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STANDARD STELLAR MODELS; α CEN A AND B

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ABSTRACT

The standard stellar models for α Cen A and B have been constructed without resorting to the arbitrary constraint of the Solar mixing length ratio. Assuming that the chemical compositions and the ages of the two stars are the same, series of models have been constructed. Using the observational constraints, $[Z/X]$, we were able to constrain the number of the 'possible' models.

We find that utilizing the observational constraints of [Z/X] the best models for α Cen system are with the initial $Z = 0.03$, $X = 0.66 \sim 0.67$. In particular, the primary and the secondary stars may have the same mixing length ratio 1.6 ~ 1.7, which is the same as that of the calibrated Solar model. And, the age of the system is about 5.4 Gyr. Finally, the large spacing of the p-modes is predicted to be $104 \pm 4\mu H_z$ for α Cen A.

Key words : stars : evolution – stars : individual(α Centauri) – stars : interior – stars : oscillations

I. INTRODUCTION

In stellar model construction, it has been a common practice to utilize the mixing length approximation for the determination of the temperature gradient within the convection zones. The approximation which was originally developed to describe incompressible terrestrial convection, is an extreme simplification of the actual physical process of stellar convection. The major source of uncertainty in this approximation is the mixing length itself. In actual model construction, the mixing length ratio, which is the mixing length scaled with local pressure scale height, is chosen in such a way that Solar models have the correct radius. Then, the same value is used for all the other stellar model construction, simply because there has been no alternative way to estimate the mixing length ratio for different cases.

If we have stars other than the Sun, whose radius is known, we can calibrate the mixing length ratio for the stars to test the uniqueness of the mixing length ratio. α Centauri system, the closest system from the Sun, provides such an opportunity. This is the closest multiple system which has been observed extensively. Especially, the primary and the secondary, α Centauri A and B, compose a visual binary system. From the parallax and astrometric investigations, the individual masses and luminosities have been accurately determined. From the spectroscopic analysis, the metallicity of the system and the effective temperature of the primary component have been also determined. The effective temperature of the secondary has been determined from the color index (eg. Neuforge-Verheecke & Magain 1997 table 1). More recently, because of its proximity, the system has been one of the prime target for asteroseismology (eg. Kjeldsen et al. 1999)

Study of α Cen A and B can be summarized as a problem with 4 knowns and 5 unknowns. From observations we know the luminosities and the surface temperatures of α Cen A and B. We have to use these 4 knowns to determine the 5 unknowns, namely, age of the system, the mixing length ratios of the A (α_A) and B (α_B), the initial helium abundance (Y_{ZAMS}), and the initial metal abundance (Z_{ZAMS}). The previous studies have been, either constraining the Z_{ZAMS} based on spectroscopic analysis (e.g. Edmonds et al. (1992) used Z=0.026 (Furenlid & Meylan 1990)) or assuming the mixing length ratios of the A and B are the same (e.g. Noels et al. 1991, Fernandes & Neuforge 1995), or removing the need of the mixing length ratio in the model construction using alternative scheme for the convection (eg. Lydon et al. 1993, Fernandes & Neuforge 1995).

In our study, we proceed by allowing the 5 unknowns to vary simultaneously. The adopted chemical compositions cover the typical galactic values, namely $0.02 \le Z_{ZAMS} \le 0.04$ and $0.61 \le X_{ZAMS} \le 0.75$, and the mixing length ratios $0.5 \le \alpha \le 2.6$. We have selected models which go through near the observed luminosity and the effective temperature of α Cen AB within the observational uncertainties on the HR diagram. Among those we tried to "collect" all possible models with the assumption of same composition and age for the A and the B. Utilizing updated observational constraint [Z/X] (Neuforge-Verheecke & Magain 1997) the models can be constrainted further.

The expected p-mode frequencies of α Cen A have been suggested here. Once the p-modes are observationally identified, they can be even tighter constraints.

In Section II, observational constraints are discussed. The input physics used for this computations are described in Section III. In Section IV we describe the evolutionary model computations for α Cen A and B. In Section V we discuss how the observational constraints, [Z/X], reduce the possible models. Finally, the results are summarized and discussed in Section VI.

II. OBSERVATIONAL CONSTRAINTS

(a) Mass

Being the closest visual binary system, the orbital elements and the parallax of α Cen A and B have been determined rather accurately (eg. Heintz 1982; Kamper & Wesselink 1978). The masses of α Cen A and B have been known to be $1.09M_\odot$ and $0.90M_\odot$, respectively. The values of masses have been adopted in most of related works, without much criticism (eq. Demarque et al. 1986, Noels et al. 1991, Fernandes & Neuforge 1995). Recently, Neuforge-Verheecke & Noels (1998) proposed the masses $1.12M_\odot$ and $0.94M_\odot$. Pourbaix et al. (1999) suggested even higher masses, which are $1.16M_\odot$ and $0.97M_\odot$ for α Cen A and B, respectively. These higher masses are not widely accepted yet. The more detailed discussions on the new masses can be found at Demarque et al. (1986). For this study, we adopt, without variation, the masses $1.09M_\odot$ and $0.90M_\odot$, for α Cen A and B, respectively.

(b) Luminosity and Temperature

The values of the luminosities of the A and B are taken from Noels et al. (1991). The uncertainties in the luminosities are caused by uncertainties in both the parallax and the bolometric correction of the two stars (Demarque et al. 1986, Lydon et al. 1993). We take the luminosities $\log(L/L_\odot) \approx 0.1853 \pm 0.006$ and -0.3065 ± 0.009 for the A and B, respectively.

For the effective temperatures, there has been no agreement on the precise values. In their table 1, Neuforge-Verheecke & Magain (1997) compiled the temperatures of the A and the B from the previous studies. For our study, we take the mean values of the temperatures in the table, which are $5770 \pm 40K$ and $5300 \pm 100K$ for the A and the B, respectively. The uncertainty in the temperature of the A has been estimated based on the standard deviation among the values in the table. On the other hand, the uncertainty in the temperature of the B has been estimated to be about 2.5 times that of the A, considering the relative errors in the individual references.

(c) Composition

Although there have been many observational attempts, the metallicity of α Centauri system is rather uncertain: It can be any value between 'about solar' and 'twice solar' metallicity. Recently, Neuforge-Verheecke & Magain (1997) derived $0.2 < [Z/X] < 0.3$, where $(Z/X)_\odot = 0.0245$ (Grevesse & Noels 1993; Grevesse et al. 1996). One important observational fact which most researchers agree upon, is $[Fe/H]_{\alpha CenA} < [Fe/H]_{\alpha CenB}$ (Chmielewsky et al. 1992). If we assume the common origin of the A and the B stars, this observational fact strongly indicates the non-negligible effect of chemical diffusion process. More detailed discussion on the observational constraints can be found at Guenther & Demarque (1999).

III. INPUT PHYSICS

We have tried to use the most updated physics for this study. The input physics taken into account for the model construction are summarized in Table 1. Using the same input physics, the standard solar model requires the mixing length ratio = 1.68087, the initial X=0.71797, Z= 0.01799, and the Solar surface hydrogen abundance of X=0.73511.

The solar mixture by Grevesse & Noels (1993) contains some significant changes to the CNO abundances in comparison to the Anders & Grevesse's values(Anders & Grevesse 1989). There was a major effort in the early 1990's to determine new and more accurate CNO and Fe abundances in the Sun which were reported by Grevesse & Noels (1993). Now, the solar Fe abundance agrees with the meteoritic abundance. The new solar value is $Z/X = 0.0245$, while the old value (with meteoritic iron) by Anders & Grevesse (1989) was $Z/X = 0.0267$.

OPAL opacities have been known as the best Rosseland mean opacities available today. Lately, the group released newer tables for the Grevesse & Noels (1993) solar mixture that include the effects of seven additional heavy elements as well as the effects of some physics changes (Rogers & Iglesias 1995, Iglesias & Rogers 1996).

The equation of state was taken from Rogers et al. (1996). The most popular equation of state has been the simple Saha solver. As the importance of the Coulomb interaction has been acknowledged, the effect has been taken care of using the Debye-Hückel approximation. Care has to be taken, however, that the Debye-Hückel correction is valid only when $\Lambda < 0.2$ (see page 23-26 for definition of Λ and discussion in Rogers (1994)). For $M=0.75M_\odot$ and above, Λ is always less than 0.2 in the post-ZAMS stellar evolution models. Thus, those models constructed using the OPAL equation of state and those of the usual Saha solver together with the Debye-Hückel approximation are very similar (Chaboyer & Kim 1995). Note that OPAL equation of state has its Coulomb interaction treatment valid for values of Λ up to 3, which is more than an order of magnitude better than the simple Debye-Hückel formula that most people use.

Chmielewsky et al. (1992) found a difference in the [Fe/H] of the two stars. Assuming the initial abundance of the two stars to be the same, this indicates that the diffusion of chemical elements has taken place in the stellar convection zones. To include the He diffusion in the calculation, we employed Loeb's formula (Bahcall & Loeb 1990, Thoul et al. 1994). For more detailed discussion on diffusion, see Chaboyer et al. (1992) where

Table 1. Input physics included in the model construction

Input physics	Reference
The solar mixture	Grevesse & Noels (1993)
Alexander-Faugerson low Temperature opacity table	Alexander & Faugerson (1994)
OPAL Rosseland mean opacity table	Rogers & Iglesias (1995) Iglesias & Rogers (1996)
OPAL equation of state tables	Rogers et al. (1996)
He diffusion	Bahcall & Loeb (1990) Thoul et al. (1994)
Eddington T-tau relation	

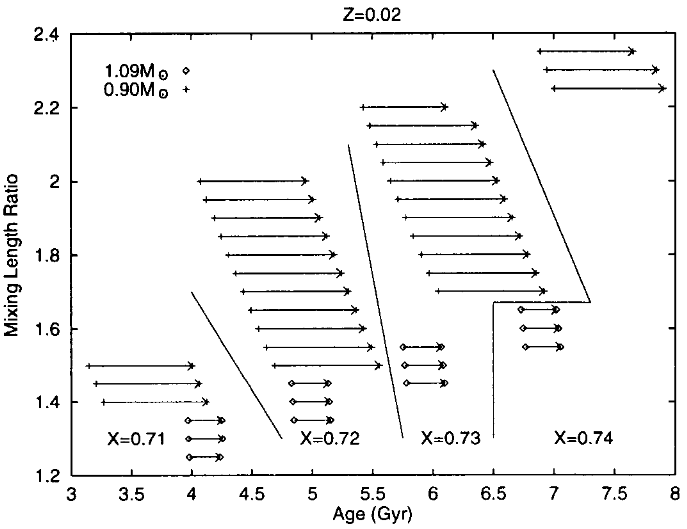

Fig. 1.— Comparison of two sets of models in Age vs. Mixing length domain for Z=0.02 case. The arrows show the duration which a particular model stays within the observational error near the location of α Cen A and B on the HR diagram. The Z and X in this plot is the initial abundance of the models. As the stars evolve the surface Hydrogen abundance comes to reduced because of the Helium diffusion process.

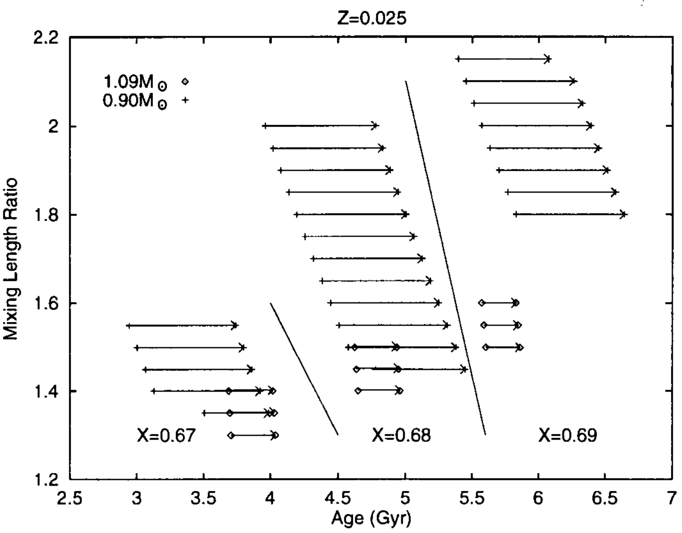

Fig. 2.— Comparison of two sets of models in Age vs. Mixing length domain for Z=0.025 case.

Bahcall & Loeb 1990 and Michaud & Proffitt (1993) have been compared. Bahcall & Loeb 1990 differs by 20 to 30% from the (more correct) Thoul et al. (1994) formulation. The diffusive coefficient of each metal element is still uncertain. Thus, the common practice is to assume that all heavy elements diffuse with the same velocity as fully ionized iron (Guenther & Demarque (1997)). Our study takes into account the helium diffusion only.

Since the upper turning points of sound waves are located in the atmosphere, the stellar atmospheric structure is very important for seismic study. The effect of the atmospheric structure is shown throughout all the frequencies of p-modes. Therefore, an empirically determined atmosphere structure has been preferred for the helioseismology. See Guenther et al. (1992) for more comments. For stellar seismology, however, we resort to the Eddington grey atmosphere.

IV. MODEL CONSTRUCTION

(a) Calibrated Solar Model

For the purpose of comparison, a modern reference standard solar model was constructed with the Yale Stellar Evolution Code. The OPAL opacities tables (Iglesias & Rogers 1996) were used together with the low-temperature opacities from Alexander & Faugerson (1994). The equation of state was taken from Rogers et al. (1996). When physical parameters are out of the table, the Yale standard implementation with the Debye-Hückel correction was used (Guenther et al. 1992, Chaboyer & Kim 1995). Helium diffusion processes were included in the model. Note that the model is evolved from the zero-age main sequence to the current solar age of 4.55 Gyr. The mixing length ratio (α) and the initial helium content (Y_{ZAMS}) have been adjusted in the usual way so as to match the solar luminosity, radius, and the observed solar ratio of heavy element to hydrogen abundances $Z/X = 0.0245$ (Grevesse & Noels 1993; Grevesse et al. 1996), at the Solar age. The standard Solar model requires the mixing length ratio = 1.68087, the initial X=0.71797, and Z= 0.01799. At the Solar age the model has the surface hydrogen abundance X=0.73511.

(b) Modeling Procedure

Throughout this study, the following three are assumed;

1. the mass of the A is $1.09M_\odot$ and that of the B is $0.90M_\odot$ (Demarque et al 1986),
2. the A and the B is coeval, and the age of the system is less than 9 Gyr,
3. the initial metal abundance is between 0.02 and 0.04.

We have taken the observed luminosity and temperature from Noels et al. (1991), as Lydon et al. (1993) did. Note that we have used the common mixing length approximation, and that we allowed the mixing length ratio of A and of B to be different, and to be different from that of the Sun.

Two five-dimensional grids (Y_{ZAMS}, Z_{ZAMS}, α_A, α_B, age of the system), one for α Cen A and one for α Cen B, were constructed. The masses of α Cen A and B are $1.09M_\odot$ and $0.9M_\odot$, respectively. The models with the initial Z (Z_{ZAMS}) ranging from 0.02 to 0.04 in increments of 0.005, the initial X (X_{ZAMS}) from 0.61 to 0.75 in increments of 0.01, and the mixing length ratios (α) from 0.5 to 2.6 in increments of 0.05, have been evolved up to the age of 9 Gyr. Among those, the models which go through the location of α Cen A or B on HR diagram within the observation uncertainty, have been selected. Then, for the same initial X and Z, the chosen models of the A and those of the B have been compared whether those models that are within the respective error box at the same time. The figure 1 through 5 show those models chosen. For each model with a given set of X_{ZAMS}, Z_{ZAMS}, and the mixing length ratio, the time period which the model is within the observed uncertainty on the HR-diagram, is indicated on the figures.

The table 2 summarized the characteristics of the models.

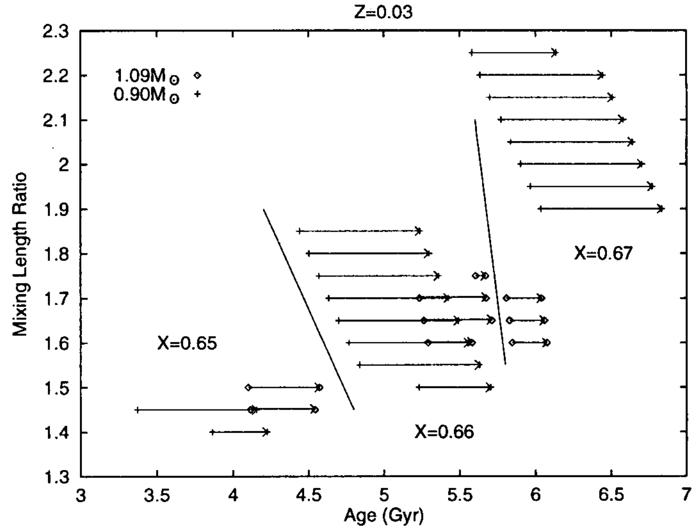

Fig. 3.— Comparison of two sets of models in Age vs. Mixing length domain for Z=0.03 case.

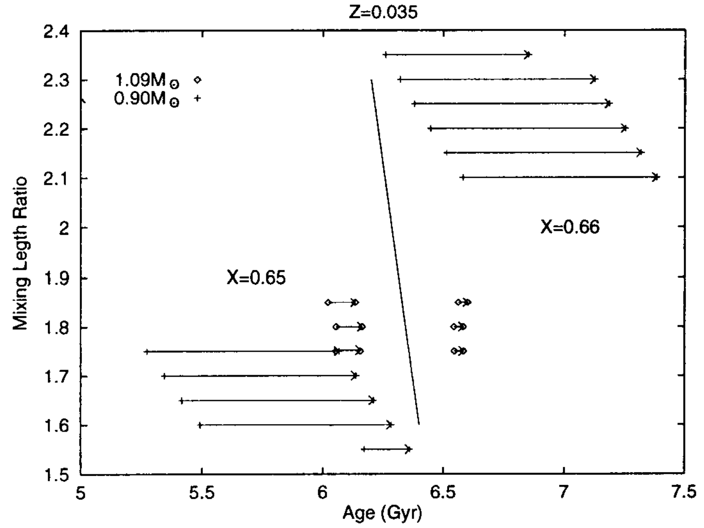

Fig. 4.— Comparison of two sets of models in Age vs. Mixing length domain for Z=0.035 case.

V. DISCUSSION

Our study reproduces the results of both Fernandes & Neuforge (1995) and Edmonds et al. (1992), and provide the clue to reconcile the contradiction between

Table 2. The surface chemical composition of the models

Z_{ZAMS}	X_{ZAMS}	$[Z/X]^1$	$\Delta Y/\Delta Z^2$
0.02	0.71	0.041 ~ 0.044	0.399 ~ 0.593
	0.72	0.036 ~ 0.039	-0.034 ~ 0.151
	0.73	0.032 ~ 0.033	-0.452 ~ -0.307
0.025	0.67	0.163 ~ 0.165	1.755 ~ 1.914
	0.68	0.157 ~ 0.159	1.384 ~ 1.526
	0.69	0.152 ~ 0.154	1.038 ~ 1.146
0.03	0.66	0.252 ~ 0.253	1.800 ~ 1.863
	0.67	0.245 ~ 0.246	1.426 ~ 1.493
0.035	0.65	0.326 ~ 0.327	1.744 ~ 1.775
	0.66	0.312 ~ 0.320	1.442 ~ 1.471
0.04	0.64	0.391 ~ 0.392	1.672 ~ 1.694

[1] $[Z/X] = \log\left(\frac{Z}{X}\right) - \log\left(\frac{Z}{X}\right)_\odot$; $0.19 \leq [Z/X] \leq 0.31$ Neuforge-Verheecke & Magain 1997

[2] $\Delta Y/\Delta Z \approx 3$ Perrin et al 1977, Reeves & Johns 1976, Peimbert 1975, Audouze & Tinsley 1976, Fernandes et al. 1998 ; cf, $\Delta Y/\Delta Z = 3 \pm 2$ Fernandes et al. 1998

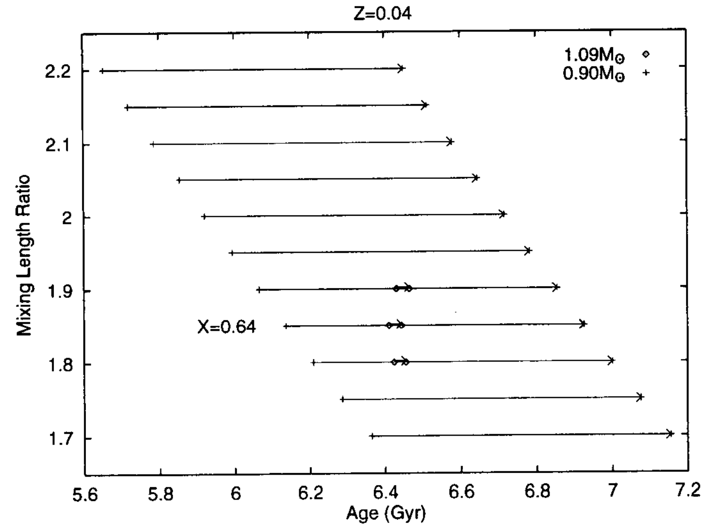

Fig. 5.— Comparison of two sets of models in Age vs. Mixing length domain for Z=0.04 case.

the two previous studies. Starting with the assumption of $\alpha_A = \alpha_B$ Fernandes & Neuforge (1995) have concluded that the metal abundance of α Cen system is about twice of the Solar value. Yet, their result was not compatible with their own observational constraint of $0.20 \leq [Z/X] \leq 0.033$ (eg, Neuforge-Verheecke & Magain 1997). On the other hand, Edmonds et al. (1992) with assumption of Z=0.026 (Furenlid & Meylan 1990) have ruled out the possibility of $\alpha_A = \alpha_B$.

Our results, the figure 1 through 5 and the table 2, can be compared with the results of Fernandes & Neuforge (1995). Assuming the same mixing length ratio for the A and the B, Fernandes & Neuforge (1995) concluded Z=0.038. As they pointed out, however, this result was not compatible with the their own observational constraint, $0.20 \leq [Z/X] \leq 0.033$ (Neuforge-Verheecke & Magain 1997). In our study, if we take the same assumption, i.e. the same mixing length ration for α Cen A and B, the models with Z=0.03, Y=0.31, and the mixing length ratio 1.6 ~ 1.7 can give the same initial composition and the same age at around 5.4 Gyr. And, these models, unlike Neuforge-Verheecke & Magain (1997), have the surface chemical composition within the $[Z/X]$ constraint. It is interesting to observe that the calibrated Solar model has about the same mixing length ratio. Furthermore, for any initial Z, except $Z_{ZAMS} = 0.02$, there is at least one pair of the A and the B models with the same mixing length ratio.

The different results come, firstly, from the inclusion of the He diffusion process in the model computations. As the He diffusion takes place, the relative abundance of the H increases, which results in the increase of the surface opacity. To compensate the increased opacity, one has to increase the mixing length ratio. Therefore, by including the diffusion process, the models for the A and the B with the same mixing length ratio can be available at lower Z. Secondly, depending on the metallicity, the models of $1.09 M_\odot$ can develop the convective core. And this affects the evolution on the HR diagram. This causes non-negligible effect on the results. As a result, when utilizing the $\alpha_A = \alpha_B$ constraint, one can find at least one solution for any initial Z except $Z_{ZAMS} = 0.02$ case.

When we fix the initial $Z = 0.025$ (cf, Edmonds et al. (1992) have used Z_{ZAMS}=0.026 of Furenlid & Meylan (1990).), one can **not** rule out the same mixing length ratio for the A and the B, as shown in the figure 2. Both models for the A and the B, with the same

Z_{ZAMS}=0.025, X_{ZAMS}=0.67 and the mixing length ration 1.35 ~ 1.4, can be within the respective error boxes on the HR diagram at the age around 3.8 Gyr. Same is possible for Z=0.025, X=0.67 models with the mixing length ratio 1.45 ~ 1.5 at around 4.8 Gyr. Therefore, one can not rule out the possibility of $\alpha_A = \alpha_B$. Because Edmonds et al. (1992) failed to recognize the existence of the multiple solution, they were not able to find the result that α Cen A and B with Z=0.026 models **can** have the same mixing length ratio.

From the table 2, one can see that the $[Z/X]$ constraint suggests Z=0.03 as the possible initial chemical composition of α Cen system. Especially, the models with the mixing length 1.6 ~ 1.7, whose initial chemical composition is Y=0.31 X=0.66 Z=0.03, can be within the respective observational error boxes on the HR-diagram at 5.4 Gyr (Figure 3). These results are consistent with all the available constraints, and do not rule out the possibility of the same mixing length ratio for the Sun and α Cen A and B. This may imply that the mixing length ratio remains constant among stars similar to the Sun, as Ludwig et al. (1999) have concluded based on their two dimensional numerical radiation hydrodynamics calculations of time-dependent compressible convection. Another interesting point in Table 2 is that $1 < \Delta Y/\Delta Z < 2$ for the models.

To predict the seismic p-mode frequencies of α Cen A, we have chosen four models of 1.09 $M_\odot$. Two models at different evolutionary stages (because of the main sequence convective core) which are at the center of the observational error box on the HR diagram. Two other models are one with maximum radius and the other with the minimum radius. The full evolutionary paths of the four models are shown in Figure 6. The evolution near the turn-off point shows that two models with higher initial Y has the convective core during the main sequence life time.

Their pulsation frequencies calculated using Guenther's non-radial non-adiabatic stellar pulsation program (Guenther 1994). The first-order frequency spacing of p-modes, $\Delta\nu = \nu_{n,l} - \nu_{n-1,l}$, is likely to be the first quantity determined by asteroseismology. The predicted first-order spacing averages to $104 \pm 4\mu H_z$ for A (Fig. 7; cf, 107 or 101 μH_z Kjeldsen et al 1999). As expected, the first-order spacing, which is an indicator of the mean density of the star, is independent to the convective core. From the similar computation we found that the first-order spacing is about $171\mu H_z$ for α Cen B.

On the other hand, the effect of the different evolutionary stages can be seen from the second-order splitting of p-modes $\delta\nu = \nu_{n,l} - \nu_{n-1,l+2}$, in Figure 8. Being more sensitive to the surface, the second-order splitting decrease as star evolves.

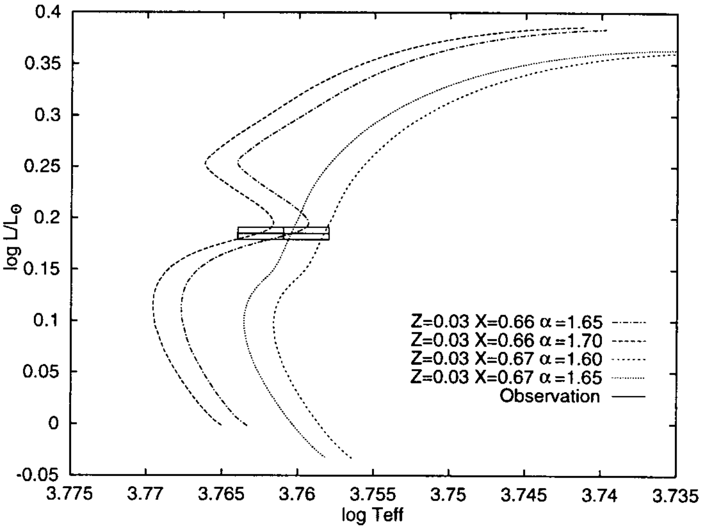

Fig. 6.— The HR diagram of four $1.09M_\odot$ models chosen. The effect of the main sequence convective core is evident for the two higher He abundance models.

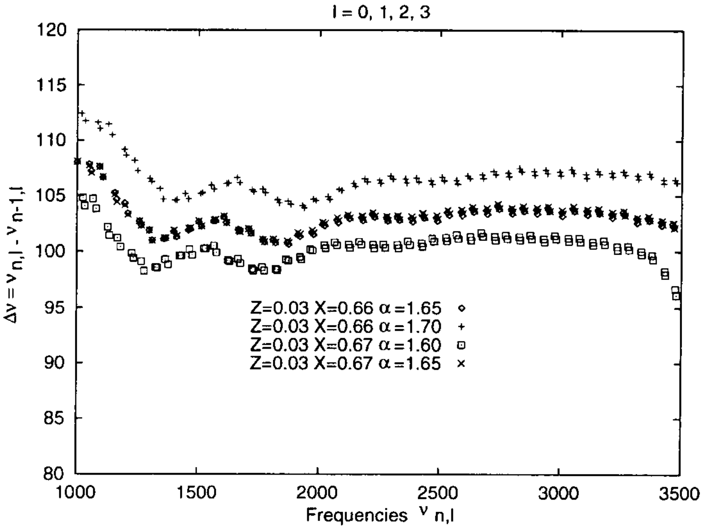

Fig. 7.— The large delta of the four models whose evolution are shown at Figure 6.

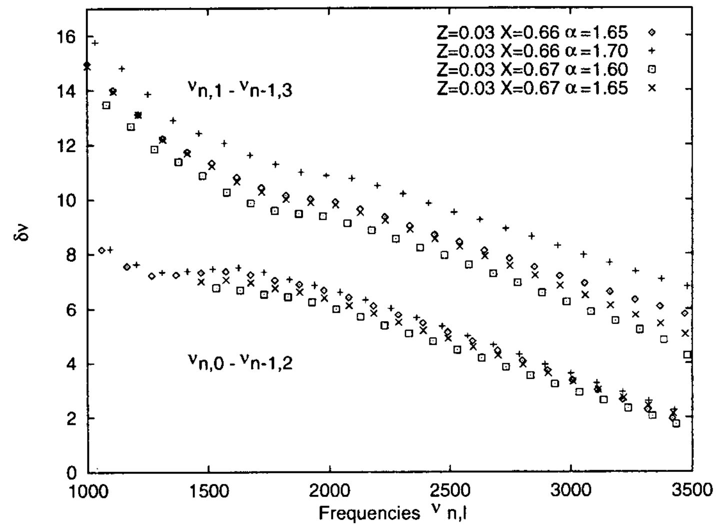

Fig. 8.— The small delta of the four models.

VI. SUMMARY

Series of 1.09 and of 0.90 $M_\odot$ models have been constructed for the α Cen binary system. The models were with Z_{ZAMS} ranging from 0.02 to 0.04 in increments of 0.005, X_{ZAMS} from 0.61 to 0.75 in increments of 0.01, and the mixing length ratios from 0.5 to 2.6 in increments of 0.05, have been calculated for the 9 Gyr evolution. Among those, the models which go through the location of α Cen A or B on HR diagram have been selected. Then, for the same X_{ZAMS} and Z_{ZAMS}, the chosen models of the A and those of the B have been compared whether those models are within the respective error box on the HR diagram at the same time.

Firstly, the major results of the previous studies were reproduced and the difference in the results were understood. Our new results are differ from Fernandes & Neuforge (1995) because of taking into account the diffusion process and the convective core. Edmonds et al. (1992) has failed to recognize the existence of the multiple solution. Thus, contrary to their conclusion, the same mixing length ratio for the A and the B can *not* be ruled out.

Secondly, utilizing the observational constraints [Z/X] the best models for α Cen system are with the initial $Z = 0.03$, $X = 0.66 \sim 0.67$. In particular, if we assume α Cen system with the initial $Z = 0.03$, $X = 0.66$, the primary and the secondary stars may have the same mixing length ratio $1.6 \sim 1.7$, which is the same as that of the calibrated Solar model. And, the age of the system is about 5.4 Gyr.

Finally, the large spacing of the p-modes is $104 \pm 4\mu H_z$ for A, $171\mu H_z$ for B.

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