The Performance Improvement of a Multicarrier DS-CDMA System Using both Time-Diversity and Frequency Offset

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In this paper, we show the robustness of a proposed system using both a frequency diversity and a time diversity effect when the channel assumptions are relaxed such that all subbands are not subject to independent fading. We take time-repetition coding with interleaving such that the repetition code symbols are spaced sufficiently far apart to guarantee time diversity. We also incorporate offset multicarriers into a proposed multicarrier DS-CDMA system in a different way compared to that of [10] to reduce the total multiple access interference. When we take a convolutional encoder instead of a time-repetition encoder, and again use both time diversity and frequency offset, then we can obtain the significant performance gain.

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I. INTRODUCTION

The current literature on multicarrier (MC) modulation as applied to DS-CDMA may be classified into two general areas, depending upon whether time domain or frequency domain spreading is employed. In the former case (often called MC DS-CDMA), transmitted symbols are multiplied by low-rate spreading sequences in time, yielding conventional, narrowband DS waveforms. The complete DS-CDMA waveforms are then transmitted at different carrier frequencies [1]–[4], such that the net bandwidth allocation is equal to that of a single carrier DS-CDMA system using a higher rate spreading sequence. In the latter case (called MC-CDMA), however, the spreading sequence is serial-to-parallel converted such that each chip modulates a different carrier frequency, and thus, the data symbol is transmitted in parallel [5]–[8]. Such systems are commonly referred to as orthogonal frequency division multiplex CDMA (OFDM-CDMA) systems.

Both classes of MC modulation systems show a similar capability to mitigate the effects of fading, however, the time spreading class will, in general, employ a smaller number of carriers relative to the frequency spreading class, and thus, be lower in complexity. The system proposed in this paper belongs to the time spreading class of systems.

Multicarrier DS-CDMA is an effective approach to combat fading and various kinds of interference. In [1], Kondo and Milstein proposed a scheme where a data sequence multiplied by a spreading sequence modulates disjoint multiple carriers. The receiver provides a correlator for each carrier, and the outputs of the correlators are combined with a maximal-ratio combiner. Bandlimited spreading waveforms are used to prevent self-interference, and system performance is evaluated in the presence of partial-band interference over an assumed slowly fading Rayleigh channel, under the assumption that all subbands are subject to independent fading. In [9], Xu and Milstein analyzed the effect of correlation among the subcarriers on the performance of the system of [1]. In [10], Han, Milstein and Kim proposed an offset multicarrier DS-CDMA system, where a frequency offset between two multicarrier systems is employed. They show that the offset multicarrier DS-CDMA system outperforms the conventional multicarrier DS-CDMA system. In [2], Rowitch and Milstein presented a multicarrier asynchronous DS-CDMA system wherein the output of a convolutional encoder modulates multiple bandlimited DS-CDMA waveforms which are transmitted in parallel at different carrier frequencies. Compared to the performance of a conventional single carrier DS-CDMA system with a rake receiver, their system demonstrated similar performance at roughly equal receiver complexity in the absence of narrowband interference, and superior performance of the multicarrier system in the presence of narrowband interference. In this paper, we will show the robustness of a proposed system using both a frequency diversity and a time diversity effect when the channel assumptions are relaxed such that all subbands are not subject to independent fading. Note that interleaving is required to achieve the time diversity. We also incorporate offset multicarriers into a proposed multicarrier DS-CDMA system in a different way compared to that of [10] to reduce the total multiple access interference. There are two frequency bands (A and B), and each band consists of M disjoint subbands. Also, a frequency offset, f_{os}, is employed between frequency band A and frequency band B. We use either a rate 1/2 convolutional encoder or a rate 1/2 time-repetition encoder, together with an interleaver, in a similar manner to [2], so that we can achieve both time diversity and frequency diversity.

II. SYSTEM MODEL

Figure 1(a) depicts a bandlimited DS waveform of a wideband single carrier system in the frequency domain, where the bandwidth, BW_S, is given by

$$BW_S = (1+\beta)\frac{1}{T_{c1}}. \tag{1}$$

In (1), β is the rolloff factor of the chip wave-shaping filter ($0 < \beta \le 1$), and T_{c1} is the chip duration of the single carrier DS system. In the conventional multicarrier DS systems, as shown in Fig. 1(b), the overall bandwidth is the same as that of the bandlimited single carrier DS system, and the entire spectrum is divided into M equi-width disjoint frequency bands. The bandwidth of each frequency subband, BW_M, is given by

$$BW_M = \frac{BW_S}{M} = (1+\beta)\frac{1}{T_{c,m}}, \tag{2}$$

where $T_{c,m}$ is the chip duration of the multicarrier system, and M is the total number of subcarriers. For ease of terminology, we refer to the conventional multicarrier system and the proposed multicarrier system as the MC system and the proposed system, respectively. In the proposed system, the entire spectral band is covered by two overlapping frequency bands (A and B) as shown in Fig. 1(c). Each user uses both bands (A and B), and each band consists of M disjoint subbands. We employ a frequency offset of f_{os} between band A and band B. Note that any given user sends each symbol through both band A and band B, but at different times after interleaving to guarantee time diversity. Let N_M and N denote the spreading gain of the MC and proposed systems, respectively. They are related by

$$\frac{N_M}{N} = \frac{T_c}{T_{c,m}} = \frac{BW_M}{BW_o} = 1 + \frac{R}{M}, \tag{3}$$

where T_c and BW_o are the chip duration and the bandwidth of each subband, respectively, for the proposed system, and $R = f_{os} / BW_o$ is the frequency offset-to-bandwidth ratio. We will consider a raised-cosine wave-shaping filter whose magnitude-squared transfer function, $|H(f)|^2 (\equiv X(f))$, is given by [12]

$$X(f) = \begin{cases} \frac{1}{W}, & |f| \le \frac{W}{2}(1-\beta) \\ \frac{1}{2W}\{1-\sin[\frac{1}{2\beta}(\frac{2\pi|f|}{W}-\pi)]\}, & \frac{W}{2}(1-\beta) \le |f| \le \frac{W}{2}(1+\beta) \\ 0, & \text{elsewhere}, \end{cases} \tag{4}$$

where W is the chip rate. Then the bandwidth of each subband is given by $(1+\beta)W$. For a given bandwidth, the spreading gain decreases as β increases, so that

$$N_M = \frac{N_1}{1+\beta}\frac{1}{M}, \tag{5}$$

and

$$N = \frac{N_1}{1+\beta}\frac{1}{R+M}, \tag{6}$$

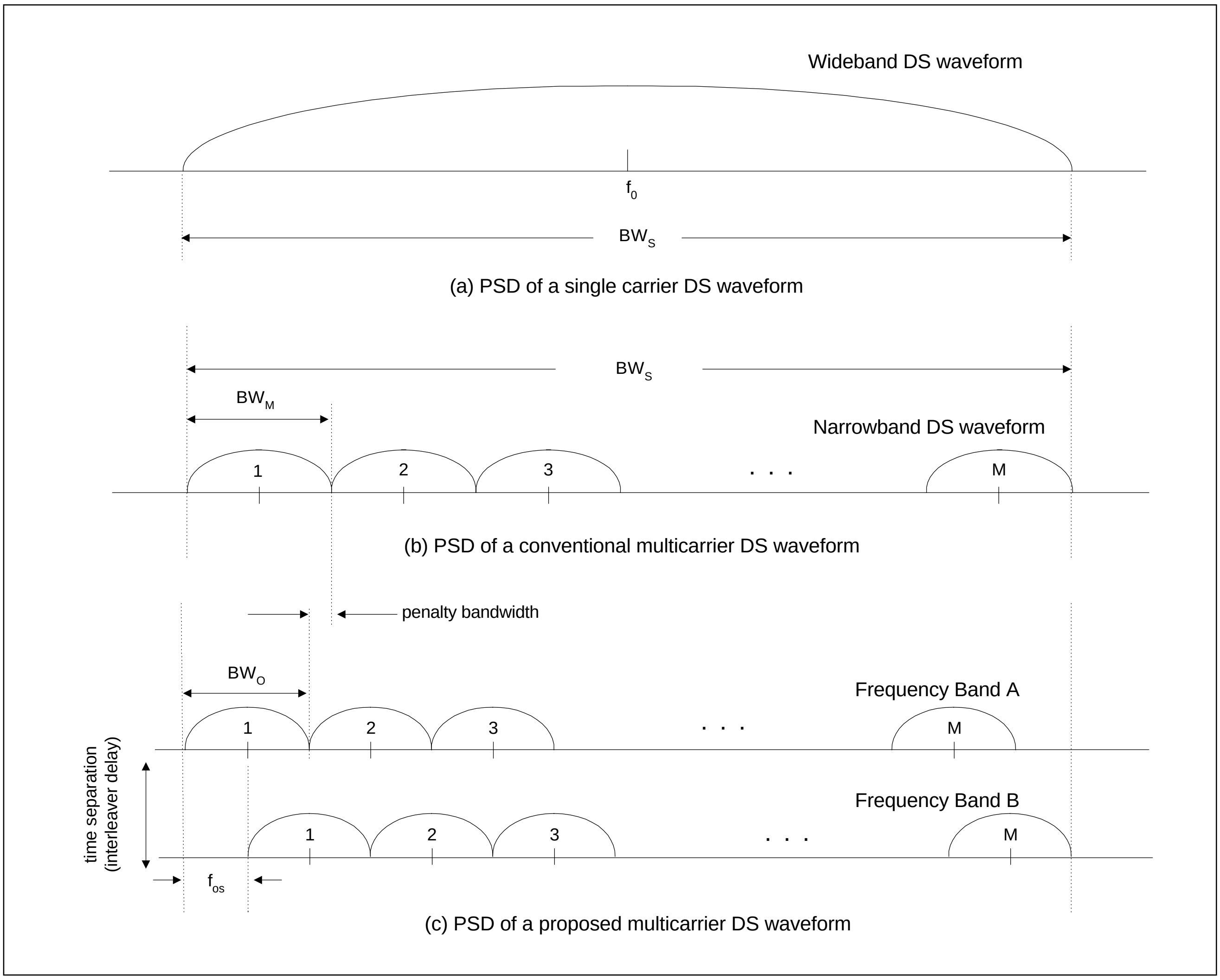

Fig. 1. Power spectral density of DS waveforms.

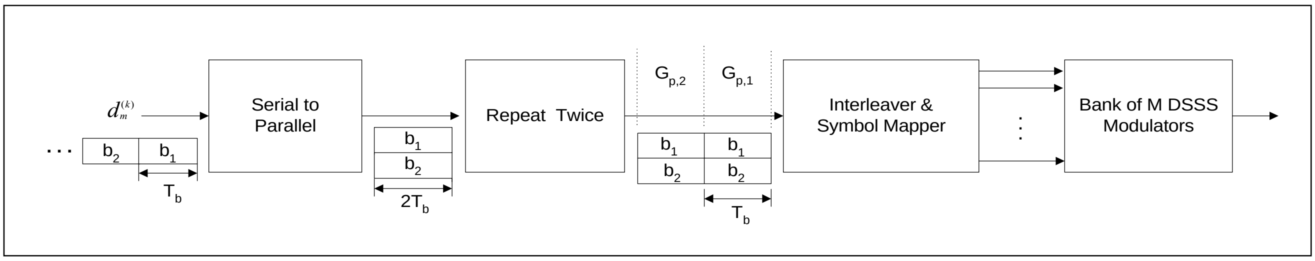

Fig. 2. Multicarrier CDMA transmitter (Q=2 Case).

where N_1 is the spreading gain in the single carrier DS-CDMA system when $\beta = 0$.

1. Transmitter

Figure 2 shows the transmitter for the k_{th} user, and Fig. 3 shows the corresponding code symbol to subcarrier mapping. The number of subcarrier slots in each frequency band per coded symbol will be *M*/2, so that, in every bit interval, two different data symbols are transmitted at a time by *M* subcarriers in either band A or band B. In Fig. 2, d_m^k, $m \equiv \lfloor n/N \rfloor$, is a random binary sequence representing the input data. The binary data sequence with bit duration T_b is serial-to-parallel converted into Q parallel streams. The symbol duration on each stream is

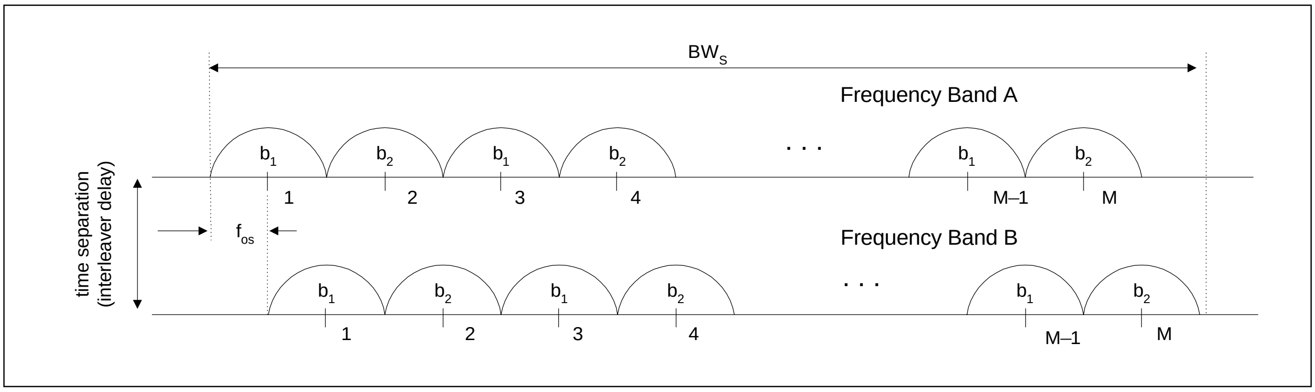

Fig. 3. Example code symbol to subcarrier mapping for the transmitter of Fig. 2.

QT_b. Next, each of the Q symbols is repeated Q times, simultaneously, such that the new symbol duration on each stream is T_b. We incorporate interleaving such that the repetition code symbols are spaced sufficiently far apart (greater than the coherence time) to guarantee time diversity. As a practical matter, we might set $Q = 2$ to minimize the increase in required interleaver delay, and that is what is shown in Fig. 2. Assuming that Q is indeed equal to 2, we let $G_{p,q}$ denote the q_{th} symbol group, where q equals one or two, corresponding to the p_{th} input data group, which consists of two consecutive data bits for each $p(=1, 2, \cdots)$. For example, the first 2 code symbols, $\{b_{i,m}^{(k)}\}_{i=1}^{2}$, initially form $G_{1,1}$, and each of two symbols is transmitted over $M/2$ subcarrier slots in band A. At a later time, these same two symbols constitute $G_{1,2}$, and they are transmitted in band B. We refer to the frequency mapping as an inner-repetition code, to distinguish it the previous repetition code in the time domain, referred to as an outer-repetition code. The two parallel code symbols are mapped to subcarriers alternately so as to maximize the separation in frequency between inner-repetition code symbols for each symbol in $G_{p,1}$. Let $\{f_v\}_{v=1}^{M}$ be an ordered set of carrier frequencies, such that $f_v < f_{v+1}$. We specify the mapping of the 2 parallel code symbols, $b_{i,m}^{(k)}, 1 \le i \le 2$, into the M output symbols, $a_{v,m}^{(k)}$, as follows:

$$a_{v,m}^{(k)} = b_{f(v),m}^{(k)}, \qquad m \equiv \lfloor n/N \rfloor, \tag{7}$$

where

$$f(v) = 1 + \{v-1, \bmod 2\}, \quad 1 \le v \le M. \tag{8}$$

It is important to emphasize that, after the time interval to guarantee time diversity, the two parallel code symbols in $G_{p,2}$ are transmitted simultaneously in the same manner as the symbols in $G_{p,1}$ except that the band B of the symbols in $G_{p,2}$ is offset with respect to the band A of the symbols in $G_{p,1}$ to reduce the total multiple access interference.

2. Channel

The multipath channel has been extensively measured and analyzed in the past. The channel model used here is from [11], [12]. The channel is assumed to be a slowly varying, frequency selective, Rayleigh fading channel with delay spread of T_m, as in [1]. We choose M such that each subband of the MC system has no selectivity, i. e., $T_m / T_{c,m} \le 1$.

We define the coherence bandwidth, $(\Delta f)_c$, as that frequency separation for which the magnitude of envelope correlation coefficient in (9), for $\tau = 0$, first drops below 0.5 [11], i.e.,

$$\rho^{(k)}(\Delta f, \tau) = \frac{J_0^2(\chi V \tau)}{1 + (\Delta f)^2 T_m^2} \tag{9}$$

where Δf and τ are the frequency separation and the time separation, respectively, and J_0 is the zero-order Bessel function of the first kind [13]. In (9), V stands for a vehicle speed, and $\chi = 2\pi/\lambda$, where λ is the wavelength. It is easy to see that the correlation function used in [9] falls out as a special case of (9) for τ set to zero (no time separation). Let δ be the subcarrier bandwidth of the MC system normalized by the coherence bandwidth, that is $\delta = BW_M / (\Delta f)_c$. Then we have

$$BW_o / (\Delta f)_c = \delta / (1 + R/M). \tag{10}$$

However, since the transmitted repetition symbols are separated by $2BW_o$, we can make all subbands fade independently as long as $2BW_o \ge (\Delta f)_c$ (or $\delta \ge (1 + R/M)/2$). This result may be of practical interest when channel assumptions are relaxed such that the coherence bandwidth varies with time.

The transfer function of the v_{th} frequency band for the k_{th} user is given by $\zeta_{k,v} = a_{k,v}\, e^{j\beta_{k,v}}$, where, under the assumption that all subbands are subject to independent fading, $\alpha_{k,v}$ and $\beta_{k,v}$ are respectively, an i.i.d. (independent, identically distributed) Rayleigh random variable with a unit second moment, and an

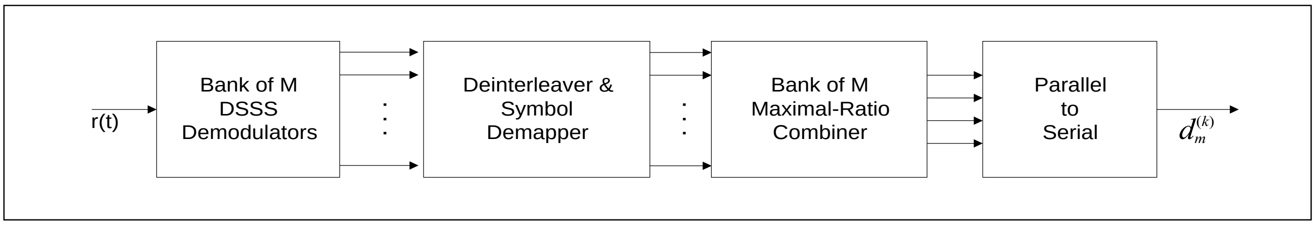

Fig. 4. Multicarrier CDMA receiver.

i.i.d. uniform random variable over $[0, 2\pi)$. However, when there is correlation in the fading of the various subcarriers, $\{\zeta_{k,v}, 1 \le v \le M\}$ are identical, but, not necessarily independent, zero-mean, complex Gaussian variables; $\alpha_{k,v}$ and $\beta_{k,v}$ are, respectively, a Rayleigh random variable with a unit second moment, and a uniform random variable over $[0, 2\pi)$. For either case, the received signal may be represented by

$$\gamma(t) = \sum_{k=1}^{K_1} \{\sqrt{2E_c} \sum_{n=-\infty}^{\infty} c_n^{(k)} h(t - nT_c - \tau_k) \cdot \sum_{v=1}^{M} \alpha_{k,v}\, a_{v,m}^{(k)} \cos(2\pi f_v t + \theta_{k,v})\}$$
$$+ \sum_{k=Ku-K_2+1}^{Ku} \{\sqrt{2E_c} \sum_{n=-\infty}^{\infty} c_n^{(k)} h(t - nT_c - \tau_k) \cdot \sum_{v=1}^{M} \alpha_{k,v}\, a_{v,m}^{(k)} \cos(2\pi (f_v + f_{os}) t + \theta_{k,v})\} + n_w(t), \quad (11)$$

where E_c is the energy per chip, $h(t)$ is the impulse response of the chip wave-shaping filter, $H(f)$, and $n_w(t)$ is AWGN with two-sided power spectral density of $\eta_0/2$. In (11), K_1 and K_2 stand for the number of users who are using band A and band B, respectively, to transmit data at an arbitrary time t, and $K_1 + K_2 = K_u$, where K_u is the total number of users. The $\{\tau_k\}$ are asynchronous delays assumed to be i.i.d. uniform random variables over $[0, T_c)$, and $\theta_{k,v} = \psi_{k,v} + \beta_{k,v}$, where $\psi_{k,v}$ is a random phase uniformly distributed over $[0, 2\pi)$ due to the transmitter.

3. Receiver

The receiver of the k_{th} user is shown in Fig. 4. Each demodulator (correlator) consists of a bandpass matched filter operation, followed by coherent demodulation, sampling, despreading and summing (from $n' = 0$ to $N-1$). After deinterleaving, the $2M$ receiver test statistics are then demapped into 2 partitions of M terms in accordance with the mapping rule of Fig. 3. Each partition is input to a maximal-ratio combiner producing a diversity combined test statistic.

III. ANALYSIS

1. Output of the Chip Matched Filter

We evaluate the performance of the first user $(k = 1)$, assuming perfect carrier, code, and bit synchronization. Without loss of generality, we assume that the desired user transmits data through frequency band A. Then the output, prior to sampling, from the correlator branch at the v_{th} subcarrier frequency, $y_v(t)$, is given by

$$y_v(t) = S_{y_v}(t) + I_{y_v}^A(t) + I_{y_v}^B(t) + N_{y_v}(t), \quad (12)$$

where

$$S_{y_v}(t) = \sqrt{E_c}\,\alpha_{1,v} \sum_{n=-\infty}^{\infty} a_{v,m}^{(1)} c_n^{(1)} x(t - nT_c) \quad (13)$$

$$I_{y_v}^A(t) = \sum_{k=2}^{K_1} \{\sqrt{E_c} \sum_{n=-\infty}^{\infty} c_n^{(k)} \alpha_{k,v} x(t - nT_c - \tau_k) a_{v,m}^{(k)} \cos\phi_{k,v}\} \quad (14)$$

$$I_{y_v}^B(t) = \sum_{k=K_u-K_2+1}^{K_u} \Bigg\{ \sqrt{E_c} \sum_{n=-\infty}^{\infty} c_n^{(k)} \alpha_{k,v} a_{v,m}^{(k)} \Big\{ [\{h(t-nT_c-\tau_k) \cdot \cos(2\pi f_{os} t)\} * h(-t)] \cdot \cos\phi_{k,v} - [\{h(t-nT_c-\tau_k) \cdot \sin(2\pi f_{os} t)\} * h(-t)] \cdot \sin\phi_{k,v} \Big\} \Bigg\}, \quad \text{for } v=1 \tag{15a}$$

$$I_{y_v}^B(t) = \sum_{k=K_u-K_2+1}^{K_u} \Bigg\{ \sqrt{E_c} \sum_{n=-\infty}^{\infty} c_n^{(k)} \cdot \Big[[\{h(t-nT_c-\tau_k)\cos(2\pi f_{os} t)\} * h(-t)] \cdot \{\alpha_{k,v} a_{v,m}^{(k)} \cos\phi_{k,v} + \alpha_{k,v-1} a_{v-1,m}^{(k)} \cos\phi_{k,v-1}\} - [\{h(t-nT_c-\tau_k)\sin(2\pi f_{os} t)\} * h(-t)] \cdot \{\alpha_{k,v} a_{v,m}^{(k)} \sin\phi_{k,v} - \alpha_{k,v-1} a_{v-1,m}^{(k)} \sin\phi_{k,v-1}\} \Big] \Bigg\}, \quad \text{for } v \neq 1, \tag{15b}$$

$$N_{y_v}(t) = Lp\{n_{w_v}(t)\sqrt{2}\cos(2\pi f_v t + \theta_{1,v})\}, \tag{16}$$

and where $\phi_{k,v} \equiv \theta_{k,v} - \theta_{1,v}$. $Lp\{\ \}$ represents a lowpass filtering operation, and $n_{w_v}(t)$ is the AWGN term at the v_{th} matched filter output. Note that $S_{y_v}(t)$ is the desired signal term, $I_{y_v}^A(t)$ and $I_{y_v}^B(t)$ represent the MAI in band A and band B, respectively, and $N_{y_v}(t)$ is the filtered AWGN.

2. Signal out of the v_{th} Correlator and its Statistics

We evaluate the statistics of the signal out of the v_{th} correlator:

$$Z_v = S_{Z_v} + I_{Z_v}^A + I_{Z_v}^B + N_{Z_v}, \tag{17}$$

where

$$S_{Z_v} = \sum_{n'=0}^{N-1} c_{n'}^{(1)} S_{y_v}(n'T_c), \quad I_{Z_v}^A = \sum_{n'=0}^{N-1} c_{n'}^{(1)} I_{y_v}^A(n'T_c)$$
$$I_{Z_v}^B = \sum_{n'=0}^{N-1} c_{n'}^{(1)} I_{y_v}^B(n'T_c), \quad N_{Z_v} = \sum_{n'=0}^{N-1} c_{n'}^{(1)} N_{y_v}(n'T_c). \tag{18}$$

The conditional mean of Z_v, conditioned on $\alpha_{1,v}$ and the code symbol, $\alpha_{v,m}^{(1)}$, can be obtained as

$$E[Z_v | \alpha_{1,v}, a_{v,m}^{(1)}] = \pm N\sqrt{E_c}\alpha_{1,v}, \tag{19}$$

as shown in [1]. The conditional variance of Z_v, conditioned on $\alpha_{1,v}$, is given by

$$Var[Z_v | \alpha_{1,v}] \equiv \sigma_v^2 = Var[I_{Z_v}^A | \alpha_{1,v}] + Var[I_{Z_v}^B | \alpha_{1,v}] + Var[N_{Z_v} | \alpha_{1,v}] \tag{20}$$

The conditional variance for the MAI components can be represented by

$$Var[I_{Z_v}^A | \alpha_{1,v}] \cong NR_{I_{y_v}^A}(0) = \frac{NE_c(K_1-1)}{2}\Lambda^A(\beta) \tag{21}$$

and

$$Var[I_{Z_v}^B | \alpha_{1,v}] \cong NR_{I_{y_v}^B}(0) = \frac{NE_c K_2}{2}\Lambda^B(\beta) \tag{22}$$

where

$$\Lambda^A(\beta) = \frac{1}{T_c}\int_{-\infty}^{\infty} |X(f)|^2 df, \tag{23}$$

$$\Lambda^{B}(\beta)=\begin{cases}\frac{1}{2T_c}\int_{-\infty}^{\infty}[|H(f)|^2\{|H(f-f_{os})|^2+|H(f+f_{os})|^2\}]df, & \text{for } v=1\\ \frac{1}{T_c}\int_{-\infty}^{\infty}[|H(f)|^2\{|H(f-f_{os})|^2+|H(f+f_{os})|^2\}]df, & \text{for } v\neq 1\end{cases} \tag{24}$$

In (21), $R_{I_{y_v}^A}(\tau)$ is the autocorrelation function of $I_{y_v}^A(t)$, and is defined as the inverse Fourier transform of the power spectral density of $I_{y_v}^A(t)$,

$$S_{I_{y_v}^A}(f)=\frac{E_c(K_1-1)}{2T_c}|X(f)|^2. \tag{25}$$

In (22), $R_{I_{y_v}^B}(\tau)$ is the autocorrelation function of $I_{y_v}^B(t)$, and is defined as the inverse Fourier transform of the power spectral density of $I_{y_v}^B(t)$,

$$S_{I_{y_v}^B}(f)=\begin{cases}\frac{E_cK_2}{4T_c}[|H(f)|^2\{|H(f-f_{os})|^2+|H(f+f_{os})|^2\}], & \text{for } v=1\\ \frac{E_cK_2}{2T_c}[|H(f)|^2\{|H(f-f_{os})|^2+|H(f+f_{os})|^2\}], & \text{for } v\neq 1\end{cases} \tag{26}$$

The conditional variance for the AWGN component can be represented by

$$Var[N_{Z_v}|\alpha_{1,v}]\cong NR_{N_{y_v}}(0)=\frac{N\eta_0}{2} \tag{27}$$

where $R_{N_{y_v}}(\tau)=\frac{\eta_0}{2}\cdot x(\tau)$ is the autocorrelation function of $N_{y_v}(t)$.

Since neither of the variance expressions depend on the fade amplitude, $\alpha_{1,v}$, the variance in (20) is unconditional. We now wish to consider the joint statistics of the inputs to a given maximal-ratio combiner. To do this, we must define the mechanism by which the symbol demapper partitions the $2M$ correlator outputs, $\{Z_v^{(q=1)}, Z_v^{(q=2)}\}_{v=1}^{M}$, into 2 partitions of M terms. Let $Z_{i,r}$ denote the correlator output corresponding to the r_{th} repetition of the i_{th} code symbol, where $1\le i\le 2$ and $1\le r\le M/2$. Then, consequent on the mapping shown in Fig. 3, we have for each i and j

$$Z_{i,j}=Z_{v(i,j)},\quad j=r+(M/2)(q-1) \tag{28}$$

where the function $v(i,j)$ maps the indices i and j into the appropriate frequency subband index $1\le v\le 2M$ and is given by

$$v(i,j)=i+(j-1)2. \tag{29}$$

Define the test statistic for the maximal-ratio combiner corresponding to the i_{th} code symbol as

$$Z_i=\sum_{j=1}^{M}g_{i,j}Z_{i,j}, \tag{30}$$

where the optimal gain coefficients, $g_{i,j}$, are selected according to

$$g_{i,j}=\frac{E[Z_{i,j}\mid\alpha_{1,v(i,j)}]}{Var[Z_{i,j}\mid\alpha_{1,v(i,j)}]}, \tag{31}$$

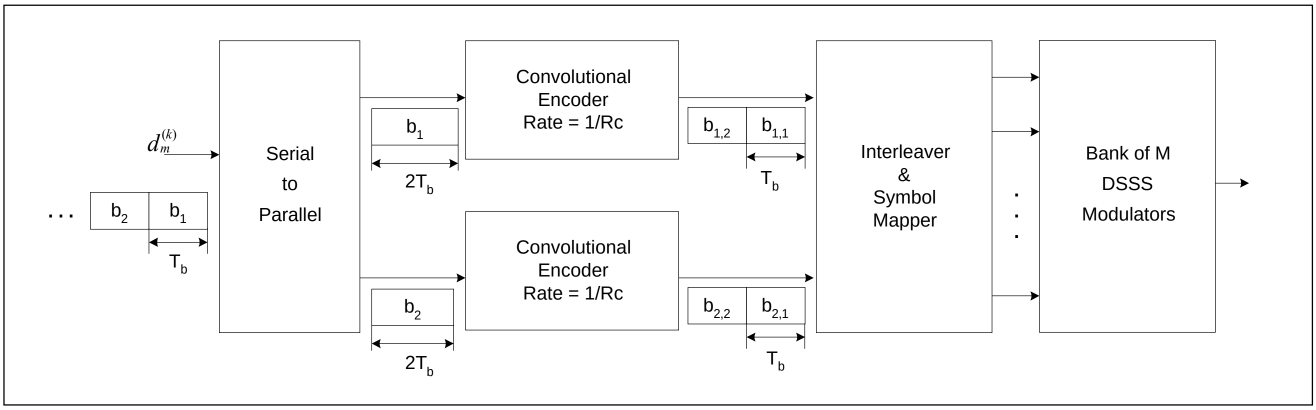

Fig. 5. Multicarrier CDMA transmitter with a convolutional encoder (R_c = 2).

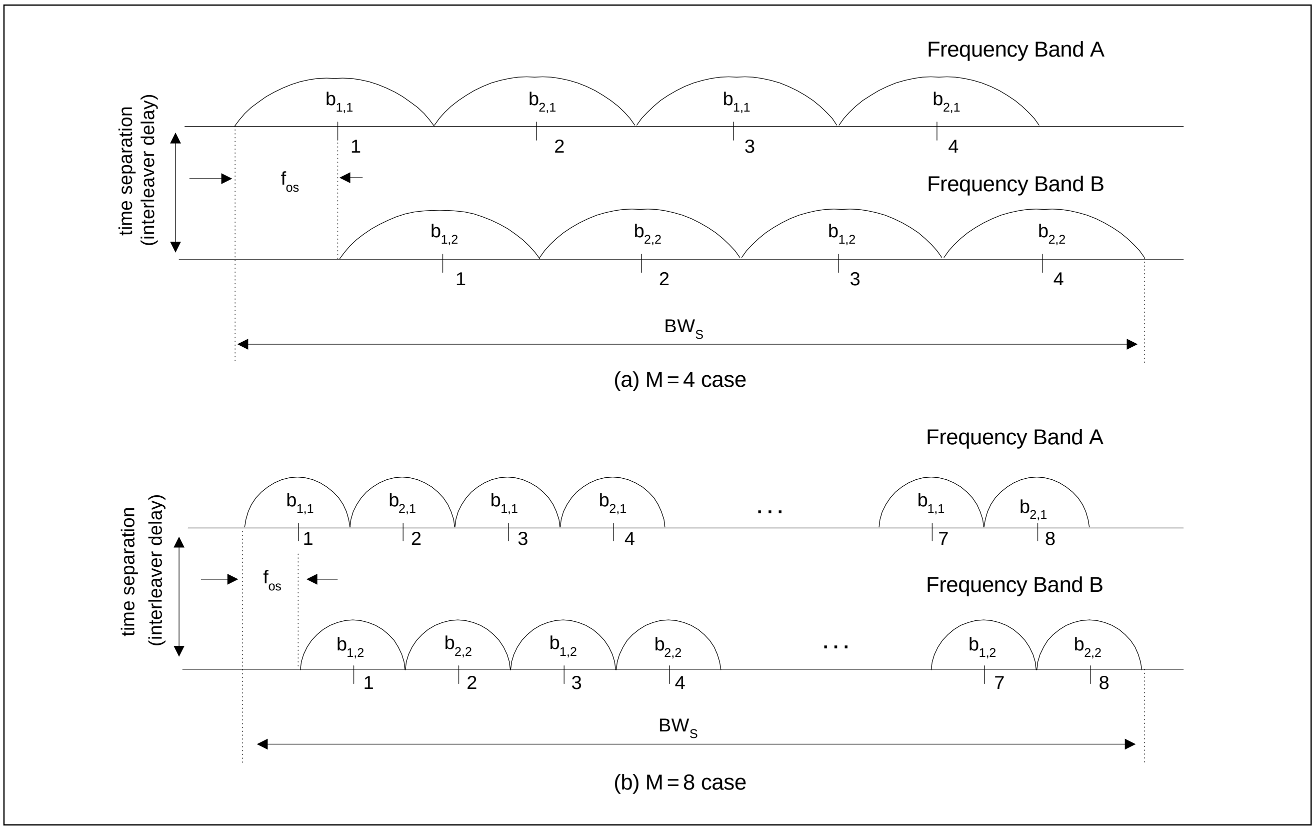

Fig. 6. Example code symbol to subcarrier mapping for the transmitter of Fig. 5.

as shown in [1]. We assume that the $\{\alpha_{1,v(i,j)}\}$ are varying sufficiently slowly so that the conditional mean and variance of $Z_{i,j}$ can be accurately estimated. Then the resulting SNR of Z_i, $1 \le i \le 2$, conditioned on $\alpha_{1,v}$, is given by

$$\Gamma_i = \sum_{j=1}^{M} \frac{E^2[Z_{i,j} \mid \alpha_{1,v}]}{Var[Z_{i,j} \mid \alpha_{1,v}]} = N^2 E_c G \tag{32a}$$

where

$$G = \sum_{j=1}^{M} \frac{\alpha_{1,v}^2}{\sigma_v^2}. \tag{32b}$$

When all subbands are subject to independent fading, (33) can be used to obtain the probability of bit error as [1]:

$$P_e = \int_0^\infty Q\left(-\sqrt{N^2 E_c G}\ \right) f_G(g)\, dg, \tag{33a}$$

where

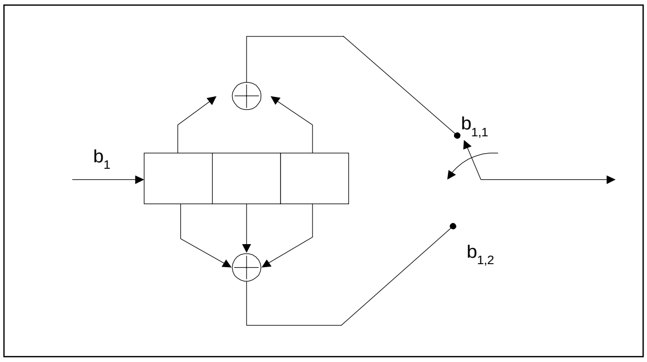

Fig. 7. Convolutional Encoder (R_c=2, K=3).

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt \qquad (33b)$$

and $f_G(g)$ is the probability density function of the random variable G in (32b). When there is correlation in the fading of the various subcarriers, we use (34) to obtain the probability of bit error as [9]:

$$P_e = \int_0^\infty Q(-\sqrt{\gamma}) f_\Gamma(\gamma) d\gamma = \frac{1}{2}(1 - \sum_{j=1}^{M} d_j \lambda_j \sqrt{\frac{\lambda_j}{1+\lambda_j}}), \qquad (34a)$$

where

$$d_j = \lambda_j^{M-2} \prod_{p=1,\, p\neq j}^{M} (\lambda_j - \lambda_p)^{-1}, \qquad (34b)$$

and where $\{\lambda_j\}$ are the eigenvalues of correlation matrix $\boldsymbol{R}$, which can be obtained from [9], [11].

Next we analyze the performance of the proposed system after replacing time repetition coding by convolutional coding. Figure 5 shows the new transmitter with a rate 1/2 convolutional encoder for the k_{th} user, and Fig. 6 shows the corresponding code symbols to subcarrier mapping on the spectra of the new system. Figure 7 depicts a rate 1/2 convolutional encoder of constraint length 3. It can be shown that the transfer function for the example encoder in Fig. 7 is given by [12]

$$T(D_1, D_2, B) = \frac{D_1^2 D_2^3 B + D_1^4 D_2^2 B^2 - D_1^2 D_2^4 B^2}{1 - 2D_2 B - D_1^2 B^2 + D_2^2 B^2}. \qquad (35)$$

For an arbitrary rate $1/R_c$ convolutional encoder, the corresponding transfer function, $T(D_1, D_2, \cdots, D_{R_c}, B)$, may be used in conjunction with the Chernoff bound in [12], [14] to union bound the probability of bit error as [2], [14]

$$P_b < \frac{\partial T(D_1, D_2, \cdots, D_{R_c}, B)}{\partial B}\Big|_{B=1,\, D_i = P_i,\, i=1,2,\cdots,R_c}, \qquad (36)$$

where

$$P_j \equiv \prod_{i=1}^{M/2} \frac{1}{1+\bar{\gamma}_{i,j}}, \quad i = 1, 2, \cdots, R_c, \qquad (37)$$

and $\bar{\gamma}_{i,j}$ is the average signal-to-noise ratio of the $v(i, j)_{th}$ frequency subband, and can be represented as

$$\bar{\gamma}_{i,j} \equiv \frac{N^2 E_c}{2\sigma_{v(i,j)}^2} = \frac{N^2 E_c}{2Var[Z_{i,j} \mid \alpha_{1,v(i,j)}]}. \qquad (38)$$

IV. NUMERICAL RESULTS

(Comparison with the Conventional Multicarrier Systems)

The amount of MAI from a user transmitting through band A and band B is proportional to $\Lambda^A(\beta)$ and $\Lambda^B(\beta)$, respectively, which, for the raised-cosine filter characteristic of (4a), are given by

$$\Lambda^A(\beta) = 1 - \frac{\beta}{4}, \qquad (39)$$

and for $v \neq 1$,

$$\Lambda^B(\beta) = \begin{cases} 1-\beta, & 0 < \beta \le \frac{1}{3} \\ \frac{3-\beta}{4} - \frac{3\beta-1}{8}\sin\frac{\pi}{2\beta} + \frac{3\beta}{4\pi}\cos\frac{\pi}{2\beta}, & \frac{1}{3} \le \beta < 1 \end{cases} \qquad (40)$$

When all subbands are subject to independent fading (i.e., $2BW_o \ge (\Delta f)_c$), we can evaluate the performance of the proposed system using (33). In the case of correlated fading (i.e., $2BW_o < (\Delta f)_c$), we use (34) to evaluate the performance of the proposed system.

Figure 8 shows the BER versus E_b/η_0 of the proposed system compared with the MC system for several values of δ. The error-floor of the proposed system is smaller than that of the MC system, because the correlation between the transmitted repetition symbols of the proposed system is less, due to the interlacing of the subcarriers corresponding to the two symbols transmitted in parallel, than that of the symbols in the MC system, and the MAI is significantly reduced by the offset multicarrier scheme.

In Fig. 9, we can see that the performance of both the MC and the proposed systems can be improved by increasing the number of carriers M (diversity order). Also, the BER difference between the MC and the proposed system increases. This is because the spreading gain penalty in (6) becomes negligible as M increases.

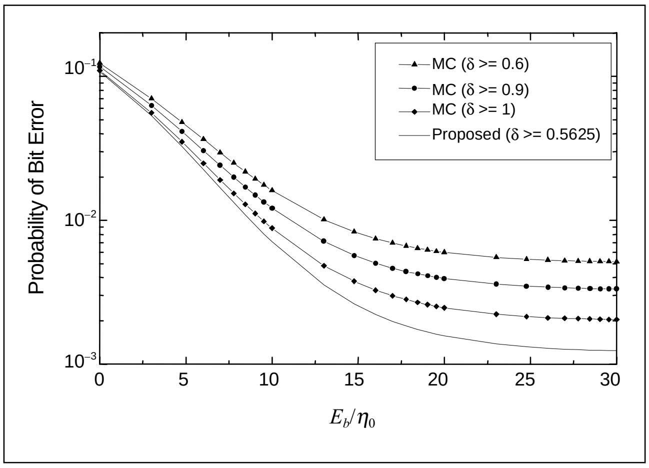

Fig. 8. BER vs. E_b/η_0 for M = 4, $K_1 = K_2 = 25$, $N_1 = 512$, and $\beta = 0.5$.

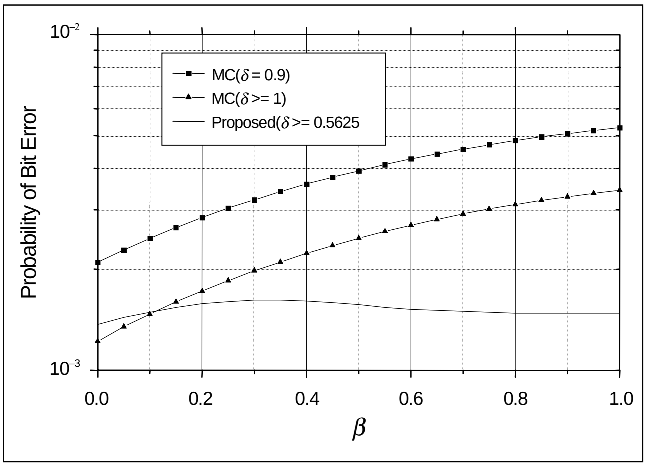

Fig. 10. BER vs. β for M = 4, $K_1 = K_2 = 25$, $N_1 = 512$, and E_b/η_0=20 [dB].

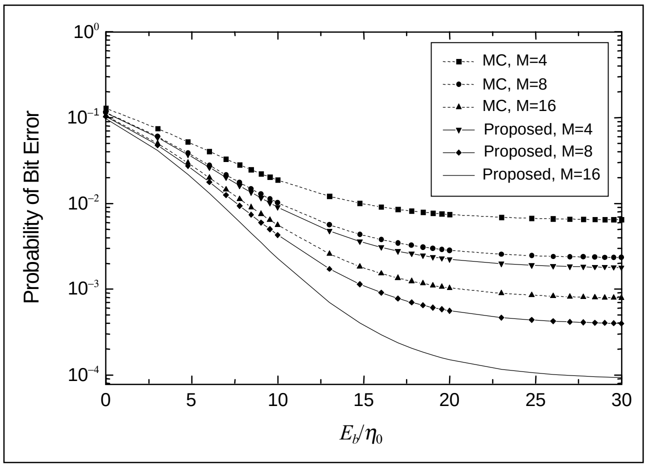

Fig. 9. BER vs. E_b/η_0 for $K_1 = K_2 = 25$, $N_1 = 512$, $\beta = 0.5$, $\delta = 0.5$, and M = 4, 8, 16.

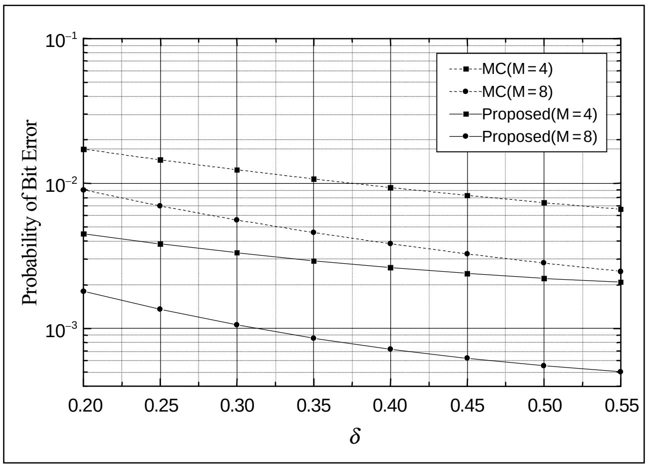

Fig. 11. BER vs. δ for $K_1 = K_2 = 25$, $N_1 = 512$, $\beta = 0.5$, and $E_b/\eta_0 = 20$[dB].

The effect of the rolloff factor β is shown in Fig. 10. The proposed system outperforms the MC system since the proposed system is robust to the rolloff factor β. In the proposed system, the total MAI decreases more rapidly than does that in the MC system. This is because the tail of the spectral density decreases with increasing β, and in the proposed system, frequency bands with low spectral density of band A are overlapped by those with high spectral density of band B. For very small values of β (i.e., where the spectral density is almost flat over each subband), note that the BER of the proposed system can be slightly higher than that of the MC system when $\delta \geq 1$ because of the penalty in bandwidth due to the offset scheme of the proposed system. The BER of the MC system, however, increases as the rolloff factor β increases because of the reduced spreading gain, N_M.

The correlation between subcarriers is governed by the parameter δ. We can obtain a frequency diversity gain with negligible correlation between the adjacent inner-repetition codes when $(\delta \geq (1 + R/M)/2)$, as can be seen in Fig. 8, because of the code symbol to subcarrier mapping in Fig. 3. Under correlated fading $(\delta < (1 + R/M)/2)$, Fig. 11 shows that the BER of the proposed system is superior to that of MC system, although the BER improvement slightly decreases as δ decreases.

When we take a rate 1/2 convolutional encoder instead of a rate 1/2 time-repetition encoder, as can be seen in Fig. 12, the performance gain is significant at the cost of the increased transmitter/receiver complexity. In Fig. 12, we plot the upper bound on the probability of bit error versus E_b/η_0, where $E_b = MNE_c$, and allow M to take on values 4 and 8. We fix the number of users at 50. This performance improvement is due to the coding gain achieved by convolutional coding over time-repetition coding.

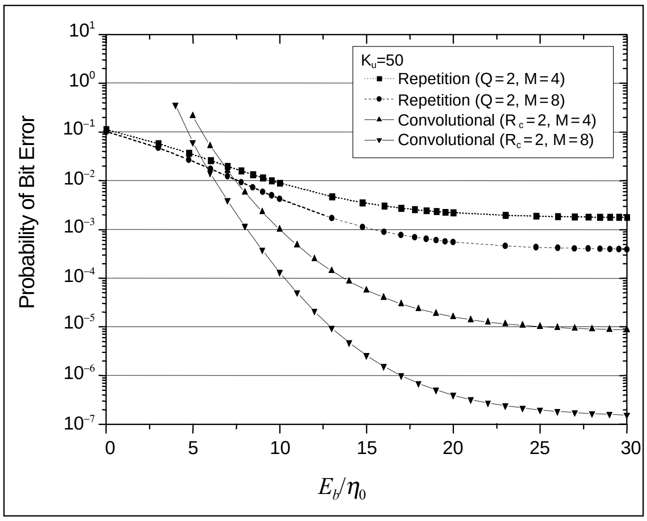

Fig. 12. BER vs. E_b/η_0 for $K_1 = K_2 = 25$, $N_1 = 512$, and $\beta = 0.5$.

V. CONCLUSIONS

We proposed a multicarrier system, where both a frequency diversity and a time diversity are used. We also incorporated offset multicarriers into a proposed DS-CDMA system, where each user has two frequency bands (A and B), and a frequency offset is employed between band A and band B. It was shown that the BER of the proposed system is smaller than that of the MC system, because the correlation between the transmitted repetition symbols of the proposed system is less, due to the interlacing of the subcarriers corresponding to the two symbols transmitted in parallel, than that of the symbols in the MC system, and the MAI is significantly reduced by the offset multicarrier scheme. In the proposed system, the total MAI decreases more rapidly than does that in the MC system. This is because the tail of the spectral density decreases with increasing β, and in the proposed system, frequency bands with low spectral density of band A are overlapped by those with high spectral density of band B. As a consequence, the proposed system becomes robust to the rolloff factor of the chip wave-shaping filter, and we can get a performance improvement over conventional MC system as the number of subcarriers increases. We also showed that the performance gain is significant at the cost of the increased transmitter/receiver complexity, when we take a rate 1/2 convolutional encoder instead of a rate 1/2 time-repetition encoder, as can be seen in Fig. 12.

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