

K_I Criteria of Surface Check under Stepwise Loadings of Drying Stresses^{*1}

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ABSTRACT

Finite element method was utilized to analyze crack tip stress and displacement field under drying stress case as stepwise loading. Opening mode of single-edge-notched model was employed and analyzed by linear elastic fracture mechanics of plane stress case. The drying stresses were applied as stepwise loads at the boundary elements of the model with 10 steps of time serial. The stress intensity factor(K_I) for opening mode reached to its maximum just prior to the stress reversal. The K_I from the displacement fields revealed 1.7 times higher than those from stress fields. By comparing the two sets of K_I from displacement and stress fields, single parameter K_I showed its validity to characterize displacement fields around the crack tip front while stress field could not be characterized due to large variations between two sets of data.

Key words fracture intensity factor, drying stress, finite element method, stress field, displacement field

INTRODUCTION

It is well known that the formation of surface checks reveals during the early stage of drying process when drying stresses exceed tensile strength of wood in the perpendicular direction. The surface checks usually propagate along weak portions of wood such as ray tissue, and ray tissue then can be supposed as a latent or potential crack when wood is exposed to such stresses conditions in lateral directions. Stress intensity factor, K_I , is now applicable to explain crack development at ray tissues since fracture of crack will occur when stress intensity is

equal to the toughness (Broek, 1986). The understanding on the stress fields around crack tip, therefore, would provide useful information in detail with relation to formation of surface checks, and possible methodological approach to restrain them during drying process.

In analysis of stress intensity at a potential crack with stress distributions in lateral direction, the classical solutions in a closed form are only available for the uniform loading case. Stress intensity can not be easily scrutinized with non-uniform distribution of stress along the boundary layers in the case of drying stress application. In this research, virtual test using

^{*1} Received on September. 27, 1999

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finite element analysis is applied to examine the stress and displacement fields for a single-edge-notched case of opening mode in which ray tissues are assumed to be a potential crack while wood are exposed to drying stresses. In this case stress field around the potential crack can be characterized with K_I for an opening mode.

MODEL FOR FINITE ELEMENT ANALYSIS

Geometry of Model

A radial crack of 1mm length is assumed to be at the center of 5cm by 10cm in cross section red oak lumber. Due to the symmetry of stress distribution along the thickness direction of the specimen, one-fourth of the cross section is considered as a model in geometry. The model is meshed with 216 elements as shown in Figure 1 and Figure 2. Two sets of elements are incorporated in the geometrical model for a finite element analysis. In ANSYS system, crack tip solid elements so called STIF-85 which have a common edge at the crack tip enclose around the crack tip. STIF-85 element is defined by eight nodal points having three degree of freedom: translation in the nodal x , y and z direction. The crack tip solid elements are then enclosed by isoparametric quadratic quadrilateral element so called STIF-45 element. In this geometrical model a stress intensity factor for stress field is valid only in the crack tip front element while that for displacement field is valid beyond the crack tip. Linear Elastic Fracture Mechanics (LEFM) is considered in this research and visco-elastic and mechanosorptive creep of wood are neglected. For the analysis, the following material properties of red oak (Bodig and Jayne, 1982) are incorporated to the geometrical model:

- E_T (Modulus of Elasticity): 0.63GPa
- G_{TR} (Modulus of Rigidity): 0.24GPa
- ν_{TR} (Poisson's Ratio): 0.33

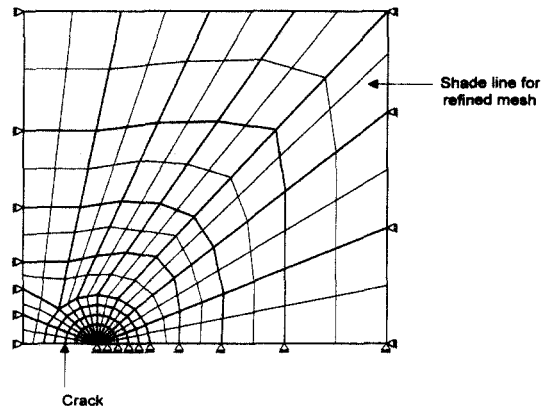


Fig. 1. Geometrical model of finite element analysis for a single edge notched crack.

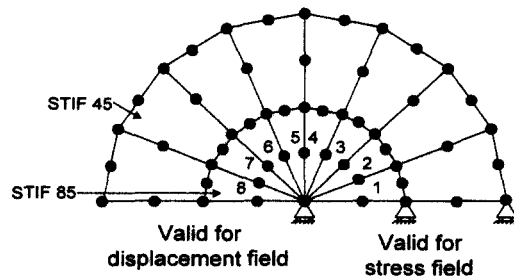


Fig. 2. Detail of crack solid elements around the crack tip.

Loading Conditions

Since surface checks are developed in the early stage of drying process, loading conditions applied to the model by stepwise load function from the stress distribution in the early stage of drying with specified time steps. In this study drying stress development by the work of Kawai(1979) was applied for the early stage of drying including stresses reversal. Application of stepwise load function is because of difficulties in continuous iterations of drying stress curves with time serials. Loading conditions including stress reversals in drying process are shown in Figure 3.

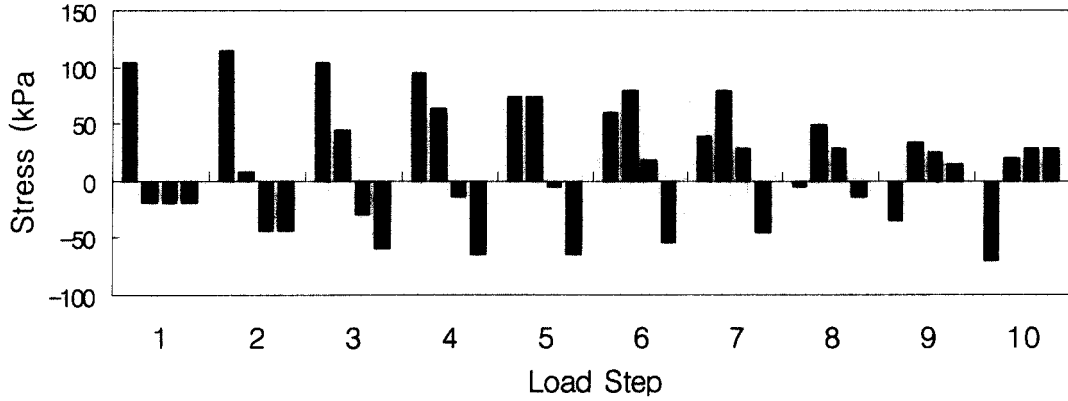


Fig. 3. Applied loads at boundary elements with load steps.

Analysis for Stress and Displacement Fields

The stress fields around crack tip are given by Griffith Criterion (Brook, 1986, 1989) for open mode as follows;

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \theta \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \quad [1]$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \theta \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \quad [2]$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \theta \cos \frac{3\theta}{2} \quad [3]$$

where σ_x , and σ_y are normal stresses in x and y direction (MPa), τ_{xy} is shear stress (MPa), K_I is stress intensity factor ($\text{MPa}\sqrt{\text{m}}$), and r is distance from crack tip (mm), respectively. From these equations stress intensity factor can be obtained by simple substitution of specified stress condition. At given loading condition σ_y is found from the finite element analysis, and stress fields can then be characterized by stress intensity factor.

Displacement fields can also be characterized by stress intensity factor. For plane stress case which condition is valid for drying stress development case since longitudinal stresses are negligible, displacement fields can be obtained as follows;

$$U_x = \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \cos \frac{\theta}{2} \left(\frac{1-\nu}{1+\nu} + \sin^2 \frac{\theta}{2} \right) \quad [4]$$

$$U_y = \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \sin \frac{\theta}{2} \left(\frac{2}{1+\nu} - \cos^2 \frac{\theta}{2} \right) \quad [5]$$

and

where U_x and U_y are displacement in x and y directions (mm), G is modulus of rigidity (MPa) and ν is Poisson's ratio. From Equations (1) to (5), it is clear that single parameter K_I can characterize the entire crack tip stress and displacement fields since stress and displacement field can be known explicitly if the parameter K_I is available.

RESULTS AND DISCUSSION

The stress intensity factors K_I obtained from stress and displacement fields are shown in Table 1. The values of K_I from this research appears about 10 to 20 times higher than those obtained from laboratory tests for single-edge-notched neck-down tensile specimen of red oak in Tangential-Radial cleavage plane (Park, 1995). This is believed to be caused by different loading conditions as well as different

geometrical propensities such as localized material properties of elements in the geometrical model. This result also may infer the inappropriate approach of finite element analysis without incorporation of rheological and mechno-sorptive phenomena for non-equilibrium moisture condition under drying process and local material properties of elements even though the loading conditions may also result the differences in stress intensities.

K_I tends to decrease by refining the meshes of the geometrical model. This is a possible indication of convergence criterion to the closed form solution even though one more refining would be necessary for clarification. K_I values from the displacement field were generally higher than those from the stress field as a factor of 1.7. In general K_I is expected to see its maximum value when the outermost shell is exposed to the highest tensile stress such as load step 2. This is because the potential crack tip is located at outermost layer of the geometry. It is quite interesting that K_I values reach to their maximum just prior to stress reversal of drying stress. This indicates that crack tip plays a role of a pivot in opening mode of fracture due to tensile stress on the surface element of geometrical model and consequent compressive stress at the inner element. When the surface stress reversed to

compression from tensile, K_I values decrease rapidly. This trend of K_I criteria explain when the surface check is exposed to the most critical condition to start to open.

Since no closed form solution is available for stepwise loading condition for single edge crack geometry, approximations of stress and displacement fields around crack tip from finite element analysis were compared to those from Equation (2). In application of Equation (2) as theoretical approach, K_I values obtained from the crack tip element are utilized where θ is zero. It is obvious that stress field of finite element model is valid only for crack tip element. In this case it is recommended that the crack tip element have angle less than 22.5 degree in triangular element (De Salvo, 1985). However it is also clear that displacement field is valid at the element which is 180 degree away from the crack tip element.

Stress and displacement field from approximation and theoretical approach are compared in Figure 4 for load step 1, 3, 5, 7. Since the geometrical model with stepwise loading is an opening mode of fracture mechanics, stress and displacement in Y direction are considered in Cartesian Coordinate. It is clearly observed that the displacement fields from K_I are more likely to converge to theoretical analysis, while stress fields show large deviations. Even LEFM

Table 1. Stress Intensity Factors (MPa \sqrt{m})

Load Step	Stress Field		Displacement Field	
	Coarse Model	Refined Model	Coarse Model	Refined Model
1	5.81	4.66	10.01	8.01
2	6.19	5.58	10.26	9.35
3	7.68	6.81	12.95	11.63
4	8.22	7.40	14.39	12.81
5	7.88	6.90	13.52	11.98
6	7.90	6.38	13.82	11.99
7	7.16	6.13	12.71	11.01
8	2.43	2.46	5.48	4.64
9	0.46	0.44	1.25	1.06

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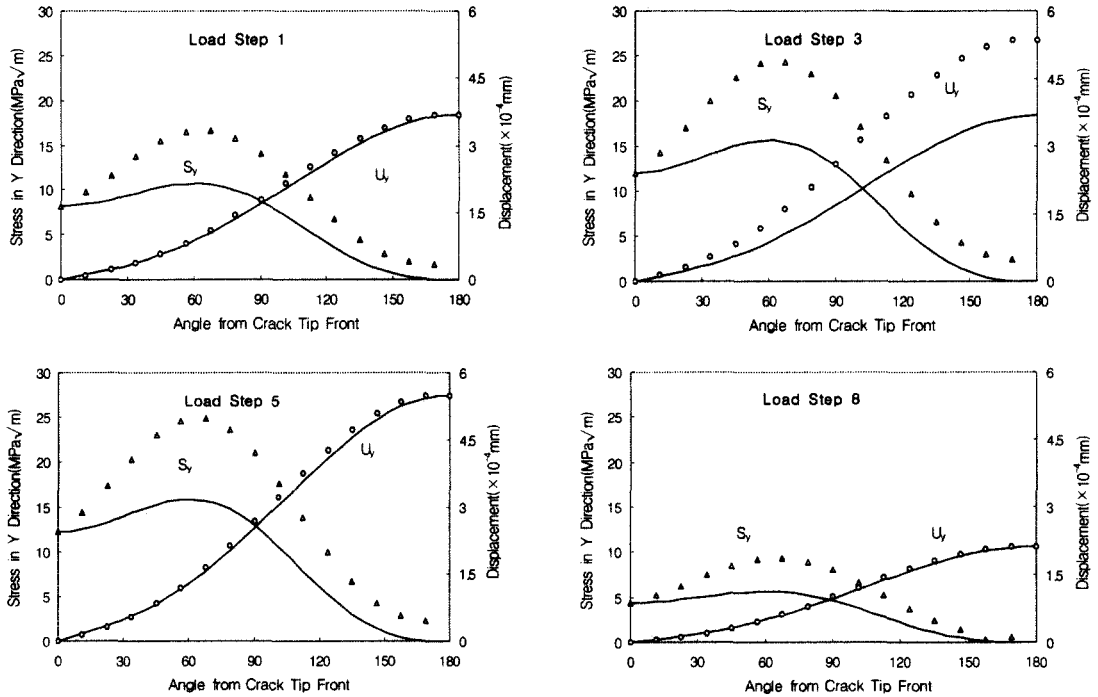


Fig. 4. Stresses and displacements fields in y-direction around the crack tip from finite element and theoretical analysis.

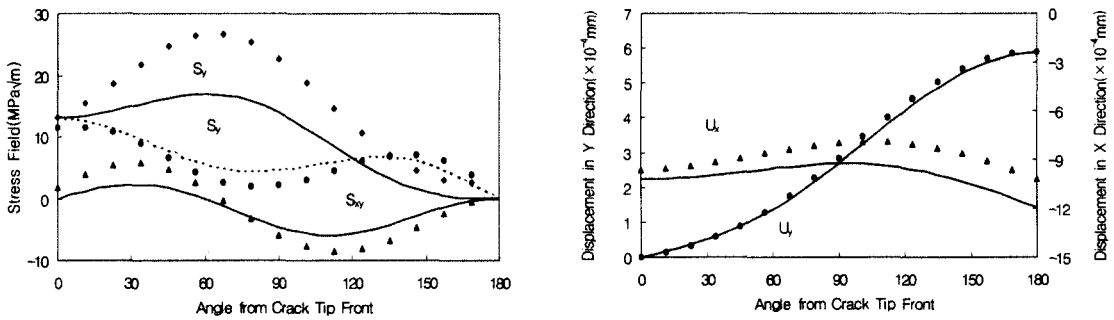


Fig. 5. Stresses and displacements fields around the crack tip from finite element and theoretical analysis at the load step 4.

describes that single parameter K_I generally characterize the entire stress, strain and displacement fields, the necessary condition for LEFM is uniform loading. For stepwise loading condition in this research K_I is not valid for stress field.

Figure 5 describes the stresses and displacements fields when K_I reached to it's maximum at load step 4. These fields also state the validity of displacement field rather than stress field. Invalidity of stress fields from K_I in this research could be the results of stepwise

loading condition. Even the failure criteria are not available from the finite element analysis due to no informations on critical stress intensity factors, K_{IC} , it is very useful to predict the trend of stress intensities since these are equivalent to material toughness. It is, however, recommended that localized material properties such as Young's modulus of every single element in the geometrical model should be incorporated with the progress of drying process for more precise analysis of stress and displacement field. Time series application of loading by finite layer of the model would also be desirable for further detail of K_I criteria.

CONCLUSIONS

Finite element method was utilized to analyze crack tip stress and displacement field for drying stress case as stepwise loading. The stress intensity factors for opening mode reached to their maximum prior to the stress reversal. By comparing the two sets of K_I from theoretical and finite element approximations, single parameter K_I , show its validity to characterize displacement fields around the

entire crack tip front while stress field could not be useful due to large variations between two sets of data.

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