expression (9) is equivalent to the minimization of

$$\sum_{i=1}^{n}[c_i - E(C_i \mid C_{i-1})]^2, \tag{10}$$

since $E(C_i \mid C_{i-1}) = C_{i-1} + E(N_i \mid C_{i-1})$ and $C_i = C_{i-1} + N_i$. We should note that because of sequential application of test instances, C_{i-1} has been already realized and observed at the time when t_i is applied. The distribution of N_i or C_i thus depends on C_{i-1}. Therefore, it seems that the minimization of Expression (9) or (10) is more appropriate than the minimization of Expression (8). The above least squares criteria assume that the variabilities of C_i's or N_i's are homogenous. But this assumption does not hold for the HGDM. For example, the variance of N_i is obtained as

$$Var(N_i \mid C_{i-1}) = \frac{(m-C_{i-1})C_{i-1}p_i(1-p_i)}{m-1}.$$

Clearly $Var(N_i \mid C_{i-1})$ is not constant for all i. In the circumstances the weighted least squares method is generally known to be adequate. Park et al.[11] thus suggested that the estimates be computed by minimizing

$$\sum_{i=1}^{n}\frac{[x_i - E(N_i \mid C_{i-1})]^2}{Var(N_i \mid C_{i-1})}.$$

Next we consider the problem of estimating parameters of the generalized HGDM. It is not difficult to obtain from Expressions (4), (6) and (7) that

$$E(N_i \mid C_{i-1}) = (m - C_{i-1})p_i,$$

$$E(N_i) = mp_i \prod_{j=1}^{i-1}(1-p_j)$$

and

$$E(C_i) = m\left[1-\prod_{j=1}^{i}(1-p_j)\right].$$

These expected values are identical to those for the HGDM. If we estimate parameters by the least squares method, the estimation and prediction results for the generalized HGDM are the same with the corresponding results for the HGDM. This implies that the generalized HGDM performs at least as well as the HGDM.

The previous studies on the HGDM employed the least squares method mainly due to the mathematical difficulty of the maximum likelihood method. However, sometimes the maximum likelihood estimates can be computed by the least squares method. This approach is illustrated by means of the generalized HGDM. The log likelihood function is obtained from Expression (5) as

$$L(m, p)=\sum_{i=1}^{n}[\ln\Gamma(m-C_{i-1}+1)+x_i\ln p_i+(m-C_i)\ln(1-p_i)] = \sum_{i=1}^{n} l_i$$

where p is the vector of p_i's and $\Gamma(\cdot)$ denotes the gamma function. Instead of maximizing $L(m, p)$, we may minimize $\tilde{L}(m, p)=-L(m, p)$ with respect to m and p. Note that l_i's are all negative and $\tilde{L}(m, p)=\sum_{i=1}^{n}(z_i-\sqrt{-l_i})^2$ where $z_i=0$ for all i. Therefore the maximum likelihood estimates are the least squares estimates for the nonlinear regression model $z_i=\sqrt{-l_i}+\varepsilon_i$, where ε_i is the error term. The available nonlinear least squares procedures can be used for computing the maximum likelihood estimates.

6. Prediction for the Generalized HGDM

Suppose that we want to predict the number of faults newly detected by next d test instances. Such a prediction problem occurs when we determine the software release time or further test instances required to meet the given software reliability objective. This prediction problem can be solved by estimating $E(N_{n+1}+\cdots+N_n+d|C_n)$.

Replacing n in Expression (6) with $n+d$, the joint distribution of N_i, $i=1,2,\cdots,n+d$ is obtained as

$$P(N_i=x_i, i=1,2,\cdots,n+d) = \binom{m}{x_1,\cdots,x_{n+d}}\prod_{i=1}^{n+d}\left[p_i\prod_{j=1}^{i-1}(1-p_j)\right]^{x_i}\cdot\left[\prod_{i=1}^{n+d}(1-p_i)\right]^{m-\sum_{i=1}^{n+d}x_i} \tag{11}$$

Division of Expression (11) by Expression (6) results in the conditional distribution of N_{n+i}, $i=1,2,\cdots,d$.

$$\begin{aligned}P(N_{n+i}&=x_{n+i}, i=1,\cdots,d|N_j, j=1,\cdots,n)\\ &=P(N_{n+i}=x_{n+i}, i=1,\cdots,d|C_n)\\ &=\binom{m-C_n}{x_{n+1},\cdots,x_{n+d}}\prod_{i=1}^{d}\left[p_{n+i}\prod_{j=1}^{i-1}(1-p_{n+j})\right]^{x_{n+i}}\cdot\left[\prod_{i=1}^{d}(1-p_i)\right]^{m-C_n-\sum_{i=1}^{d}x_{n+i}}\end{aligned}$$

This is also a multinomial distribution of which parameters are $(m-C_n)$ and $p_{n+i}\prod_{j=1}^{i-1}(1-p_{n+j})$, $i=1,2,\cdots,d$. Then

$$P(N_{n+1}+\cdots+N_{n+d}=x|C_n) = \binom{m-C_n}{x}\left[1-\prod_{i=1}^{d}(1-p_{n+i})\right]^{x}\cdot\left[\prod_{i=1}^{d}(1-p_{n+i})\right]^{m-C_n-x}$$

Thus

$$E(N_{n+1}+\cdots+N_{n+d}|C_n) = (m-C_n)\left[1-\prod_{i=1}^{d}(1-p_{n+i})\right]. \tag{12}$$

By replacing the parameters in Expression (12) with the corresponding estimates, we can predict the number of faults newly discovered by next d test instances.

7. Conclusions

SRGMs are useful statistical tools for monitoring and evaluating the quality of a software system. It is necessary to develop new SRGMs and modify existing SRGMs in order to model the test-and-debug process more realistically. Thus we generalized the HGDM by assuming that the sensitivity factor is a binomial random variable, not a constant. The generalization enables us to easily apply the HGDM and characterize its statistical properties. Methods for parameter estimation and prediction were discussed. Further generalization will be to incorporate the concept of imperfect debugging into the generalized HGDM.

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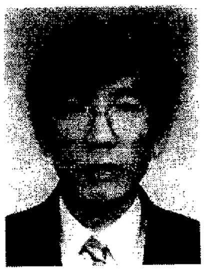

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