Journal of the Korea Society of Mathematical Education Series D:
Research in Mathematical Education
Vol. 2, No. 2, November 1998, 71–78

韓國數學教育學會誌 시리즈 D:
<數學教育研究>
제 2 권 제 2 호 1998 년 11 월, 71–78

An Analysis of the Practice of Proof Education in Korea — Focused on the Middle School Geometry

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This paper investigates the practices of proof education in Korea by analyzing the teaching and learning of proofs in classes in the second year of middle school. With this purpose, this study examines the features and deficiencies of the ways of teaching proofs and investigates the difficulties which students have in learning them. Furthermore, it suggests methods for the improvement of teaching proofs.

1. INTRODUCTION

Since the days of ancient Greece, proof has played a central role in school mathematics, not to mention being a fundamental part of mathematics in general. In school mathematics, proofs have been authorized as a main method for developing deductive reasoning ability and promoting the understanding of mathematics.

This thesis investigates the practices of proof education in Korea by analyzing the teaching and learning of proofs in classes in the second year of middle school. With this purpose, this study examines the features and deficiencies of the ways of teaching proofs and investigates the difficulties which students have in learning them. Furthermore, it suggests methods for the improvement of teaching proofs.

In this study, the descriptive observation method of the qualitative ethnographic research method is used to the general view on proof lesson. Also the method of informal interview is used, which is to collect the responses of teachers and students using descriptive questions about proofs and proof lessons. Moreover, the details of the proof lessons and interviews are audiotaped and videotaped to facilitate later analysis.

The subjects of the analysis of proof lessons are the instructions of teacher-Y in K-middle school and teacher-J in S-middle school. Ten hours of three classes in S-middle school and twenty hours of five classes in K-middle school are recorded.

2. The Ways of Teaching Proofs

The analysis of the methods of teaching proofs reveals that mathematics teachers make every effort to teach them to students who have difficulties and reluctances in learning them. Mathematics teachers explain the content of proofs as fully as possible in order to help students learn, but tend to overlook the essential feature of proofs. From now on we will examine the various ways by which mathematics teachers attempt to teach proofs as easily as possible, then investigate the part that should complement teaching proofs by discussing the insufficient points in proof teaching.

First, mathematics teachers attempt to contextualize in various situations to help in the learning of proof by students. They contextualize the meaning of the statements to be proved with various figures before they start the proof, which can be thought to help students understand the meaning of statements represented in the form of a sentence. For example, the following figures were drawn by teacher-Y in order to help students understand the meaning of statement, "in an isosceles triangle, the bisector of the vertical angle bisects the base side and meets vertically."

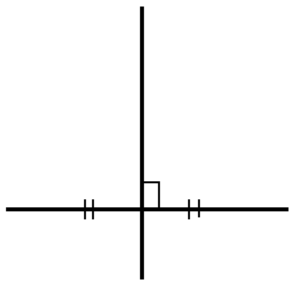

Figure 1. Perpendicular bisector

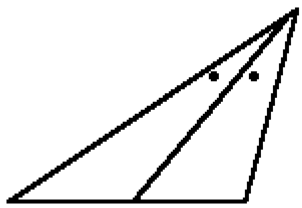

Figure 2. Interior angles

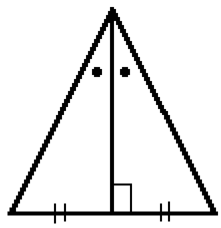

Figure 3. An isosceles triangle

Mathematics teachers attempt continually to contextualize while they prove a statement. Most of the proof in middle school geometry depends on figures. Mathe-matics teachers use colored chalk to help students understand the figures. This device can be said that teachers regard for the "duplication obstacle" of students.[1]

Furthermore, mathematics teachers attempt to contextualize after they complete a proof. In explaining the proved statement, mathematics teachers use more flexible signals instead of the symbols used in proving. For example, in explaining the proved statement,

[1] In this study, "duplication obstacle" means the difficulties which learners have in thinking of points, lines, faces, and angles as if they exist twice in geometric figures.(Fischbein 1987, p. 93)

"The base angles of an isosceles triangle are the same size", teacher-J represented the interior angles of triangle with signals "1, 2, 3", and indicated the place to look at a triangle for convincing students of the result of the proof.

Second, mathematics teachers use simple and common words to explain mathematical concepts. In the textbook, the mathematical concepts are explained with terms as academic as possible. However, mathematics teachers explain the mathematical concept more easily by using simple and common words which are familiar to students. In response to the question, "What do you think about the description of a proof in the textbook?", teacher-Y said that she had difficulties in making students understand the description of the proof in the textbook because the description given in textbook is too difficult and complex. In addition, teacher-Y said that it was sufficient for students to have some sense about mathematical concepts.

Third, mathematics teachers attempt to proceduralize the process of proofs and contrive their own devices of teaching to help students perform them. For example, teacher-J suggested the proceduralized order of proving, "dividing a statement into assumption and conclusion $\Rightarrow$ drawing a figure $\Rightarrow$ applying the theorems $\Rightarrow$ arranging the method systematically". Also teacher-Y suggested a more or less fixed method to help students devide a statement into assumption and a conclusion. Teacher-Y helped students choose the most natural and correct sentence when "if" is among the words of which the statement consists.

Above we looked at the various ways of teaching that mathematics teachers attempt to teach proofs as easily as possible to students who have difficulties and reluctances in learning proofs. In the following, we will inquire into the part that should complement in teaching proofs by discussing the insufficient points in proof teaching.

First, it is found that mathematics teachers unilaterally follow the synthetical manner in discussing proofs. That is, mathematics teachers do not deal with the analytical feature of proofs. Mathematics teachers begin with the assumption and arrive at the conclusion, which is a typical synthetical method. Fig. 4 shows the synthetical manner which mathematics teachers use to explain proofs. It can be said that teachers' synthetical manner in explaining proofs is caused by the synthetical manner in the textbook.

However, a proof is a dynamic reasoning process unifying analytical thought and synthetical thought. In order to prove a statement, we must unify the process of analysis which investigates the prerequisite conditions to satisfy a conclusion by analysing the conclusion to be proved and the process of synthesis which leads into the prerequisite conditions from the assumption.

That is, the method of proof is found in the analytical method, and then the process of arranging the whole proof by the synthetical method follows. Hence there is no way to explain why the proof becomes so by the synthetical manner alone or to show the deep structure of the proof.

Second, mathematics teachers emphasize the result of the proof more than the process of the proof itself. For example, teacher-J emphasized and reminded students of statements proved in the previous lessons prior to the regular lesson. Also, teacher-J asked students to memorize statements proved in previous lessons during the class. Similarly teacher-Y organized the statements proved in previous lessons systematically and explained them repeatedly.

There might be actual and inevitable reasons why teachers emphasize the result of proof more although they fully explain the process of the proof. The theorems of the results of a proof will be importantly used and applied in learning mathematics later, in particular geometry, but the process of the proof itself will be rarely applied. Of course mathematics teachers know well that students will undergo great difficulties in learning mathematics later if they are not acquainted with the theorems of the results of the proofs. Accordingly mathematics teachers repeat and emphasize theorems of the results of proofs while they explain the process of a proof in detail.

However, the teaching methods that emphasize the result of a proof more than its process can have a tacit influence on the learning of students. Students concentrate on memorizing the mathematical statements by taking notice of the theorems of the result of a proof more than the process of the proof without knowing why the statements hold. They can miss the opportunity to foster the power of mathematical thought through proofs education.

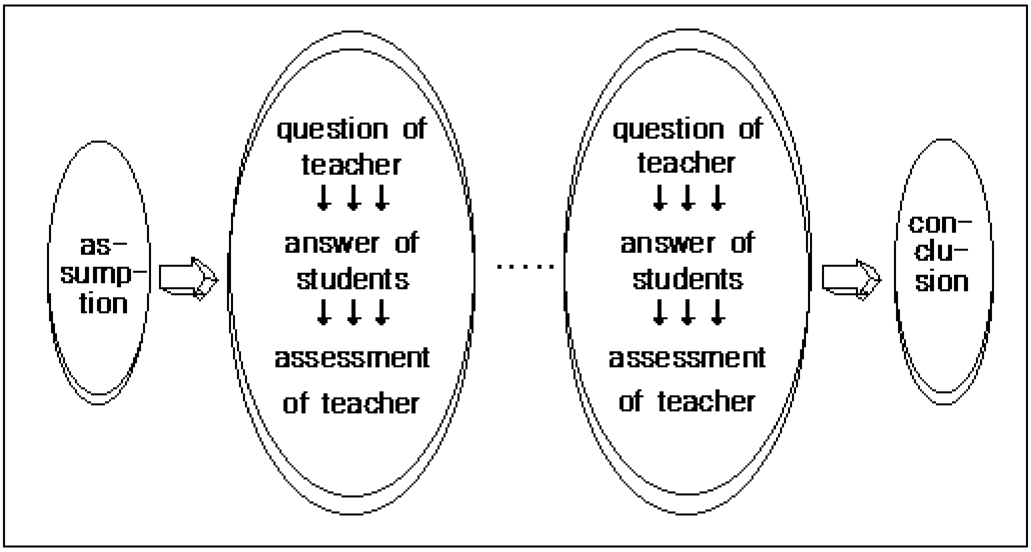

Figure 4. Teacher's method of discussing a proof: synthetical manner

3. The Actual State of Learning Proofs

In the following, we will examine the actual state of students' understanding of proofs, focusing on the difficulties which are revealed in the learning process. The difficulties that have been revealed through the analysis of the actual state of learning proofs suggest concrete and practical implications on the direction of improvement of teaching proofs.

First, the difficulty that students most often undergo in the learning of proofs is that they do not find the method of the proof at all. Most students do not attempt to prove new statements at all, and they only repeat the method of proofs explained by the teacher. Most students recognize a proof as something that can not be found by their own thought. Moreover, most students think that he or she can only memorize the proofs explained by the teacher and the textbook.

This phenomenon can be caused by the teaching of proofs unilaterally in a synthetical manner. The synthetical manner that begins with an assumption and leads to a conclusion linearly does not show students why the proofs can be done. Proof education focusing on the synthetical manner unilaterally makes students feel that they cannot help memorizing a proof because the proof already exists without their effort. Hence, the analytical manner should be introduced in teaching proofs. Proof as a dynamic reasoning process that finds the clue of the proof method in an analytical manner and completes the proofs in a synthetical manner must be given to students. We should help students perform proofs by themselves by using both the analytical manner and the synthetical manner properly.

Second, most students have difficulties in interpreting the proofs problem of the form "if A, then B". Most students do not know the exact meaning of the assumption and the conclusion in a proof problem of this kind, and they show the cognitive obstacles of using the conclusion arbitrarily or restating the statement to be proved during its proving. This may be caused by the formal teaching of assumptions and conclusions. Therefore, we should teach the statement of the form "if A, then B" gradually rather than teach it directly through the formal explanation of assumptions and conclusions.

In addition, the proof problem of the form "if A, then B" is different from the usual problem which students are already familiar with. In a usual problem, students must find the solution from the given condition, focusing on whether the solution is correct or not rather than the process leading to the solution. On the other hand, in a proof problem, the conclusion which students should arrive at is already given, and the students should focus on the process used to arrive at the conclusion. Accordingly, the proof problem is very strange and difficult to students who have been familiar with the usual problem. Hence we should give students a learning environment which will help them begin with familiar problem type and work into the proof learning. Rather than giving both assumption and conclusion, we need to give the assumption alone that corresponds to the condition of the usual problem, then have students examine the conclusion that follows from the assumption of a given condition, which can help students recognize the need of proofs naturally in the course of examining why their own conclusion holds

Third, most students have difficulties in representing the assumption and conclusion with symbols and in using symbols when they perform proofs. Most students have been familiar with "sign as signal" in which they must find the value and range of a sign or arrange it from a complex form into a simple one, such as "$2x + 4 = 2$".[2] But the sign in a proof is "sign as symbol" in which students have to think about what meaning the sign represents and, at the same time, must express the relationship between concepts via the sign, such as "$\angle$A $\angle$B". Students must perform proofs by developing a thought activity via the means of symbols, which is a difficult factor in the learning of proofs. Hence, we should introduce the symbols slowly instead of using them impetuously in teaching proofs. For example, we can think of a device of teaching that makes students explain the assumption, conclusion, and proofs in words, then express those with symbols again.

Fourth, enough time is not being allotted for students to investigate the method of proofs during classes. Most students complain that they do not have enough time to prove by themselves. This phenomenon requires us to reconsider the quality and quantity of proofs education.

Above, we analyzed the practice of proofs education being performed in mathematics classrooms via the descriptive observation method of qualitative ethnographic research methods. Through this analysis, it was revealed that mathematics teachers made every effort to teach proofs to students who have difficulties and reluctances in learning proofs. However, the essential features of proofs were overlooked in proof teaching, and most students had great difficulties in learning proofs and did not perform the proofs at all in spite of mathematics teachers' every effort.

4. The Direction for Improvement of Teaching Proofs

In this section, we will examine the direction of the improvement of teaching proofs on the basis of the results analyzed in the previous sections. The direction for improvement of teaching proofs will be helpful to researchers of curriculum development, authors of textbooks, and mathematics teachers.

First, we should teach proofs as a dynamic reasoning activity that unifies the analytical thought and the synthetical thought. In section 2, we identified that proof teaching at

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[2] It follows the classification of Van Dormolen(1986) that we classify the signs with "sign as signal" and "sign as symbol".

present is dealing with the synthetical face of proofs only. However, the synthetical manner alone can not explain the reasoning behind the proofs and can not show the deep structure of the proof. Moreover, the proof teaching by the synthetical manner alone brings about the result of imposing the proof as mere record of knowledge on students instead of being a real mathematical thought activity. Hence, we should reflect on both the analytical manner and the synthetical manner in proof teaching in order to teach proofs as a real mathematical thought process rather than a fixed ritual form of mathematics. We should teach students the method of analysis which examines the prerequisite conditions to satisfy a conclusion by analysing the conclusion to be proved and the method of synthesis which leads into the prerequisite conditions from the assumption, which can help students perform a proof by themselves.

Second, we should have students guess the conclusion by themselves by giving the assumption alone instead of giving both assumption and conclusion, then perform the proof by having them justify the truth of their own conclusion. That is, we need to search for more meaningful proofs education by unifying the context of rediscovery which makes students guess the various conclusions by themselves from the given assumption and the context of justification which examines whether their own conjecture is right or not. This device for the improvement of proof teaching will be helpful in making students recognize the need of proof naturally in the course of examining why the guessed conclusion holds and making students fully understand the meaning of assumption and conclusion in proofs.

Third, we should teach the statements of the form “if A, then B” gradually. In section 2, we identified the fact that the assumption and conclusion were formally given in the statements of the form “if A, then B” without the contextual meaning of them being explained. Also, in section 3, we identified the fact that most students had great difficulties in the statements of the form “if A, then B” and went through the cognitive obstacle of performing proofs by misunderstanding the meaning of assumptions and conclusions.

Hence we should make students familiar with the statements of the form “if A, then B” by teaching it gradually from elementary school. In fact, we can identify the statements of the form “if A, then B” tacitly appearing in the textbook of elementary school in Korea. In higher grades of elementary school, we should help students experience comparing the statements, “if A, then B”, “A is B”, and “in A, B holds”. Also, in middle school, we should give students various activities such as “Does B hold only if A?”, “In the statements of the form “if A, then B”, how will B be if some conditions of A are eliminated?”, “In the statements of the form “if A, then B”, how will B be if some conditions are added to A ?”, and “In the wrong statements of the form “if A, then B”, how can the wrong statement be changed into a right statement by amending A or B?”

Finally, the most important activity in learning a proof is that students themselves perform the proof by contemplating the statement to be proved because proofs is not the type of knowledge that can be obtained by mechanical repetition. Hence it is desirable that students have enough time during classes for exploring the method of proofs on a few statements rather than having the teacher present proofs on many statements to students one-sidely. Moreover, choosing contents worthy of rediscovery and justification, we should help students foster the power of thinking mathematically and reasoning logically through proof education by making them contemplate proofs enough. This device for the improvement of teaching proofs requires us to reconsider the quality and quantity of proof education, and ask for the qualitative change in the mathematics curriculum.

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