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CORRECTIONS TO "A UNIFIED FIXED POINT THEORY OF MULTIMAPS ON TOPOLOGICAL VECTOR SPACES"

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ABSTRACT. This is to correct Section 4 of our previous work [1].

Section 4 of our previous work [1] is incorrectly stated and our aim in this note is to replace the first part of Section 4 (from the beginning to the line 23 of page 815) by the following:

4. New fixed point theorems for condensing multimaps

In this section, we deduce new theorems for condensing maps.

Let X be a closed convex subset of a t.v.s. E and C a lattice with a least element, which is denoted by 0. A function $\Phi : 2^X \to C$ is called a *measure* of *noncompactness* on X provided that the following conditions hold for any $A, B \in 2^X$:

(1) $\Phi(A) = 0$ if and only if A is relatively compact;
(2) $\Phi(\overline{\text{co}}\, A) = \Phi(A)$; and
(3) $\Phi(A \cup B) = \max\{\Phi(A), \Phi(B)\}$.

It follows that $A \subset B$ implies $\Phi(A) \leq \Phi(B)$.

The above notion is a generalization of the set-measure γ and the ball-measure χ of noncompactness defined in terms of a family of seminorms or a norm.

For a measure Φ of noncompactness on E, a map $T : X \multimap E$ is said to be *Φ-condensing* provided that if $A \subset X$ and $\Phi(A) \leq \Phi(T(A))$, then A is relatively compact; that is, $\Phi(A) = 0$.

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From now on, we assume that Φ is a measure of noncompactness on the given set X in a t.v.s. E or on E if necessary.

Note that any map defined on a compact set or any compact map is Φ-condensing. Especially, if E is locally convex, then a compact map $T : X \multimap E$ is γ- or χ-condensing whenever X is complete or E is quasi-complete.

The following is well-known; for example, see Mehta *et al.* [1997].

LEMMA. *Let X be a nonempty closed convex subset of a t.v.s. E and $T : X \multimap X$ a Φ-condensing map. Then there exists a nonempty compact convex subset K of X such that $T(K) \subset K$.*

Note that even if X is admissible, we can not say that K is admissible in E. Therefore, we need the following concept:

A nonempty subset X of a t.v.s. E is said to be *q-admissible* if any nonempty compact convex subset K of X is admissible. We give some examples of q-admissible sets as follows:

(1) Any nonempty locally convex subset of a t.v.s.

(2) Any nonempty subset of a locally convex t.v.s.

(3) Any nonempty subset of a t.v.s. E on which its topological dual E^* separates point. Note that any compact convex subset of such a space E is affinely embeddable in a locally convex t.v.s.; see Weber [1992b].

It should be noted that an admissible t.v.s. (in the sense of Klee [1960]) and a q-admissible t.v.s. can be also defined.

From Theorem 1 and Lemma, we have the following:

THEOREM 2. *Let X be a q-admissible closed convex subset of a t.v.s. E. Then any Φ-condensing map $F \in \mathfrak{B}^\kappa(X,X)$ has a fixed point.*

Proof. By Lemma, there is a nonempty compact convex subset K of X such that $F(K) \subset K$. Since $F \in \mathfrak{B}^\kappa(X,X)$, there exists a closed map $\Gamma \in \mathfrak{B}(K,K)$ such that $\Gamma(x) \subset F(x)$ for all $x \in K$. Since Γ is compact and K is admissible, by Corollary 1.1, it has a fixed point $x_0 \in K$; that is, $x_0 \in \Gamma(x_0) \subset F(x_0)$. This completes our proof. □

COROLLARY 2.1. *Let X be a q-admissible closed convex subset of a t.v.s. E. Then any closed Φ-condensing map $F \in \mathfrak{B}(X,X)$ has a fixed point.*

COROLLARY 2.2. *Let X be a q-admissible closed convex subset of a t.v.s. E. Then any Φ-condensing map $F \in \mathfrak{B}^\sigma(X,X)$ has a fixed point.*

In the remainder of this section, we list more than ten papers in chronological order, from which we can deduce particular forms of Theorem 2.

Darbo [1955]: Recall that Kuratowski defined the measure of noncompactness, $\alpha(A)$, of a bounded subset A of a metric space (X,d):

$$\alpha(A) = \inf\{\varepsilon > 0 : A \text{ can be covered by a finite number of sets of diameter less than or equal to } \varepsilon\}.$$

Let $T : X \to X$ be a continuous map. Darbo calls T an *α-contraction* if given any bounded set A in X, $T(A)$ is bounded in X and

$$\alpha[T(A)] \leq k\alpha(A),$$

where the constant k fulfills the inequality $0 \leq k < 1$.

Darbo [1955] showed that if G is a closed, bounded, convex subset of a Banach space X and $T : G \to G$ is an α-contraction, then T has a fixed point.

Sadovskii [1967]: Introduced the notion of condensing maps in Banach spaces and obtained a form of Corollary 2.1 extending the above result of Darbo.

This is the end of our corrections.

REMARKS. 1. Our failure in [1] is mainly based on the unjustified fact that every admissible set is q-admissible. It would be interesting to prove or disprove this statement.

2. Until now, the results in Section 4 were used for locally convex t.v.s. only. There exists a measure of noncompactness on a certain subset in a more general t.v.s.

3. Similarly, in our another previous work [2, Theorems 3 and 4], the admissibility of X should be replaced by the q-admissibility. Moreover, in [3, Theorem 1], $\mathrm{cl}\, f(D)$ should be replaced by D. Further, each of [3, Theorems 2-4] can be slightly improved by replacing the admissibility of K by that of D.

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