

LAYER OSCILLATIONS IN A FREE BOUNDARY
PROBLEM WITH PUSHCHINO DYNAMICS
SATISFYING THE DIRICHLET BOUNDARY
CONDITION

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ABSTRACT. In this paper, we consider a free boundary problem with Pushchino dynamics satisfying the Dirichlet boundary condition. We shall show the stationary solutions $(v^*(x), s^*)$ exist and the Hopf bifurcation occurs at a critical point τ when $s^* \in (1/3, 1)$.

1. Introduction

In [5], they showed the existence of the Hopf bifurcation in a free boundary problem with the Pushchino dynamics satisfying the Neumann boundary conditions. In this paper, we shall consider this free boundary problem which satisfy the Dirichlet boundary condition at the boundaries in x -axis:

$$(1.1) \quad \begin{cases} v_t = Dv_{xx} - (c_1 + b)v + c_1 H(x - s(t)) & \text{for } (x, t) \in \Omega^- \cup \Omega^+, \\ v(0, t) = 0 = v(1, t) & \text{for } t > 0, \\ v(x, 0) = v_0(x) & \text{for } 0 < x < 1, \\ \tau \frac{ds}{dt} = C(v(s(t), t)) & \text{for } t > 0, s(0) = s_0, \end{cases}$$

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where $\Omega = (0, 1) \times (0, \infty)$, $\Omega^- = \{(x, t) \in \Omega : 0 < x < s(t)\}$ and $\Omega^+ = \{(x, t) \in \Omega : s(t) < x < 1\}$ and, in addition, $v(x, t)$ and $v_x(x, t)$ are assumed continuous in Ω . The diffusion constant D is assumed to be finite (so, let $D = 1$) and $H(y)$ is the Heaviside function. By the bistable assumption, a constant b must satisfy $-c_1 < b < \frac{c_1(c_2-a)}{c_1+a}$ with $0 < a < c_1$. The velocity function of a free boundary is

$$C : I \rightarrow \mathbf{R}; I := \left(-\frac{a}{c_1 + c_2}, \frac{c_1 - a}{c_1 + c_2} \right), \quad 0 < a < c_1$$

and given by

$$C(v) = \frac{2(c_1 + c_2)v - (c_1 - 2a)}{\sqrt{\left(\frac{c_1-a}{c_1+c_2} - v\right)\left(v + \frac{a}{c_1+c_2}\right)}}.$$

In section 2, we shall state the well-posedness of (1.1) by using a change of variables and gives enough regularity of the solution for an analysis of the bifurcation. In section 3, we show that a Hopf bifurcation occurs at a critical value of τ as τ decreases.

2. The Well-posedness

The problem (1.1) may be written by an abstract evolution equation:

$$(F) \quad \begin{cases} \frac{d}{dt}(v, s) + \tilde{A}(v, s) = F(v, s) \\ (v, s)(0) = (v_0(\cdot), s_0) \end{cases}$$

where

$$\tilde{A} := \begin{pmatrix} -\frac{d^2}{dx^2} + (c_1 + b) & 0 \\ 0 & 0 \end{pmatrix}$$

and the nonlinear operator F by

$$F(v, s) = \begin{pmatrix} F_1(v(\cdot, t), s(t)) \\ F_2(v(\cdot, t), s(t)) \end{pmatrix} := \begin{pmatrix} c_1 H(\cdot - s(t)) \\ \frac{1}{\tau} C(v(s(t), t)) \end{pmatrix}.$$

We let the differential operator $A = -\frac{d^2}{dx^2} + (c_1 + b)$ together with Dirichlet boundary conditions $v(0) = v(1) = 0$. The combination of the jump discontinuity of a Heaviside function in the first component of F and the nature of the dependence of v on s in the second component of F makes it impossible to find a function space of the form $X = L_p$, $1 \leq p \leq \infty$ such that F satisfies a Lipschitz condition on $\tilde{X} \subset X \times \mathbf{R}$. As a first step we obtain more regularity for the solution by semigroup methods, considering A as a densely defined operator

$$\begin{cases} A : D(A) \subset_{\text{dense}} X \longrightarrow X \\ X := L_2((0, 1)) \text{ with norm } \|\cdot\|_2 \\ D(A) := \{v \in H^{2,2}(0, 1) : v(0) = 0 = v(1)\}. \end{cases}$$

Since $H(\cdot - s)$ is not regular enough, it is impossible to get differential dependence on initial conditions that is needed for an application of the Hopf bifurcation theorem.

We now decompose v in (F) into a part u , which is a solution to a more regular problem, and a part g , which is worse, but explicitly known in terms of the Green's function G of the operator A .

PROPOSITION 2.1. *Let $G : [0, 1]^2 \rightarrow \mathbf{R}$ be a Green's function of the operator A . Define $g : [0, 1]^2 \rightarrow \mathbf{R}$*

$$g(x, s) := c_1 \int_s^1 G(x, y) dy = A^{-1}(c_1 H(\cdot - s))(x)$$

and $\gamma : [0, 1] \rightarrow \mathbf{R}$

$$\gamma(s) := g(s, s).$$

Then $g(\cdot, s) \in D(A)$ for all s , $\frac{\partial g}{\partial s}(x, s) = -c_1 G(x, s)$ is in $H^{1,\infty}((0, 1) \times (0, 1))$, and $\gamma \in C^\infty([0, 1])$.

Proof. Everything follows from the fact that G is in $H^{1,\infty}$ and C^∞ on either $\{x \leq y\}$ or $\{x \geq y\}$, and that $H(\cdot - s) \in L^2$. □

Using these preliminary observations, we decompose a solution (v, s) of (F) into two parts by defining

$$u(t)(x) := v(x, t) - g(x, s(t)).$$

We denote the space $X \times \mathbf{R}$ by \tilde{X} and define

$$\begin{cases} D(\tilde{A}) := D(A) \times \mathbf{R}, \\ \tilde{A} : D(\tilde{A}) \subset_{\text{dense}} \tilde{X} \longrightarrow \tilde{X}, \tilde{A}(u, s) := (Au, 0) \end{cases}$$

The initial value problem for (u, s) can then be written as

$$(R) \quad \begin{cases} \frac{d}{dt}(u, s) + \tilde{A}(u, s) = \frac{1}{\tau} f(u, s) \\ (u, s)(0) = (u(0), s(0)) = (u_0, s_0). \end{cases}$$

The nonlinear reaction terms f is defined on a set

$$W := \{(u, s) \in C^1([0, 1]) \times (0, 1) : u(s) + \gamma(s) \in I\} \subset_{\text{open}} C^1([0, 1]) \times \mathbf{R}$$

and given by

$$\begin{aligned} f : W &\rightarrow L_2(0, 1) \times \mathbf{R}, \\ f(u, s) &:= \begin{pmatrix} f_1(s) \cdot f_2(u, s) \\ f_2(u, s) \end{pmatrix} \end{aligned}$$

where $f_1(s) = c_1 G(\cdot, s)$ and $f_2(u, s) = C(u(s) + \gamma(s))$. The advantage of (R) over (F) is, that the right hand side of (R) is one step more regular than that of (F), since it involves $G(x, s)$ instead of $H(x - s)$. More precisely, we can show the following (refer to [5]):

LEMMA 2.2. *The functions $f : W \rightarrow \tilde{X}$ is continuously differentiable with derivatives given by*

$$\begin{aligned} f'_1(s) &= c_1 \frac{\partial G}{\partial y}(\cdot, s) \\ Df_2(u, s)(\hat{u}, \hat{s}) &= C'(u(s) + \gamma(s)) \cdot (u'(s)\hat{s} + \gamma'(s)\hat{s} + \hat{u}(s)) \\ Df(u, s)(\hat{u}, \hat{s}) &= f_2(u, s) \cdot (f'_1(s), 0) \hat{s} + Df_2(u, s)(\hat{u}, \hat{s}) \cdot (f_1(s), 1). \end{aligned}$$

We can now apply semigroup theory to (R) using domains of fractional powers $\alpha \in [0, 1]$ of A and \tilde{A} :

$$X^\alpha := D(A^\alpha), \quad \tilde{X}^\alpha := D(\tilde{A}^\alpha), \quad \tilde{X}^\alpha = X^\alpha \times \mathbf{R}.$$

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For this we need to find an $\alpha \in (0, 1)$ such that $X^\alpha \subset C^1([0, 1])$, because then $f : W \cap \tilde{X}^\alpha \rightarrow \tilde{X}$ is continuously differentiable. Theorem 1.6.1 in [3], for example, ensures that this is the case for $\alpha > 3/4$. Standard applications of theorems for existence, uniqueness and dependence on initial conditions (cf. [3]) together with the starting regularity of solutions to (F) as well as the regularity of the functions g and γ (Proposition 2.1) then give the following result:

THEOREM 2.3. i) For any $3/4 < \alpha < 1$, $(u_0, s_0) \in W \cap \tilde{X}^\alpha$ and $\tau \in \mathbf{R}$ there exists a unique solution

$$(u, s)(t) = (u, s)(t; u_0, s_0, \tau)$$

of (R). The solution operator

$$(u_0, s_0, \tau) \mapsto (u, s)(t; u_0, s_0, \tau)$$

is continuously differentiable from $\tilde{X}^\alpha \times \mathbf{R}$ into \tilde{X}^α for $t > 0$. The functions $v(x, t)$

$$v(x, t) := u(t)(x) + g(x, s(t))$$

and s then satisfy (F) with $v(\cdot, 0) \in X^\alpha$, $v(s_0, 0) \in I$.

ii) If (v, s) is a solution of (F) for some $\tau \in \mathbf{R}$ with initial condition $v_0 \in X^\alpha$, $1 > \alpha > 3/4$, $s_0 \in (0, 1)$, $v_0(s_0) \in I$, then $(u_0, s_0) := (v_0 - g(\cdot, x_0), s_0) \in \tilde{X}^\alpha \cap W$ and

$$(v(\cdot, t), s(t)) = (u, s)(t; u_0, s_0, \tau) + (g(\cdot, s(t)), 0)$$

where $(u, s)(t; u_0, s_0, \tau)$ is the unique solution of (R).

iii) For any $1 > \alpha > 3/4$, $\tau \in \mathbf{R}$, $(v_0, s_0) \in U := \{(v, s) \in X^\alpha \times (0, 1) : v(s) \in I\}$ the problem (F) has a unique solution

$$(v(x, t), s(t)) = (v, s)(x, t; v_0, s_0, \tau).$$

Additionally, the mapping

$$(v_0, s_0, \tau) \mapsto (v, s)(\cdot, t; v_0, s_0, \tau)$$

is continuously differentiable from $X^\alpha \times \mathbf{R}^2$ into $X^\alpha \times \mathbf{R}$.

3. A Hopf Bifurcation

In this section, we shall show the Hopf bifurcation occurs for some τ . The stationary problem, corresponding to (R) by

$$\begin{aligned} Au^* &= \frac{c_1}{\tau} G(x, s^*) C(u^*(s^*) + \gamma(s^*)), \quad u^*(0) = 0 = u^*(1) \\ 0 &= \frac{1}{\tau} C(u^*(s^*) + \gamma(s^*)) \end{aligned}$$

for $(u^*, s^*) \in D(\tilde{A}) \cap W$. The function $\gamma(s) = c_1 \int_s^1 G(s, y) dy$ then becomes

$$\gamma(s) = \frac{1}{(c_1 + b) \sinh(c_1 + b)} \sinh((c_1 + b)s) \left(\cosh((c_1 + b)(1 - s)) - 1 \right).$$

For $\tau \neq 0$, this system is equivalent to the pair of equations

$$(3.1) \quad u^* = 0, \quad C(\gamma(s^*)) = 0.$$

We thus obtain

PROPOSITION 3.1. *If $0 < \frac{c_1 - 2a}{2(c_1 + c_2)} < \frac{2 \sinh^3 \frac{c_1 + b}{3}}{(c_1 + b)^2 \sinh(c_1 + b)}$, then (R) has a unique stationary solution $(0, s^*)$ for all $\tau \neq 0$ with $s^* \in (0, 1)$. The linearization of f at $(0, s^*)$ is*

$$Df(0, s^*)(\hat{u}, \hat{s}) = \frac{4(c_1 + c_2)}{c_1} \left(\hat{u}(s^*) + \gamma'(s^*)\hat{s} \right) \cdot \left(c_1 G(\cdot, s^*), 1 \right).$$

The pair $(0, s^*)$ corresponds to a unique steady state (v^*, s^*) of (F) for $\tau \neq 0$ with

$$v^*(x) = g(x, s^*).$$

Proof. We note $C(r) = 0$ iff $r = \frac{c_1 - 2a}{2(c_1 + c_2)}$. We define $\Gamma(s) := \gamma(s) - \frac{c_1 - 2a}{2(c_1 + c_2)}$. Then $\Gamma'(s) > 0$ if $0 < s < \frac{1}{3}$ and $\Gamma'(s) < 0$ if $\frac{1}{3} < s < 1$. The equation $\Gamma(s) = 0$ have solutions for s if $\Gamma(0) < 0 < \Gamma(\frac{1}{3})$ or $\Gamma(1) < 0 < \Gamma(\frac{1}{3})$. This implies that $0 < \frac{c_1 - 2a}{2(c_1 + c_2)} < \gamma(\frac{1}{3})$ with $\gamma(\frac{1}{3}) = \frac{2 \sinh^3 \frac{c_1 + b}{3}}{(c_1 + b)^2 \sinh(c_1 + b)}$. Thus, there exists solutions $s^* \in (0, 1)$.

The formula for $Df(0, s^*)$ follows from Lemma 2.2 and the relation $C' \left(\frac{c_1 - 2a}{2(c_1 + c_2)} \right) = \frac{4(c_1 + c_2)}{c_1}$ and thus we now define a new parameter $\mu =$

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$\frac{4(c_1+c_2)}{rc_1}$. The corresponding steady state (v^*, s^*) for (F) is obtained using Theorem 2.3. \square

In order to show the occurrence of Hopf bifurcations at some μ^* in (R), we must show the stationary solution $(u^*(x), s^*, \mu^*)$ is a Hopf point and then there is a periodic solution near the stationary point by the Hopf bifurcation theorem in [1] and [5]. We introduce the definition of Hopf points.

DEFINITION 3.2. Under the assumptions of Proposition 3.1, define (for $1 \geq \alpha > 3/4$) the operator $B \in L(\tilde{X}^\alpha, \tilde{X})$

$$B := \frac{c_1}{4(c_1 + c_2)} Df(0, s^*).$$

We then define $(0, s^*, \mu^*)$ to be a Hopf point for (R) if and only if there exists an $\epsilon_0 > 0$ and a C^1 -curve

$$(-\epsilon_0 + \mu^*, \mu^* + \epsilon_0) \mapsto (\lambda(\mu), \phi(\mu)) \in \mathbf{C} \times \tilde{X}_{\mathbf{C}}$$

($Y_{\mathbf{C}}$ denotes the complexification of the real space Y) of eigendata for $-\tilde{A} + \mu B$ with

- i) $(-\tilde{A} + \mu B)(\phi(\mu)) = \lambda(\mu)\phi(\mu)$, $(-\tilde{A} + \mu B)\overline{\phi(\mu)} = \overline{\lambda(\mu)}\overline{\phi(\mu)}$;
- ii) $\lambda(\mu^*) = i\beta$ with $\beta > 0$;
- iii) $\text{Re}(\lambda) \neq 0$ for all $\lambda \in \sigma(-\tilde{A} + \mu^* B) \setminus \{\pm i\beta\}$;
- iv) $\text{Re} \lambda'(\mu^*) \neq 0$ (transversality).

We now check (R) for Hopf points. For this we have to solve the eigenvalue problem

$$-\tilde{A}(u, s) + \mu B(u, s) = \lambda(u, s)$$

which is equivalent to

$$(3.2) \quad \begin{aligned} (A + \lambda)u &= \mu \cdot (\gamma'(s^*)s + u(s^*)) \cdot c_1 G(\cdot, s^*) \\ \lambda s &= \mu \cdot (\gamma'(s^*)s + u(s^*)). \end{aligned}$$

We now shall show that there exists a unique, purely imaginary eigenvalue $\lambda = i\beta$ of (3.2) with $\beta > 0$ for some μ^* in order for $(0, s^*, \mu^*)$ to be a Hopf point. As a first result, we show the $(0, s^*, \mu^*)$ satisfy the condition of (i) and (ii) in the Definition 3.2.

LEMMA 3.3. Assume that the operator $-\tilde{A} + \mu^* B$ has a unique pair $\{\pm i\beta\}$ of purely imaginary eigenvalues for $\mu^* \in \mathbf{R} \setminus \{0\}$. Suppose that ϕ^* be the (normalized) eigenfunction corresponding to the eigenvalue $i\beta$. Then there exists a C^1 -curve $\mu \mapsto (\phi(\mu), \lambda(\mu))$ of eigendata such that $\phi(\mu^*) = \phi^*$ and $\lambda(\mu^*) = i\beta$.

The proof is similar to the proof in [5].

It remains to show the transversality and so, we shall use the Fourier sine transformation for the linearized eigenvalue problem of (F). We let $v = u - c_1 G(\cdot, s^*)$ then we have the linearized eigenvalue problem of (F) which is represented by

$$(3.3) \quad (A + \lambda)v = -c_1 \delta_{s^*}, \quad v(0) = 0 = v(1)$$

and

$$(3.4) \quad \lambda = \mu^* \cdot \left((v^*)'(s^*) + v(s^*) \right).$$

The next lemma gives the solution of (3.3):

LEMMA 3.4. The solution of (3.3) is obtained by

$$v(x) = -2 \sum_{k=1}^{\infty} \frac{\sin(k\pi s^*)}{(k\pi)^2 + (c_1 + b)^2 + \lambda} \sin(k\pi x).$$

Proof. If we take a Fourier sine transformation of (3.3) then we obtain

$$\int_0^1 \left(\frac{d^2 v}{dx^2} - ((c_1 + b)^2 + \lambda)v \sin(k\pi x) \right) dx = c_1 \int_0^1 \delta_{s^*} \sin(k\pi x) dx.$$

Let $c(k)$ be the Fourier coefficient of v , given by

$$c(k) = \int_0^1 v(x) \sin(k\pi x) dx$$

Thus, we obtain the solution of (3.3)

$$\begin{aligned} v(x) &= c(0) + 2 \sum_{k=1}^{\infty} c(k) \sin(k\pi x) \\ &= -2c_1 \sum_{k=1}^{\infty} \frac{\sin(k\pi s^*)}{(k\pi)^2 + (c_1 + b)^2 + \lambda} \sin(k\pi x). \end{aligned}$$

□

We now show $(0, s^*, \mu^*)$ is a Hopf point.

THEOREM 3.5. Assume that for $\mu^* \in \mathbf{R} \setminus \{0\}$ the operator $-\tilde{A} + \mu^* B$ has a unique pair $\{\pm i\beta\}$ of purely imaginary eigenvalues. Then $(0, s^*, \mu^*)$ is a Hopf point for (\mathbf{R}) .

Proof. The equation (3.4) may be written by

$$(3.5) \quad \frac{\lambda}{\mu} = (v^*)'(s^*) - c_1 G_\lambda(s^*, s^*).$$

where $-G_\lambda(x, s^*)$ is a Green's function of $-\frac{d^2}{dx^2} + (c_1 + b)^2 + \lambda$ satisfying the Dirichlet boundary condition in (3.3). If we take a derivative the equation (3.5) with respect to μ and evaluate at μ^* , then we have

$$(3.6) \quad \lambda'(\mu^*) \left(\frac{1}{\mu^*} + c_1 \frac{d}{d\mu} G_\beta(s^*, s^*) \right) = \frac{i\beta}{(\mu^*)^2}.$$

We now calculate the real part of $\frac{d\lambda}{d\mu}(\mu^*)$. Let

$$\begin{aligned} a &= \operatorname{Re} \left(\frac{d\lambda}{d\mu}(\mu^*) \right), & b &= \operatorname{Im} \left(\frac{d\lambda}{d\mu}(\mu^*) \right), \\ c &= \operatorname{Re} \left(\frac{d}{d\lambda} G_\beta(s^*, s^*) \right) \text{ and } d = \operatorname{Im} \left(\frac{d}{d\lambda} G_\beta(s^*, s^*) \right). \end{aligned}$$

Substituting these values into (3.6), we obtain

$$(a + ib) \left(\frac{1}{\mu^*} + c_1(c + id) \right) = \frac{i\beta}{(\mu^*)^2}.$$

The real part a of $\frac{d\lambda}{d\mu}(\mu^*)$ is given by

$$a = \operatorname{Re} \left(\frac{d\lambda}{d\mu}(\mu^*) \right) = \frac{\frac{\beta c_1 d}{(\mu^*)^2}}{\left(\frac{1}{\mu^*} + c_1 c \right)^2 + (c_1 d)^2}$$

and

$$\begin{aligned} d &= \operatorname{Im} \left(\frac{d}{d\lambda} G_\beta(s^*, s^*)(\mu^*) \right) \\ &= 4\beta \sum_{k=1}^{\infty} \frac{(k\pi)^2 + (c_1 + b)^2}{((k\pi)^2 + (c_1 + b)^2)^2 + \beta^2} (\sin k\pi s^*)^2. \end{aligned}$$

Therefore, we have the transversality condition

$$\operatorname{Re} \left(\frac{d\lambda}{d\mu}(\mu^*) \right) > 0.$$

Therefore, by the Hopf-bifurcation theorem in [5], there exists a family of periodic solutions which bifurcates from the stationary solution as μ passes μ^* . \square

We now show the existence and uniqueness of pure imaginary eigenvalues, and the critical point μ^* .

THEOREM 3.6. *There exists a unique, purely imaginary eigenvalue $\lambda = i\beta$ of (3.2) with $\beta > 0$ for a unique critical point μ^* in order for $(0, s^*, \mu^*)$ to be a Hopf point with $\frac{1}{3} < s^* < 1$.*

Proof. We need to find a point μ such that the linearized eigenvalue problem (3.3) has a pair of pure imaginary complex conjugate eigenvalues. Letting $\operatorname{Re} \lambda = 0$ and $\operatorname{Im} \lambda = \beta > 0$ in (3.4), we obtain the real part

$$(3.7) \quad (v^*)'(s^*) - c_1 \operatorname{Re} G_\beta(s^*, s^*) = 0.$$

The imaginary part is

$$(3.8) \quad \beta + \mu c_1 \operatorname{Im} G_\beta(s^*, s^*) = 0.$$

We have to check that the equation (3.7) has a solution for β . So define

$$T(\beta) := (v^*)'(s^*) - c_1 \operatorname{Re} G_\beta(s^*, s^*)$$

then $T(\beta)$ is a strictly increasing continuous function of β^2 . Furthermore,

$$\lim_{\beta \rightarrow \infty} T(\beta) = \frac{\cosh \sqrt{c_1 + b} s^* (\cosh(\sqrt{c_1 + b}(1 - s^*)) - 1)}{\sqrt{c_1 + b} \sinh \sqrt{c_1 + b}} > 0$$

and

$$\begin{aligned} T(0) &= (v^*)'(s^*) - c_1 \operatorname{Re} G_0(s^*, s^*) \\ &= \gamma'(s^*) \\ &= \frac{2}{\sqrt{c_1 + b} \sinh \sqrt{c_1 + b}} \sinh \sqrt{c_1 + b} \left(\frac{3s^* - 1}{2} \right) \sinh \sqrt{c_1 + b} \left(\frac{s^* - 1}{2} \right). \end{aligned}$$

In order to have a solution of β , $T(0)$ must be negative. Thus there is a pair of pure imaginary complex conjugate eigenvalues if $\frac{1}{3} < s^* < 1$.

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Thus, by the intermediate value theorem, there exists a unique point β , $0 < \beta < \infty$ such that $T(\beta) = 0$ for $\frac{1}{3} < s^* < 1$. The corresponding value μ can be found by substituting β into the equation (3.8)

$$\frac{1}{\mu^*} = 2c_1 \sum_{k=1}^{\infty} \frac{(c_1 + b)^2 + (k\pi)^2}{((c_1 + b)^2 + (k\pi^2))^2 + \beta^2} (\sin k\pi s^*)^2.$$

Therefore there exists a unique μ^* such that $\lambda(\mu^*) = \pm i \operatorname{Im} \lambda(\mu^*) = \pm i \beta$. □

The following theorem summarizes what we have proved for the free boundary problem with the Dirichlet boundary condition:

THEOREM 3.7. *Assume that $0 < \frac{c_1 - 2a}{2(c_1 + c_2)} < \frac{2 \sinh^3 \frac{c_1 + b}{3}}{(c_1 + b)^2 \sinh(c_1 + b)}$ with $0 < a < c_1 < 1$, so that (R), respectively (F), has a unique stationary solution (u^*, s^*) where $u^* = 0$ and $\frac{1}{3} < s^* < 1$ respectively (v^*, s^*) , for all $\mu > 0$. Then there exists a unique $\mu^* \geq 1$ such that the linearization $-\tilde{A} + \mu^* B$ has a purely imaginary pair of eigenvalues. The point $(0, s^*, \mu^*)$ is then a Hopf point for (R) and there exists a C^0 -curve of nontrivial periodic orbits for (R), (F), respectively, bifurcating from $(0, s^*, \mu^*)$, (v^*, s^*, μ^*) , respectively.*

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