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Mathematics Teachers' Perspective of Their Students' Learning in Traditional Calculus and Its Teaching Strategies[1]

AHUJA, O.P.
Division of Mathematics, National Institute of Education, Nanyang Technological University, 469 Bukit Timah Road, Singapore 259756; Email: ahujaop@nievax.nie.ac.sg

LIM-TEO, SUAT KHOH
Division of Mathematics, National Institute of Education, Nanyang Technological University, 469 Bukit Timah Road, Singapore 259756; Email: teosk@nievax.nie.ac.sg

LEE, PENG YEE
Division of Mathematics, National Institute of Education, Nanyang Technological University, 469 Bukit Timah Road, Singapore 259756; Email: leepy@nievax.nie.ac.sg

We conducted a survey with the objective of studying the teachers' perspective of their students' learning in traditional calculus and its teaching strategies. This survey was targeted at mathematics teachers of the junior colleges, polytechnics and universities in Singapore.

In this paper we present findings of the first part of our survey. The first part addresses various issues related to the mathematics teachers' perception of the current status of calculus education in Singapore. The findings of this study will be unique in Singapore context because, according to the best of our knowledge, no such survey on calculus education has ever been undertaken in Singapore.

INTRODUCTION

A traditional calculus course and its teaching strategies such as the lecture method are basically skills-based which result in rote, manipulative learning. In this style of teaching and learning, there is too much drill and recitation and many students may retain little of both techniques and ideas in calculus in the long run. Thus, this results in instrumental understanding or unsupported procedural knowledge. Skemp (1976) describes "instrumental understanding" as knowing rules without knowing why they work. As pointed out

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by Skemp, skills-based courses are very efficient if and only if criterion is the ability to perform routine manipulations. It means if an application problem requires conceptual knowledge, then most students might find it difficult to solve the problem. In fact, it is found that such students suffer from their pursuit of a meaningless, ritualistic manipulation of symbols. These findings are supported by Ferrini-Mundy & Lauten (1993), Schoenfeld (1990), Skemp (1976), White & Mitchelmore (1996) and others.

The traditional calculus and its teaching strategies have also become a 'litany of procedures and template problems which too often results only in giving students some rather routine practice in algebraic manipulations. In addition, students with weaker backgrounds are usually driven away in frustration over the manipulations required, even if they are able to understand the basic ideas of calculus' (Tucker & Leitzel 1995, p. 57).

RESEARCH FOCUS AND METHODOLOGY

Three general research questions arising from the above-mentioned main objective guided the design of the questionnaire for this part of the survey. Each of these questions suggests several more specific questions. These general questions were:

1. What is the teachers' perspective of their students' learning calculus?
2. What are the main problems that plague teachers in the present state of calculus education in Singapore?
3. What are the views of the teachers about improving calculus education in Singapore?

A questionnaire was developed to seek information about teacher qualifications and experience, teacher's perception about students' attitude towards calculus, methods of teaching calculus, and others. The open-ended question sought teachers' difficulties and suggestions for improvement in calculus education. The survey was conducted in April/May 1997. There were twelve short-answer items and one open-ended question in the first part the questionnaire. The following samples from three different populations were included in the survey:

Group 1: The Principals of all the fourteen Junior Colleges in Singapore were requested to ask six to seven of their teachers engaged in teaching calculus to complete the questionnaires. In all, 91 teachers from all the Junior Colleges completed the questionnaire.

Group 2: The Heads of Mathematics Departments of all the four polytechnics in Singapore were requested to ask most of their lecturers engaged in teaching calculus to complete the questionnaire. In all, 77 lecturers from all the polytechnics completed the questionnaire.

Group 3: Ten lecturers, engaged in teaching first year calculus to mathematics students at both the universities in Singapore, were requested to complete the questionnaire. Responses were received from all the ten lecturers.

Overall, the response rate for the survey was 100% and about one-fifth of the respondents gave comments and suggestions for one open-ended question.

For the purpose of this study, we wanted to see the main trends in responses. For most of the questions, categories of answers were provided and respondents selected one of those categories. Percentages of respondents choosing each category were then calculated. The percentages did not add always up to 100 because of non-responses.

Throughout this paper, we shall write 'JC', 'Poly' and 'Univ' for 'junior colleges', 'polytechnics' and 'universities' respectively. Also, the term 'teachers' is used to refer to teachers/lecturers from JC, Poly or Univ.

FINDINGS OF THE SURVEY

In this section, we present the results of the survey. The numerical results are summarised in the tables and/or figures. While the teachers' comments and suggestions are too numerous to be included in detail in this paper, their essence is extracted and used in various parts of the paper. Also, the summary of the respondents' comments and suggestions are highlighted in some of the subsections in this section.

The first subsection of this section provides a general description of the respondents in terms of their mathematics background, teaching experience, and their perception as calculus students. Other subsections are the write-up of the results of questions related to issues and problems of the calculus education in Singapore.

Population characteristics

The survey results show that most of the calculus teachers surveyed (86.5%) are Singaporeans or Permanent Residents. Only 1.7% of 178 respondents stated that they were expatriates, while 11.8% chose not to offer this information. The other characteristics about the population in all three groups are given below.

Table 1 summarises the respondents' highest mathematics background. Most of the JC respondents (79.1%) have Honours or a higher degree in Mathematics. But, at least 41.6% of Poly respondents (18.2% with Minor Maths and 23.4% with Major Maths) have qualifications lower than Honours in Mathematics. We notice that 28.6% of Poly respondents have other qualifications such as a degree or diploma in engineering. As expected, all university respondents have Ph. D. as the highest degree in mathematics.

Table 1. Highest mathematics background

Qualification	JC (91)	Poly (77)	Univ (10)	All (178)
B.A./B.Sc. Minor Maths	1 (1.1%)	14 (18.2%)	0 (0.0%)	15 (8.4%)
B.A/B.Sc. Major Maths	18 (19.8%)	18 (23.4%)	0 (0.0%)	36 (20.2%)
Hons Maths	67 (73.6%)	10 (13.0%)	0 (0.0%)	77 (43.3%)
M.Sc. Maths	4 (4.4%)	12 (15.6%)	0 (0.0%)	16 (9.0%)
Ph.D. Maths	1 (1.1%)	1 (1.3%)	10 (100%)	11 (6.2%)
Others	1 (1.1%)	22 (28.6%)	0 (0.0%)	23 (12.9%)

As Table 2 shows, approximately half of the JC respondents surveyed have more than ten years of teaching experience in mathematics and a small number of JC respondents (6.6%) have two or less than 2 years of teaching experience in mathematics. On the other hand, 34% of Poly respondents have less than six years of teaching experience in Mathematics.

Table 2. Respondents' teaching experience in mathematics

Years	JC	Poly	Univ	All
0 to 2	6 (6.6%)	18 (23.4%)	1 (10.0%)	25 (14.0%)
3 to 5	15 (16.5%)	16 (20.8%)	6 (60.0%)	37 (20.8%)
6 to 10	23 (25.3%)	16 (20.8%)	2 (20.0%)	41 (23.0%)
> 10	47 (51.6%)	27 (35.1%)	1 (10.0%)	75 (42.1%)
Total	91 (100%)	77 (100%)	10 (100%)	178 (100%)

Teachers were asked *whether they have taught calculus in 1996/1997*. Overall 92.1% of the respondents reported that they are currently teaching calculus. This shows that almost all the teachers' surveyed are currently involved in teaching calculus.

In another question, they were asked *about the number of semesters they have taught calculus (or a part of it).* Responses shown in Table 3 indicate that most of the respondents (68.5%) from all three groups have taught calculus for more than 3 semesters.

Table 4 summarises the teachers' responses of a question "*When you were a mathematics student, what was your perception of calculus?*" A majority of the respondents from all three groups (54.5%) used to find calculus interesting. Not surprisingly, almost all the teachers reported that calculus was neither hard nor boring for them. However, 23.6% of all respondents had perceived calculus as of average difficulty when they were students.

Table 3. Respondents' teaching experience in calculus, in percent*

No. of Semesters	JC	Poly	Univ	All
1	7.7	13.0	10.0	10.1
2	16.5	10.4	10.0	13.5
3	4.4	7.8	0.0	5.6
> 3	68.1	67.5	80.0	68.5

* Column for each group does not add up to 100 because no information was provided by some respondents

Table 4. Respondents' perception of calculus as students, in percent*

Attitudinal Behaviour	JC	Poly	Univ	All
Interesting	47.3	63.6	50.0	54.5
Easy	46.2	31.2	40.0	39.3
Average	30.8	15.6	20.0	23.6
Hard	2.2	7.8	0.0	4.5
Boring	4.4	2.6	0.0	3.4
No opinion	5.5	6.5	10.0	6.2

* Each column total is more than 100 because of more than one choice ticked by many respondents

Student characteristics — attitudinal variables

The findings reported here are on two important attitudinal variables: students' liking calculus and students' attitude towards calculus.

The responses of the teachers to a question "*What percentage of your students like calculus?*" are summarised in Figure 1. Indeed, this figure manifests that a significant number of students do not like calculus. We notice that typically between 50–74% of all calculus students in junior colleges, polytechnics, and universities like calculus.

In another question asking "*teachers' perception about a majority of their students' attitudes towards calculus*", around 31% teachers from all three groups perceive that a majority of their students regarded calculus as hard. The figure is much higher (49.4%) for polytechnic respondents. In fact, Table 5 indicates that very few teachers (6%) perceive that a majority of their students look at calculus as an easy and interesting subject: approximately 57% of the respondents observe that their students' attitude towards calculus is average. The results show that even university students do not regard calculus as easy and interesting: 80% of university lecturers reported that most of their students found calculus as average. It is also clear from Table 5 that the polytechnic students have more serious problems in learning calculus than students in junior colleges and universities.

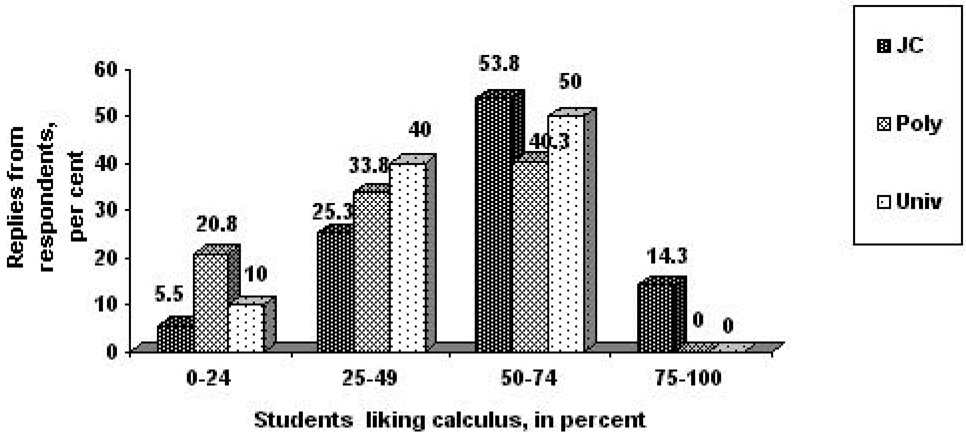

Figure 1. Teachers' perception of their students' liking calculus

Table 5. Replies of respondents about their students' attitude towards calculus, in percent*

Attitude towards calculus	JC	Poly	Univ	All
Average	65.9	44.2	80.0	57.3
Hard	16.5	49.4	20.0	30.9
Boring	15.4	19.5	20.0	17.4
Interesting	6.6	6.5	0.0	6.2
Easy	8.8	2.6	0.0	5.6
No option	5.5	6.5	0.0	5.6

* Each column total is not 100 because more than one choice was allowed to respondents

Students' performance in calculus

In this subsection, we report the findings of the questions related to the teachers' perception about their students' performance in calculus. More precisely, we look at the survey findings associated with success rate, difficulties in recalling pre-calculus material, retention, and approaches as used by the students.

The teachers were asked to *estimate the proportion of their students who attained grade C or better in the first course in calculus (or mathematics paper for which calculus is a part) in the last two years'*. As specified in Figure 2, a majority of the respondents from polytechnics (59.7%) and universities (60%) estimated that the overall success rate (C or better) of their students in calculus (in the last two years) was 50–74%. However, 20% of polytechnic teachers estimated that less than 50% of their students would attain grade C or better in calculus.

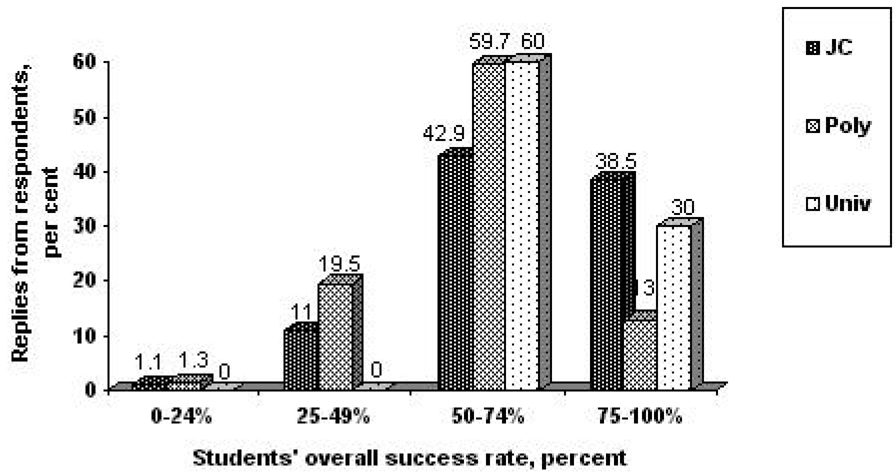

Figure 2. Teachers' perception of their students' overall success rate in calculus, in percent

Figure 3 exhibits summary of the replies from teachers to the question "*To what extent do most of your students face difficulties in recalling pre-calculus material needed for calculus?*" Overall 46.6% of the respondents from all three groups announced that the extent to which most of their students faced difficulties in recalling pre-calculus materials was too much or most often.

In particular, this percentage was much higher for poly-technic students because 70.1% of their teachers surveyed reported that most of their students often faced difficulties in recalling pre-calculus material. In brief, Figure 3 shows that recalling pre-calculus material is a major problem with many calculus students.

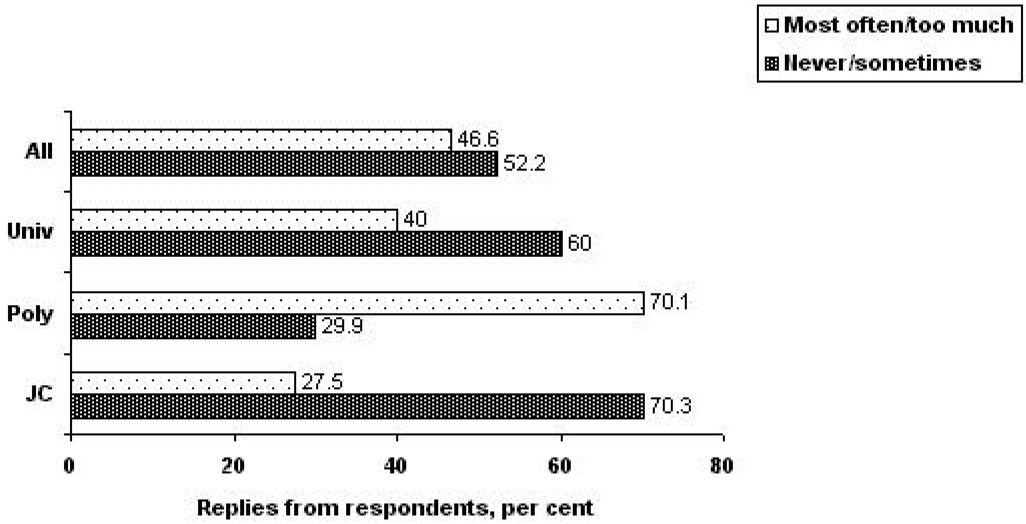

Figure 3. Respondents' perception of the extent of students' difficulties in recalling pre-calculus material, in percent

Teachers were asked, *"What do your students retain* **most** *in calculus in the long run?":*

(i) Techniques only
(ii) Ideas only
(iii) Most of both, and
(iv) Little of both.

From Figure 4, the results do not seem to indicate a vast difference in perception of the respondents of three groups: 60% of university, 50.6% of Poly and 47.3% of JC respondents reported that most of their students retained techniques only in the long run. Notice that overall 18% of the teachers from all the groups perceive that their students retain little about techniques and ideas in calculus after a period of time. Thus, Figure 4 demonstrates that students' retention in the long run is also a matter of concern for most of the calculus teachers.

The teachers were asked: "*What extent is your students' approach to calculus based on the factors:*

(i) Techniques,
(ii) Concepts,
(iii) Reasoning skills, and
(iv) Solving applications related problems?"

For each of the factors, the respondents indicated if their students' approach was based on that factor very often, sometimes, very little or never. Tables 6 to 8 summarise replies of the respondents from different groups.

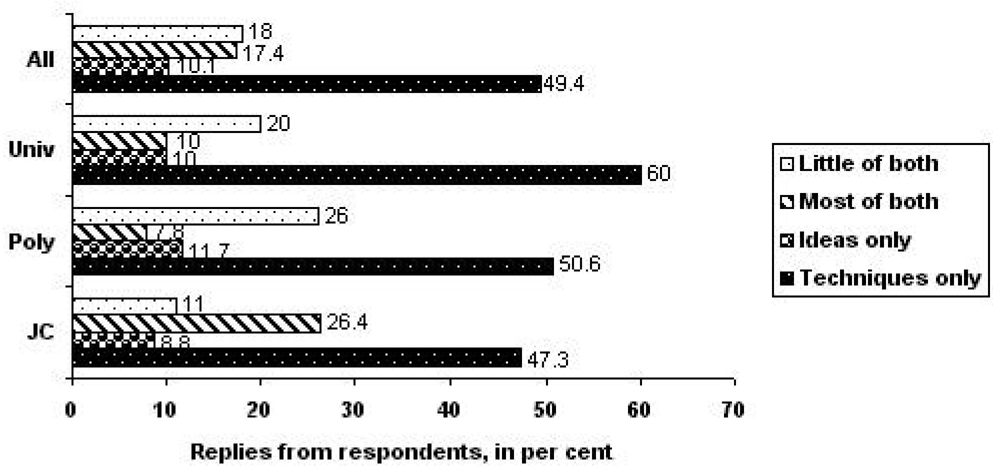

Figure 4. Teachers' views about the extent of their students' retention in calculus, in percent

Table 6. JC respondents' replies about their students' approach to calculus, in percent*

Factors	Very Often	Some-times	Very Little	Never/ No choice	Row Total
(i) Techniques	75.8	14.3	1.1	8.8	100
(ii) Applications Related Problems	14.3	51.6	26.4	7.7	100
(iii) Concepts	12.1	64.8	22.0	1.1	100
(iv) Reasoning Skills	7.7	58.2	31.9	2.2	100

* Column total is not a 100 because the respondents ticked more than one choice

Table 7. Polytechnics' respondents' replies about their students' approach to calculus, in percent*

Factors	Very Often	Some-times	Very Little	Never/ No Choice	Row Total
(i) Techniques	72.7	24.7	2.6	0.0	100
(ii) Applications Related Problems	11.7	59.7	24.7	3.9	100
(iii) Concepts	3.9	50.6	37.7	7.8	100
(iv) Reasoning Skills	3.9	39.0	51.9	5.2	100

* Column total is not a 100 because the respondents ticked more than one choice

Table 8. Universities' respondents' replies about their students' approach to calculus, in percent*

Factors	Very Often	Some-times	Very Little	Never/ No Choice	Row Total
(i) Techniques	90.0	10.0	0.0	0.0	100
(ii) Applications Related Problems	0.0	70.0	20.0	10.0	100
(iii) Concepts	0.0	60.0	40.0	0.0	100
(iv) Reasoning Skills	0.0	50.0	50.0	0.0	100

* Column total is not a 100 because the respondents ticked more than one choice

Figure 5 exhibits the opinions of respondents from the entire population. This figure shows that for an approach to calculus based on techniques, three quarters of the respondents felt that their students used such an approach very often. This is not surprising because Figure 4 exhibits that overall one half of respondents believe that most of their students generally retain techniques only in the long run. On the other hand, for the other approaches, approximately one-half of all the respondents stated that their students sometimes made use of these approaches. Among the approaches based on the other factors, the least favoured approach was to use reasoning skills with 45 per cent of all respondents disclosing that their students made very little (or no) use of reasoning skills. About one third of the respondents perceived that their students also seldom used approaches based on solving application-related problems and based on concepts.

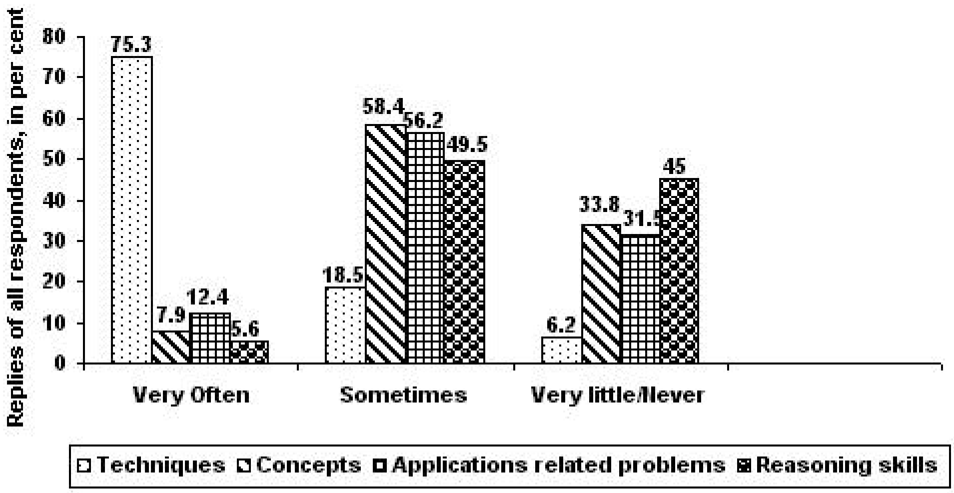

Figure 5. Respondent's' replies about their students' approach to calculus, in percent

Barriers in teaching and learning calculus

The survey sought teachers' views about major issues and problems in teaching calculus. The survey also asked for their observations on obstacles in improving calculus education in Singapore. In this subsection, teachers' comments from all groups are summarised. Wherever necessary, it may be indicated whether the views are those of the respondents from junior colleges, polytechnics or universities.

(a) Barriers due to students' weak foundation

Many of the respondents claimed that the standard of their students in pre-calculus or secondary level mathematics was an important factor hindering them from learning higher level calculus. If the students have the necessary knowledge and skills in the fundamentals, learning calculus would not be a problem. The respondents' suggestions are summarised as follows:

1. Students with good maths background learn calculus without problems and vice versa. However, many students lack the prerequisite for learning calculus. They do not have strong foundation in pre-calculus, in particular, in algebra and trigonometry.
2. There's a simplification in the Cambridge 'O-Level' syllabus which has led to a weaker foundation in the students' foundation, which makes it difficult to achieve a higher standard at subsequent levels.

3. There has been deteriorating standards of the students. In the words of a respondent, “Too much maths fundamentals have been removed from the maths syllabus at secondary level, mostly on pretext of irrelevancy and heavy workload of students” and “The repercussions will emerge in a couple of years time when we will see students with weak analytical skills, poor logical thinking and poor reasoning power.”
4. Many students develop wrong concepts in mathematics before entering into tertiary institutions. Some teachers believe that it may be due to some secondary schools providing non-maths teachers to teach lower secondary students. Once the wrong concept is formed, it is very difficult to erase it from young people’s minds. Hence it becomes a hindrance for them to learn the right mathematics.
5. Students lack algebraic skills. In fact, most students could learn quite well if they had a good background in algebra. As suggested by a respondent, “We should look into the algebra education first, than calculus”.
6. Students forget basics, lack reasoning skills and are not used to asking questions.
7. At the polytechnic level, there are many students who are doing engineering courses with only E Maths background, not Additional Maths in Cambridge O-Level. It will help if the E Maths syllabus is adjusted to include some basics in Calculus that is necessary in adapting them to engineering maths.

(b) Barriers due to students’ attitude, and lack of motivation

1. Students lack interest towards learning calculus. They are not used to asking questions. How to motivate weaker students? —This is a great problem in teaching calculus.
2. Students just want to memorise formulas to solve problems with minimal understanding.
3. It is difficult to encourage students to be interested in learning concepts rather than just technical parts. They have mental block. They usually pay attention to the tech-niques only.
4. Many students do not have good aptitude for numbers, lack common sense and are not willing to learn.
5. Without having discipline and positive attitude towards exercises, it would be difficult for them to actually learn calculus.
6. Students feel that the objective of learning calculus is for the sake of examination only.

(c) Barriers due to communication skills

1. Most poor students (in calculus) do not know how to communicate logically their

solutions in standard format. Use of technology may enhance superficial knowledge but the most important things are manipulative and communicative skills.

2. Teaching of calculus (and mathmatics in general) has to be closely linked to language abilities. Polytechnic students generally have problems extracting information and formulating the Maths in many engineering based situational problems.

(d) Barriers due to time constraint

1. There is time constraint because of heavy emphasis on the completion of the syllabus. There is too much calculus content and students do not find enough time to practise to get themselves familiar with the techniques.
2. There is not enough tutorial time per week.
3. There is not enough time for JC teachers to prepare for new teaching materials to think over its implementation.
4. More time may be required for poly students as most of them do not have Additional Maths background at Cambridge O- or A-Level. Most of these students scored either grade 5 or 6 in their E Maths.

(e) Barriers due to traditional pedagogy and learning

1. Different teaching styles at different levels coupled with students' rote learning style confuse students. They are told that "Maths is a highly beautiful picture and whatever they learn should enable them to see a bigger portion of this picture". Instead they are given a lot of "loose pieces" of Maths that they don't know (never been told) how to connect together.
2. In terms of tertiary education, the students cannot 'transcend' themselves from 'apply the formula' and 'follow the steps' style of learning which they have been so used to in their secondary school. It becomes quite hard to change their mindset at this stage. This will remain an obstacle to the study of calculus or any other subjects unless something is being done at their earlier stages of learning.
3. At the secondary level, students are generally taught 'by rote'. Students tend to memorise formula but may not know how to apply. Too much drill and recitation. Weak students often don't know how to begin. For some students, the teachers need to re-teach the topics.

(f) Barriers due to present curriculum

1. The present JC syllabus is straightforward for students of a certain calibre. Derivations of formulas are difficult to teach and often left out. Certain concepts like limits are hard to grasp. There is lack of practicality/non immediate application; concepts are somewhat abstract. Students find it hard to manipulate

particularly due to lack of experience/exposure and foundation. They usually learn mechanical skills due to syllabus.

2. Students feel that there are too many formulas and techniques to learn. They know the simple techniques but when complicated, they are easily confused. There is too much reliance on techniques and not enough emphasis on basics carried over from secondary 3 or 4 teaching of Additional Mathematics.
3. Students can't see the applications in real life.

Main objectives of teaching calculus

Teachers were invited to *check their main objective of teaching calculus, by ticking only one box*. Responses are summarised in Table 9.

Here, the objective (i) seems to be the most favourable objective of teachers in all three groups. This means that most teachers' main objective is to help their students in developing their problem solving and reasoning skills. Preparing students for university education (objective (ii)) is the second favoured objective among the JC teachers. On the other hand, polytechnic teachers appear to favour objective (iii) more than any other objective, except (i).

Most teachers from junior colleges and polytechnics gave very low priority to objective (iv), that is, 'developing higher order thinking skills' (15.4% for JC and 5.2% for poly). This percentage was higher for university teachers (40%)

Table 9. Teachers' main objective of teaching calculus, in per cent and rank

Main Objective		JC		Poly		Univ		All	
		%	Rank	%	Rank	%	Rank	%	Rank
i)	Helping students develop problem-solving and reasoning skills	40.7	1	50.6	1	50.0	1	45.5	1
Ii)	Preparing students for Univ education	24.2	2	5.2	5	10.0	3	15.2	2
iii)	Developing Manipulative Skills	15.4	3	15.6	2	0.0	-	14.6	3
iv)	Helping students develop higher-order thinking skills	15.4	3	5.2	5	40.0	2	12.4	4
v)	Preparing students for jobs/careers	0.0	-	10.4	4	0.0	-	4.5	5
vi)	Others/no choice	4.4	4	13.0	3	0.0	-	7.9	6
	Total	100		100		100		100	

Suggestions for improving calculus education

The information sought in the survey also included comments and suggestions related to appropriate role and use of technology and appropriate pedagogy for teaching and learning calculus. Many of the respondents gave detailed comments and suggestions. The following subsections summarise the teachers' comments that cover a range of issues such as curriculum, facilities, teaching/learning material, use of technology, teaching methods, infusing thinking and creativity, and others.

(A) Suggestions for improving curriculum

Some of the JC teachers feel that the current calculus curriculum is adequate, more topics should not be added in and that it is sufficient to prepare students for university requirements. On the other hand, many of respondents of all three groups pointed out that the current calculus curriculum places a lot of emphasis on techniques and theories. In particular, the calculus course in the 'A' level is too theoretical. Teachers complained that many students are not able to appreciate the beauty of some topics like family curves, polar curves and functions. We list below various suggestions of the respondents from all three groups.

(i) Students should be strong in pre-calculus materials before they are allowed to follow up with calculus content.
(ii) The A-level students should have sound knowledge of basic mathematical functions such as algebraic, exponential, trigonometric, and logarithmic functions. They should have ability to recognise and manipulate all functions according to their properties.
(iii) The calculus curriculum at every level should be changed and geared towards the direction of enabling students to appreciate the application/ideas of calculus in the real life. Cut down on the number of topics in the syllabus so that each topic can be dealt with in greater depth and so that it is possible to incorporate the use of technology and project based work. However, basic theory should be covered.
(iv) There is a need for more integrated interdisciplinary curriculum i.e., pace topics to go hand in hand with some topics in Physics (like complex numbers with circuits, curvilinear motion with projectiles etc.)
(v) Give more stress on understanding of basic concepts than familiarity with techniques only. The students need to conceptualise before applying the formulae.
(vi) Calculus should be made more interesting by including a short history of Calculus problems/projects into the curriculum.

(vii) Reduce weightage of final examination —assign some marks to tutorials, projects, laboratory work, and class participation. There should be at least two tests and an examination.

(viii) (For polytechnic students) Too little time is given to cover too wide the range of the applications. Calculus curriculum in polytechnics should be more focussed, some of the applications should be cut, curriculum should be spread over a longer period and linked closely to engineering application problems.

(ix) (For Polytechnic students) Too much emphasis is placed on the theoretical aspects, not enough on the actual engineering based applications. Include more engineering applications and modelling in the tutorial session.

(B) Suggestions for teaching strategies

It is now a well-known fact that understanding the concepts more clearly (instead of just learning manipulative skills) will facilitate analytical skills in applications. Several teachers stressed on conceptual understanding but the main difficulty is how to liven up content and make students see the applicability of the concepts. Their views are summarised as follows:

(i) It's important for mathematics teachers to make the subject easy and interesting. They should use approaches that are both interesting and effective and stimulate their students to think more creatively about each topic.

(ii) Teachers should give students lots of help to reinforce their basic skills, help them to appreciate mathematics as a whole, enable students to see the usefulness of this subject, and help them to relate calculus to real life problems.

(iii) Teachers have to use a "right easy method" (sic) to introduce difficult topics. In fact, the way we go about approaching the topics and teaching them to see different ways is crucial to the students' learning process.

(iv) Develop concepts first if possible before embarking on a technique in problem solving. Concepts must be clearly taught.

(v) Employ "enquiry" method of teaching. Make use of co-operative learning in tutorial classes. Use more interactive approaches.

(vi) Recognise the need that teachers' handouts/notes should flow in a more logical sequence. This will ease students' learning.

CONCLUSION AND OVERVIEW OF THE SURVEY

Calculus is a crucial course for any student enrolled for a polytechnic engineering diploma and a science, engineering or mathematics degree. It is also becoming important for students in economics, management, or biological sciences. Because of its importance, calculus is now being taught right from the high school in Singapore and several other countries. However, a growing number of mathematicians have expressed their distress over the low-pass rate and the manner in which calculus is taught or appreciated by many students (Barrett & Teles 1988; Cipra 1988; Culotta 255; Douglas 1986; Ferrini-Mundy & Graham 1991; Ferrini-Mundy & Lauten 1993; Klein & Rosen 1997; Tucker & Leitzel 1995; Solow 1994).

The findings of this survey carry a sobering message to the mathematics community and curriculum planners. Our study indicates that there are many mathematics teachers in JC and Poly who have qualifications lower than BA/BSc (Hons) in Mathematics: About 20% respondents in junior colleges and 42% in polytechnics do not have honours in mathematics. As expected, all university teachers surveyed have Ph.D. in mathematics. The survey also indicates that one-fourth of the teachers used to perceive calculus as average when they were students; this percentage is higher for JC teachers. On the other hand, overall 86% of the teachers surveyed have more than two years of teaching experience, 68.1% have taught calculus for more than three semesters and a 54.5% of them (as mathematics students) used to find calculus as an interesting subject.

We remark that most mathematics teachers in Singapore have better mathematics background than those of many other places; for example, in California in 1990–91, 50% of public high school teachers did not have even a minor in the subject (Jackson 1997a, p. 819). Nevertheless, we agree with Stein (1997) who wrote, "If all teachers were mathematically well prepared, I for one would stop worrying about the age-old battle still raging between 'back to basics' and 'understanding'". Earlier, it was found by Lim (1991) that while pre-service secondary mathematics teachers were generally capable in manipulative skills in calculus, there was in nearly half of thirty-one trainees a lack of deeper understanding of concepts and an inability to explain these concepts clearly. We, therefore, believe that the teachers' knowledge of the subject is most important for a meaningful mathematics education. In order to help secondary, JC, and poly mathematics teachers to develop a deeper understanding of mathematics, more workshops, in-service courses, and advanced diplomas in mathematics need to be made available.

The study shows that only about 6% of 178 respondents perceive that a majority of their students consider calculus as an easy and interesting subject. On the other hand, 31% teachers claim that most of their students find calculus as hard, and further 17% complain that this subject is boring for most of their students. Many of the other teachers (overall 57.3%) have perceived that a majority of their students view calculus as of average difficulty. There is also some indication of a sizeable proportion of students who do not like calculus and of students who may not achieve a C grade or better in the subject.

The findings of the survey point out that a majority of students have two main difficulties in learning calculus: (i) Recalling pre-calculus material and (ii) Retention of the material learnt in calculus in the long run. Overall 46.6% of the teachers have revealed that most of their students face difficulties in recalling pre-calculus material, while 18% have reported that a majority of their students retain a little of both techniques and ideas. However, the first problem seems to be more serious for polytechnic students: 70 per cent of the polytechnic teachers surveyed have complained that most of their students face difficulties in recalling pre-calculus material learnt in O-level.

Students' approach to learning is one of the most important factors in learning calculus. Overall three quarters of the teachers surveyed have reported that most of their students' approach to learning calculus is based on techniques only. However, many of the teachers claim that their students generally do not make use of concepts, applications or reasoning skills (45% for reasoning skills and over 30% for concepts or applications

All three groups of respondents cited many barriers in improving calculus education in Singapore. Firstly, many students enter JC, Poly, or Univ with a weak foundation in mathematics: they are weak in algebra and trigonometry, have poor retention, forget basics, lack reasoning skills, have weak analytical skills, are poor in concept formation and are not used to asking questions. Secondly, there are obstacles due to students' attitude and fears: they show a lack of interest, have a habit of memorising with minimal understanding, exhibit lack of motivation and have a negative attitude towards problem solving. Thirdly, there are obstacles due to communication skills: students generally do not know how to communicate logically their solution in standard format. Fourthly, there is time constraint because of heavy emphasis on the completion of the syllabuses. Fifthly, there are several barriers due to traditional calculus curriculum, pedagogy and approaches. Present syllabuses focus too much on techniques —there is insufficient emphasis on basics, there is a lack of practicality (applications are in the distant future), and there are very few proofs in calculus. Different teaching styles and different expectations of students' styles of learning at different levels confuse students. At secondary and JC levels, the students get used to the style of applying the formula, following the steps and memorising procedures. Thus, at tertiary level, it is hard to change their mindset and practices.

The approaches used in teaching any subject depends on the teacher's main objective of teaching it. The most commonly chosen main objective of teaching calculus for all three groups of teachers is that of helping students develop problem-solving and reasoning skills. This was chosen by 50% of the university and polytechnic teachers and by 40% of the junior college teachers. For university teachers, a close second choice (50%) was developing higher order thinking skills. However, this was chosen as main objective by only 15% of the JC teachers and 5% of the poly teachers.

The teachers' comments have covered a range of issues such as improving calculus curriculum, teaching methods, use of technology, infusing thinking and creativity, need for providing training, and others. The following paragraphs summarise the main suggestions offered by the respondents of all three groups.

1. The calculus curriculum at every level should be changed and geared towards the direction of enabling students to appreciate the application/ ideas of calculus in the real life.). In any revised calculus curriculum, there should be a balance in emphasis on basic skills, conceptual understanding as well as in problem solving and applications. Efforts should be made to infuse thinking and creativity in calculus curriculum.
2. Give stress on understanding of basic concepts rather than familiarity with techniques only.
3. Reduce weightage of final examination —assign some marks to tutorials, projects, tests, laboratory work, and class participation.
4. Employ "enquiry" method of teaching. Make use of co-operative learning in tutorial classes. Give them open-ended projects and writing in calculus.
5. Make use of technology coupled with new and innovative pedagogical techniques towards calculus education. Use of technology for drill and practice is useful for lower level of learning. Use of technology for self-exploration and problem solving is suitable for higher level of learning for better students.
6. Have computer laboratories, provide facilities in lecture theatres, provide for hardware/technical support to the teachers, and have appropriate textbooks that require the use of some software and have problems real to life.
7. Teachers should develop formulas with students to involve thinking rather than just dump the formula on them. They must encourage students to think more —don't spoon feed and don't encourage them to memorise.
8. Conduct in-service and/or certificate/post diploma courses for teaching calculus.

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