J. Korean Math. Soc. **36** (1999), No. 2, pp. 421–430

# PROJECTIVE REPRESENTATIONS OF WREATHED 2-GROUPS

KILSOO CHUN AND SEUNG AHN PARK

ABSTRACT. In this paper we investigate representation groups of wreathed 2- groups and explicitly determine all the linearly inequivalent irreducible projective representations of wreathed 2-groups.

## 1. Introduction

Let $G$ be a finite group and let $F$ be an algebraically closed field of characteristic zero with its multiplicative group $F^* = F - \{0\}$. A mapping

$$T : G \longrightarrow \mathrm{GL}_n(F)$$

of $G$ into the general linear group $\mathrm{GL}_n(F)$ is called a *projective representation* of $G$ of degree $n$ over $F$ if there exists a function $\alpha : G \times G \to F^*$ such that

$$T(g)T(h) = \alpha(g,h)T(gh)$$

for all $g, h \in G$. The function $\alpha : G \times G \to F^*$ is called the *factor set* of $T$. If $\alpha(g,h) = 1$ for all $g, h \in G$, then $T$ is called a *linear representation* of $G$ over $F$. We say that $T$ is *irreducible* if the vector space $V = F^n$ has no nontrivial proper subspace invariant under all $T(g),\ g \in G$.

Let $T : G \to \mathrm{GL}_n(F)$ and $S : G \to \mathrm{GL}_n(F)$ be projective representations of $G$ with factor sets $\alpha$ and $\beta$, respectively. We say that $T$ and $S$ are *projectively equivalent* if there exists a nonsingular matrix $P \in \mathrm{GL}_n(F)$ and a function $c : G \to F^*$ such that

$$S(g) = c(g)P^{-1}T(g)P, \quad g \in G.$$

In this case, $\alpha$ and $\beta$ are *equivalent*, that is, the following holds:

Received November 5, 1998.

1991 Mathematics Subject Classification: 20C25.

Key words and phrases: wreathed 2-groups, projective representations, representation groups, Schur multipliers.

$$\beta(g,h) = \alpha(g,h)c(g)c(h)c(gh)^{-1}, \quad g,h \in G.$$

If $c(g) = 1$ for all $g \in G$, then $T$ and $S$ are said to be *linearly equivalent*. Linearly equivalent projective representations have the same factor set.

A 2-group $G$ is said to be *wreathed* if $G$ is isomorphic to the wreath product $\mathbb{Z}_{2^m} \operatorname{wr} \mathbb{Z}_2$ for some $m \geq 2$. In fact, the wreathed 2-group $G$ can be presented as follows :

$$G = \langle\, x, y, z \,|\, x^{2^m} = y^{2^m} = z^2 = 1,\ xy = yx,\ x^z = y\,\rangle, \quad m \geq 2.$$

The significance of wreathed 2-groups comes from the fact that they occur as Sylow 2-subgroups of known simple groups such as

$$\mathrm{L}_3(q) = \mathrm{PSL}_3(q), \quad q \equiv 1 \pmod 4$$

and

$$\mathrm{U}_3(q) = \mathrm{PSU}_3(q), \quad q \equiv -1 \pmod 4.$$

It is also well-known that Sylow 2-subgroups of

$$\mathrm{GL}_2(q), \quad q \equiv 1 \pmod 4$$

are wreathed (*cf.* [1]).

The purpose of this paper is to explicitly determine all the irreducible projective representations of wreathed 2-groups.

## 2. Some properties of wreathed 2-groups

We investigate some properties of wreathed 2-groups in this section.

First we classify the conjugacy classes of the wreathed 2-group $G$. The proof of the following can be easily established.

LEMMA 1. *Let*

$$G = \langle\, x, y, z \,|\, x^{2^m} = y^{2^m} = z^2 = 1,\ xy = yx,\ x^z = y\,\rangle$$

*be the wreathed 2-group of order $2^{2m+1}$, $m \geq 2$. Then $G$ has $2^{2m-1} + 3 \cdot 2^{m-1}$ conjugacy classes and they are*

$$\{1\},$$
$$\mathcal{C}_i = \{(xy)^i\} \quad (1 \leq i \leq 2^m - 1),$$
$$\mathcal{C}'_j = \{x^j, y^j\} \quad (1 \leq j \leq 2^m - 1),$$
$$\mathcal{C}_{ij} = \{x^i y^j, x^j y^i\} \quad (1 \leq i < j \leq 2^m - 1),$$
$$\mathcal{D}_k = \{x^i y^j z \mid i + j \equiv k \pmod{2^m}\} \quad (0 \leq k \leq 2^m - 1).$$

PROPOSITION 2. *Let*

$$G = \langle\, x, y, z \,|\, x^{2^m} = y^{2^m} = z^2 = 1,\ xy = yx,\ x^z = y\,\rangle$$

*be the wreathed 2-group of order* $2^{2m+1}$, $m \geq 2$ *and let* $F$ *be an algebraically closed field of characteristic zero. Then the following hold.*

(1) $G' = [G, G] = \langle xy^{-1}\rangle$ *and* $|G/G'| = 2^{m+1}$.

(2) *We have*

$$F[G] \cong F_1 \oplus \cdots \oplus F_{2^{m+1}} \oplus M_1 \oplus \cdots \oplus M_{2^{m-1}(2^m-1)},$$

*where* $F_1 = \cdots = F_{2^{m+1}} = F$ *and* $M_1 = \cdots = M_{2^{m-1}(2^m-1)} = \mathrm{Mat}_2(F)$.

*Proof.* It is easy to show that $G' = \langle xy^{-1}\rangle$ and $|G/G'| = 2^{m+1}$.

Since $U = \langle x, y\rangle$ is an abelian normal subgroup of $G$ and $|G : U| = 2$, the degrees of the irreducible linear representations of $G$ over $F$ are at most 2. Since $|G/G'| = 2^{m+1}$ and $G$ has $2^{2m-1} + 3 \cdot 2^{m-1}$ conjugacy classes, we have

$$|G| = \underbrace{1 + \cdots + 1}_{2^{m+1}\text{ times}} + \underbrace{2^2 + \cdots + 2^2}_{2^{m-1}(2^m-1)\text{ times}}.$$

Thus the assertion holds. □

## 3. Representation groups of wreathed 2-groups

We now determine a representation group of the wreathed 2-group

$$G = \langle\, x, y, z \,|\, x^{2^m} = y^{2^m} = z^2 = 1,\ xy = yx,\ x^z = y\,\rangle, \quad m \geq 2.$$

First we consider the Schur multiplier of the wreathed 2-group $G$.

PROPOSITION 3. *Let* $G$ *be the wreathed 2-group of order* $2^{2m+1}$, $m \geq 2$. *Then the Schur multiplier of* $G$ *is*

$$M(G) \cong C_2.$$

*Proof.* The proof can be found in [7]. □

Using the above proposition, we can determine a representation group of $G$ as follows.

THEOREM 4. *Let*

$$G^* = \langle s, t, u, v | s^{2^m} = t^{2^m} = u^2 = v^2 = 1, [s, u] = [v, u] = 1, [s, t] = u, s^v = t\rangle,$$

*where* $m \geq 2$. *Then* $G^*$ *is a group of order* $2^{2m+2}$,

$$[G^*, G^*] = \langle st^{-1}, u\rangle, \quad Z(G^*) = \langle s^2t^2, u\rangle$$

*and* $G^*$ *is a representation group of the wreathed 2-group* $G$.

*Proof.* It is easy to prove that $[G^*, G^*] = \langle st^{-1}, u\rangle$, $Z(G^*) = \langle s^2t^2, u\rangle$. Note that $\langle u\rangle \subseteq Z(G^*) \cap [G^*, G^*]$, $|\langle u\rangle| = |M(G)|$ and $G^*/\langle u\rangle \cong G$. Hence $G^*$ is a representation group of $G$. □

Now we classify the conjugacy classes of $G^*$. The following result can be obtained by an easy calculation.

PROPOSITION 5. *Let $G^*$ be a representation group of the wreathed 2-group $G$ defined by*

$$G^* = \langle s, t, u, v | s^{2^m} = t^{2^m} = u^2 = v^2 = 1, [s, u] = [v, u] = 1, [s, t] = u, s^v = t\rangle,$$

*where $m \geq 2$. Then $G^*$ has exactly $5 \cdot 2^{2m-3} + 9 \cdot 2^{m-2}$ conjugacy classes and they are*

$\{s^it^i\}$, $\{s^it^iu\}$ $(i = 0, 2, \cdots, 2^m - 2)$,
$\{s^it^i, s^it^iu\}$ $(i = 1, 3, \cdots, 2^m - 1)$,
$\{s^it^j, s^jt^i\}$, $\{s^it^ju, s^jt^iu\}$ $(i, j = 0, 2, \cdots, 2^m - 2, i \neq j)$,
$\{s^it^j, s^jt^i, s^it^ju, s^jt^iu\}$ $(i = 0, 1, \cdots, 2^m - 1, j = 1, 3, \cdots, 2^m - 1, i \neq j)$,
$\{s^it^jv \mid i + j \equiv l \pmod{2^m}\}$ $(l = 1, 3, \cdots, 2^m - 1)$,
$\{s^it^juv \mid i + j \equiv l \pmod{2^m}\}$ $(l = 1, 3, \cdots, 2^m - 1)$,
$\{s^it^jv, s^it^juv \mid i + j \equiv l \pmod{2^m}\}$ $(l = 0, 2, \cdots, 2^m - 2)$.

PROPOSITION 6. *Let $G^*$ be a representation group of the wreathed 2-group $G$ defined by*

$$G^* = \langle s, t, u, v | s^{2^m} = t^{2^m} = u^2 = v^2 = 1, [s, u] = [v, u] = 1, [s, t] = u, s^v = t\rangle,$$

*where $m \geq 2$ and let $F$ be an algebraically closed field of characteristic zero. Then we have*

$$F[G^*] \cong F_1 \oplus \cdots \oplus F_{2^{m+1}} \oplus M_1 \oplus \cdots \oplus M_{2^{m-1}(2^m+1)} \oplus N_1 \oplus \cdots \oplus N_{2^{m-2}(2^{m-1}-1)},$$

*where*

$$F_1 = \cdots = F_{2^{m+1}} = F, \quad M_1 = \cdots = M_{2^{m-1}(2^m+1)} = \mathrm{Mat}_2(F)$$

*and*

$$N_1 = \cdots = N_{2^{m-2}(2^{m-1}-1)} = \mathrm{Mat}_4(F).$$

*Proof.* Since $V = \langle s^2, st, u\rangle$ is an abelian normal subgroup of $G^*$ and $|G^* : V| = 4$, the degrees of the irreducible linear representations of $G^*$ over $F$ are at most 4. Since $|G^*/[G^*, G^*]| = 2^{m+1}$ and $G^*$ has $5 \cdot 2^{2m-3} + 9 \cdot 2^{m-2}$ conjugacy classes, we have

$$|G^*| = \underbrace{1+\cdots+1}_{2^{m+1}\text{ times}} + \underbrace{2^2+\cdots+2^2}_{2^{m-1}(2^m+1)\text{ times}} + \underbrace{4^2+\cdots+4^2}_{2^{m-2}(2^{m-1}-1)\text{ times}} .$$

Thus the assertion holds. □

## 4. Projective representations of wreathed 2-groups

In this section we explicitly determine all the inequivalent irreducible projective representations of wreathed 2-groups using the previous results.

We first consider factor sets of the wreathed 2-group $G$.

LEMMA 7. *Let*

$$G = \langle x, y, z | x^{2^m} = y^{2^m} = z^2 = 1, xy = yx, x^z = y\rangle$$

*be the wreathed 2-group of order* $2^{2m+1}$, $m \geq 2$ *and let* $F$ *be an algebraically closed field of characteristic zero. Then there exists a factor set* $\beta : G \times G \to F^*$ *of* $G$ *over* $F$ *such that*

$$\beta(x^iy^j, x^ky^lz) = \beta(x^iy^j, x^ky^l) = (-1)^{jk},$$
$$\beta(x^iy^jz, x^ky^lz) = \beta(x^iy^jz, x^ky^l) = (-1)^{(j+k+1)l-k}$$

*for all* $0 \leq i, j, k, l \leq 2^m - 1$ *and* $\beta$ *is not equivalent to the trivial factor set* 1.

*Proof.* Let

$$G^* = \langle s, t, u, v | s^{2^m} = t^{2^m} = u^2 = v^2 = 1, [s, u] = [v, u] = 1, [s, t] = u, s^v = t\rangle$$

be a representation group of $G$. Define a (non-homomorphic) map $\tau : G \to G^*$ (inverse to the obvious homomorphism that goes in the other direction) by setting

$$\tau(x^iy^j) = s^it^j, \ \tau(x^iy^jz) = s^it^jv.$$

As the regular representation of $G^*$ is faithful and is a direct sum of irreducibles, $G^*$ has at least one irreducible representation $\rho$ which does not have $u$ in its kernel. Then $T(g) = \rho(\tau(g))$ $(g \in G)$ defines a projective representation of $G$. The corresponding factor set $\beta$ may be calculated from

$$T(g)T(h)T(gh)^{-1} = \beta(g, h)I, \quad g, h \in G,$$

where $I$ is the identity matrix.

Note that

$$\beta(y,x)I = T(y)T(x)T(yx)^{-1} = \rho(\tau(y)\tau(x)\tau(yx)^{-1}) = \rho(u),$$
$$\beta(y,x)^2 I = \rho(u)^2 = \rho(u^2) = \rho(1) = I.$$

Since $\rho(u) \neq I$, we have $\beta(y,x) = -1$ and so $\rho(u) = -I$. Using that

$$\beta(g,h)I = T(g)T(h)T(gh)^{-1} = \rho(\tau(g)\tau(h)\tau(gh)^{-1})$$

and $\rho(u) = -I$, we can show that $\beta$ has the following properties :

$$\beta(x^i y^j, x^k y^l z) = \beta(x^i y^j, x^k y^l) = (-1)^{jk},$$
$$\beta(x^i y^j z, x^k y^l z) = \beta(x^i y^j z, x^k y^l) = (-1)^{(j+k+1)l-k}$$

for all $0 \leq i,j,k,l \leq 2^m - 1$.

Now suppose that $\beta$ is equivalent to the trivial factor set 1. Then there exists a function $c : G \to F^*$ such that

$$\beta(g,h) = c(g)c(h)c(gh)^{-1}, \quad g,h \in G.$$

Since $\beta(x,y) = 1$, we have $c(xy) = c(x)c(y)$. But

$$-1 = \beta(y,x) = c(y)c(x)c(yx)^{-1} = 1.$$

This is a contradiction. Thus the assertions hold. □

Now we determine all the irreducible projective representations of wreathed 2-groups. The following is our main theorem.

THEOREM 8. *Let*

$$G = \langle x,y,z \mid x^{2^m} = y^{2^m} = z^2 = 1, xy = yx, x^z = y\rangle$$

*be the wreathed 2-group of order* $2^{2m+1}$, $m \geq 2$ *and let* $F$ *be an algebraically closed field of characteristic zero. Let* $\xi$ *be a primitive* $2^m$*-th root of unity and* $\epsilon$ *a primitive* $2^3$*-th root of unity in* $F$. *Then the following hold.*

(1) *For any factor set* $\beta$ *of* $G$ *over* $F$ *which is not equivalent to the trivial factor set,*

$$F^\beta[G] \cong M_1 \oplus \cdots \oplus M_{2^m} \oplus N_1 \oplus \cdots \oplus N_{2^{m-2}(2^{m-1}-1)},$$

*where* $M_1 = \cdots = M_{2^m} = \mathrm{Mat}_2(F)$ *and* $N_1 = \cdots = N_{2^{m-2}(2^{m-1}-1)} = \mathrm{Mat}_4(F)$.

(2) *Let* $\alpha : G \times G \to F^*$ *be a factor set of* $G$ *over* $F$ *such that*

$$\alpha(x^i y^j, x^k y^l z) = \alpha(x^i y^j, x^k y^l) = (-1)^{jk},$$
$$\alpha(x^i y^j z, x^k y^l z) = \alpha(x^i y^j z, x^k y^l) = (-1)^{(j+k+1)l-k}$$

*for all* $0 \leq i,j,k,l \leq 2^m - 1$. *Then*

$$M(G) = \{\{1\}, \{\alpha\}\}.$$

(3) *If* $m \geq 3$, *then there are* $2^m$ *inequivalent irreducible projective representations of* $G$ *over* $F$ *of degree* 2 *with factor set* $\alpha$ *and they are projective representations*

$$T_i : G \longrightarrow \mathrm{GL}_2(F), \quad 0 \leq i \leq 2^m - 1$$

*defined by*

$$T_i(x) = \begin{pmatrix} 0 & \xi^i \\ \xi^{i+2^{m-2}} & 0 \end{pmatrix}, \; T_i(y) = \begin{pmatrix} 0 & \xi^{i+3\cdot 2^{m-2}} \\ \xi^{i+2^{m-1}} & 0 \end{pmatrix}, \; T_i(z) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

*and*

$$T_i(x^j y^k z^l) = T_i(x)^j T_i(y)^k T_i(z)^l.$$

*There are* $2^{m-2}(2^{m-1} - 1)$ *inequivalent irreducible projective representations of* $G$ *over* $F$ *of degree* 4 *with factor set* $\alpha$ *and they are projective representations*

$$S_{ij} : G \longrightarrow \mathrm{GL}_4(F), \quad 0 \leq i \leq 2^{m-1} - 2, \; i + 1 \leq j \leq 2^{m-1} - 1$$

*defined by*

$$S_{ij}(x) = \begin{pmatrix} 0 & \xi^i & 0 & 0 \\ \xi^{i+2^{m-2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi^j \\ 0 & 0 & \xi^{j+2^{m-2}} & 0 \end{pmatrix},$$

$$S_{ij}(y) = \begin{pmatrix} 0 & \xi^{j+3\cdot 2^{m-2}} & 0 & 0 \\ \xi^{j+2^{m-1}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi^{i+3\cdot 2^{m-2}} \\ 0 & 0 & \xi^{i+2^{m-1}} & 0 \end{pmatrix},$$

$$S_{ij}(z) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

*and*

$$S_{ij}(x^k y^l z^n) = S_{ij}(x)^k S_{ij}(y)^l S_{ij}(z)^n.$$

(4) *If* $m = 2$, *then there are* 4 *inequivalent irreducible projective representations of* $G$ *over* $F$ *of degree* 2 *with factor set* $\alpha$ *and they are projective representations*

$$T_k : G \longrightarrow \mathrm{GL}_2(F), \quad k = 1, 3, 5, 7$$

*defined by*

$$T_k(x) = \begin{pmatrix} 0 & \epsilon^k \\ \epsilon^{k+2} & 0 \end{pmatrix}, \quad T_k(y) = \begin{pmatrix} 0 & \epsilon^{k+6} \\ \epsilon^{k+4} & 0 \end{pmatrix}, \quad T_k(z) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

*and*

$$T_k(x^l y^m z^n) = T_k(x)^l T_k(y)^m T_k(z)^n.$$

*There is exactly one inequivalent irreducible projective representation of $G$ over $F$ of degree 4 with factor set $\alpha$ and it is a projective representation*

$$S : G \longrightarrow \mathrm{GL}_4(F)$$

*defined by*

$$S(x) = \begin{pmatrix} 0 & \epsilon & 0 & 0 \\ \epsilon^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \epsilon^3 \\ 0 & 0 & \epsilon^5 & 0 \end{pmatrix}, \quad S(y) = \begin{pmatrix} 0 & \epsilon & 0 & 0 \\ \epsilon^7 & 0 & 0 & 0 \\ 0 & 0 & 0 & \epsilon^7 \\ 0 & 0 & \epsilon^5 & 0 \end{pmatrix},$$

$$S(z) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

*and*

$$S(x^l y^m z^n) = S(x)^l S(y)^m S(z)^n.$$

*Proof.* Let $\beta : G \times G \to F^*$ be a factor set of $G$ which is not equivalent to the trivial factor set. Let $G^*$ be a representation group of $G$. Then we have

$$F[G^*] \cong \bigoplus_{\{\alpha\}} F^\alpha[G],$$

where the sum runs over all elements $\{\alpha\}$ in $M(G)$. On the other hand, it follows from Proposition 2 and 6 that

$$F[G] \cong F_1 \oplus \cdots \oplus F_{2^{m+1}} \oplus M_1 \oplus \cdots \oplus M_{2^{m-1}(2^m-1)}$$

and

$$F[G^*] \cong F_1 \oplus \cdots \oplus F_{2^{m+1}} \oplus M_1 \oplus \cdots \oplus M_{2^{m-1}(2^m+1)} \oplus N_1 \oplus \cdots \oplus N_{2^{m-2}(2^{m-1}-1)},$$

where

$$F_1 = \cdots = F_{2^{m+1}} = F, \quad M_1 = \cdots = M_{2^m(2^{m-1}+1)} = \mathrm{Mat}_2(F)$$

and

$$N_1 = \cdots = N_{2^{m-2}(2^{m-1}-1)} = \mathrm{Mat}_4(F).$$

Since $\beta$ is not equivalent to the trivial factor set, $F^{\beta}[G]$ is not isomorphic to $F[G]$. Hence it follows that

$$(*) \qquad F^{\beta}[G] \cong M_1 \oplus \cdots \oplus M_{2^m} \oplus N_1 \oplus \cdots \oplus N_{2^{m-2}(2^{m-1}-1)}.$$

Let $\alpha : G \times G \to F^*$ be a factor set of $G$ over $F$ such that

$$\alpha(x^i y^j, x^k y^l z) = \alpha(x^i y^j, x^k y^l) = (-1)^{jk},$$
$$\alpha(x^i y^j z, x^k y^l z) = \alpha(x^i y^j z, x^k y^l) = (-1)^{(j+k+1)l-k}$$

for all $0 \leq i, j, k, l \leq 2^m - 1$. Then it follows from Proposition 3 and Lemma 7 that

$$M(G) = \{\{1\}, \{\alpha\}\}.$$

Since $\alpha$ is not equivalent to the trivial factor set, $(*)$ holds for $\beta = \alpha$. Hence there are $2^m$ inequivalent irreducible projective representations of $G$ over $F$ of degree 2 with factor set $\alpha$ and there are $2^{m-2}(2^{m-1} - 1)$ inequivalent irreducible projective representations of $G$ over $F$ of degree 4 with factor set $\alpha$.

Define maps $T_i : G \to \mathrm{GL}_2(F)$ and $S_{ij} : G \to \mathrm{GL}_4(F)$ as in Theorem. Then it is easy to show that $T_i$ and $S_{ij}$ are projective representations of $G$ over $F$ with factor set $\alpha$. By $(*)$, every $T_i$ and $S_{ij}$ is irreducible. Since, for $i \neq j$, $T_i(xz)$ and $T_j(xz)$ don't have the same eigenvalues, $T_i$ and $T_j$ are not linearly equivalent. If $i \neq k$ or $j \neq l$, then $S_{ij}(x)$ does not have the same eigenvalues as those of $S_{kl}(x)$. Hence $S_{ij}$ and $S_{kl}$ are not linearly equivalent.

This completes the proof. □

## References


[1] J. L. Alperin, R. Brauer and D. Gorenstein, *Finite groups with quasi-dihedral and wreathed Sylow 2-subgroups*, Trans. Amer. Math. Soc. **151** (1970), 1–261.

[2] N. Blackburn, *Some homology groups of wreath products*, Illinois J. Math. **16** (1972), 116–129.

[3] C. W. Curtis and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras*, Interscience, New York, 1962.

[4] D. Gorenstein, *Finite Groups*, Harper & Rows, New York, 1968.

[5] B. Huppert, *Endliche Gruppen* I, Springer-Verlag, Berlin, 1967.

[6] N. Ito, *On the degrees of irreducible representations of a finite group*, Nagoya Math. J. **3** (1951), 5–6.

[7] G. Karpilovsky, *Group Representations*, vol. 2, Elsevier Science, North-Holland, 1993.

[8] S. A. Park, *Projective representations of some finite groups*, J. Korean Math. Soc. **22** (1985), 173–180.

[9] I. Schur, *Untersuchungen über die Darstellung der endlichen Gruppen durch gebrochene lineare substitutionen*, J. Reine Angew. Math. **132** (1907), 85–137.
[10] M. Suzuki, *Group Theory* I, II, Springer-Verlag, New York, 1982, 1986.

Department of Mathematics
Sogang University
Seoul 121-742, Korea
*E-mail*: kschun@math.sogang.ac.kr