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What Geometric Ideas Do the Preschoolers Have?

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In the article, an analysis of solutions to six problems of geometrical character at the begining of school attendance is shown. Problems were assigned to students individually and were evaluated from different points of view.
The previous research was focused mainly on their arithmetical competence. But geometry also belongs in primary education. Therefore we prepared an analogous investigation which was focused on geometrical competence.
The experiment confirms that our children have, at the beginning of school attendance, a good level of visual appreciation of their surrounding world. Our schools do not systematically develop further this skill.

0. INTRODUCTION

Let us consider these children's drawings (see Figure 1) as a starting point for this contribution. How did these drawings come into existence? How old were the children who drew them? The answers you will find within this article but try to answer them first yourselves.

Information on the previous research

At the beginning of the nineties, research was carried out in the Netherlands, Germany and Switzerland in which the students entering the first grade (i.e., of six-year-olds) completed a test prepared in the Freudenthal Institute (Selter 1993; Heuvel-Panhuizen, 1996). The test contained six problems. The main goal of the research was to find out:

(1) What is the mathematical competence of children entering the first grade?

(2) Whether the assessment of experts (in-service teachers of various school levels, student teachers, inspectors, directors of schools) on this competence corresponds to reality.

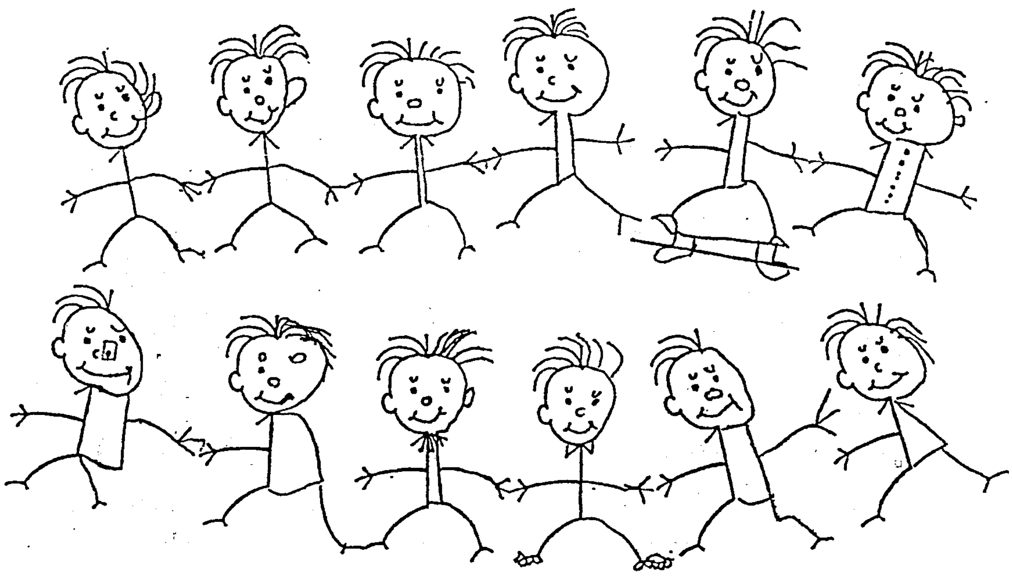


Figure 1. Drawings of six-year-olds

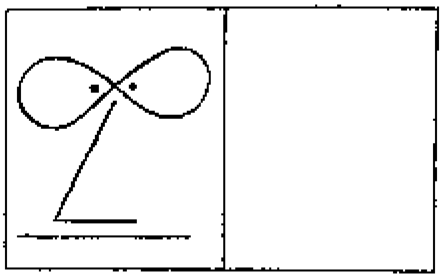
1. Investigation of geometric competence

The previous research was focused mainly on their arithmetical competence. But geometry also belongs in primary education. Therefore we prepared an analogous investigation which was focused on geometrical competence.

We joined this research in the school year 1994/95 (Grassmann et al. 1995). Our test also contained six problems. It was given to 1010 children in 27 places in the Czech Republic during October 1995 (Hošpesová, Kuřina & Tichá 1996). After evaluating the results we prepared a new set of problems for the school year 1996/97, and decided on other ways of setting problems. The research was not performed in the form of testing large groups (whole class — approximately 20–25 students), but four experimenters had individual interviews with 159 students from 7 schools (88 boys 71 girls). This research was preceded in May 1996 by a pilot investigation performed in kindergarten. Some of the problems were modified in the light of the results of this pilot research. The students worked with pictures of 16cm × 10cm sizes. On one sheet of A4 paper two such pictures were placed. We gave the sheets to the students one by one. The pictures and instructions follow:

Problems

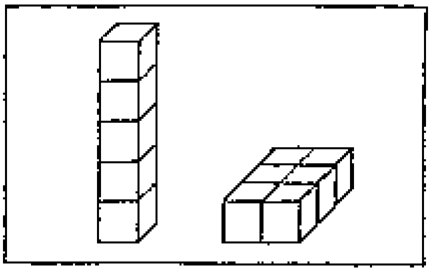
1. Face



Look carefully at the picture and draw the same one.

Figure 2a

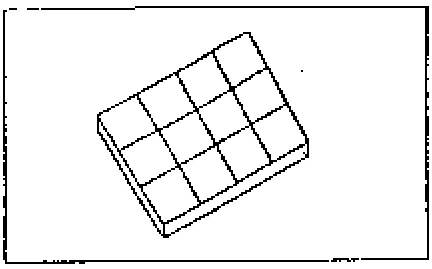
2. Cubes



You should build a house according to the picture. Colour that one for which you need more cubes.

Figure 2b

3. Chocolate



Draw how can you divide the bar of chocolate into two equal halves

Figure 2c

2. What Can We Learn from the Pictures?

In this part of the contribution we look at children's solutions to Problem 1 (face), Problem 4 (boy) and Problem 6 (umbrella). It can't be simply decided, whether they are solved correctly. But the results present a series of impulses for thinking. Some of them we will present here.

In Figure 1 we can say that although the original drawing of the left part is without gesture, children enlivened the shape in the course of drawing. We call this *phenomenon animation.*

Before reading further, look carefully at the first task in Figures 2a-f and six of the children's drawings in Figure 3.

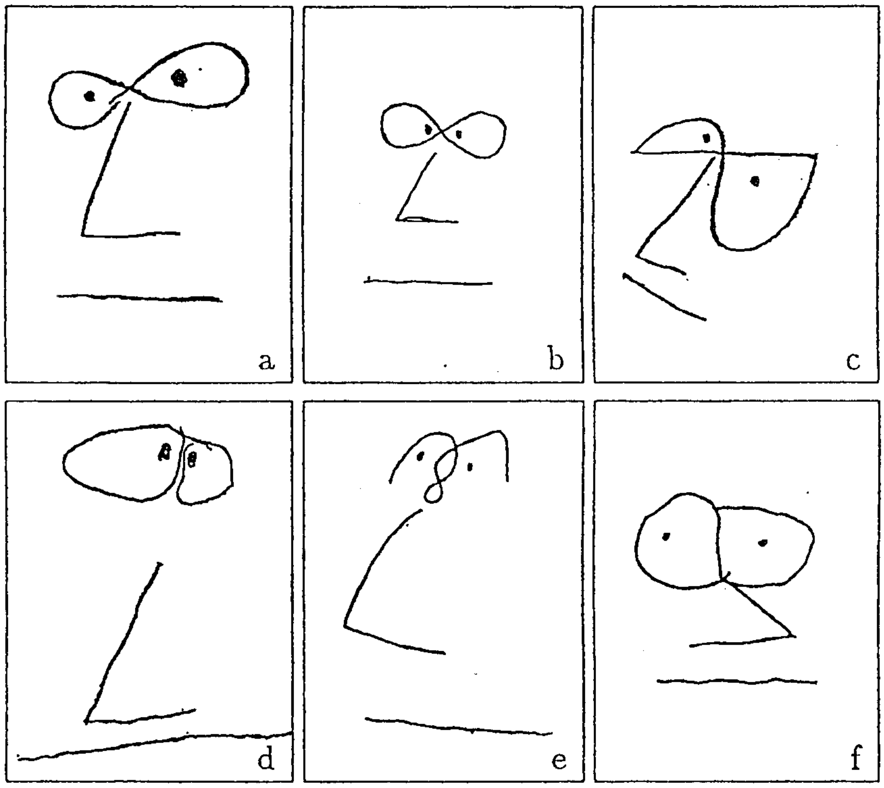


Figure 3. The children's drawings

Our intention with setting this task was to find out whether the pupils are able to perceive a relatively complicated framework of a figure and copy it. The children were not explicitly advised about its topography. Here we perceive especially a reproduction of two regions with eyes, as well as straight lines defining nose and mouth in their natural positions. Tolerating an elided mouth, which some children do not take as part of the picture, it worked out that 65% children drew a picture that can be regarded as correct. At the same time the drawing of eyes, nose and mouth varied a lot. This can be seen by comparing a picture of (we use here the names which the children gave the pictures) “director of school” (Figure 3a) and “thief” (Figure 3c), or “sad gentleman” (Figure 3d) and “kind lady” (Figure 3f).

Even here there is a distinct animation of the pictures. How can we explain it?

It appears that the picture prompts a certain image to a majority of pupils, and this gives then a foundation for reproducing it. This evidently eliminates fractional steps, and we are confident that children perceive the picture globally, as a whole. Even those pupils who are not able to copy a topologically faithful copy of the original (e.g., “a lady, who fears the dark” in Figure 3e and “kind lady” in Figure 3f), gave the picture expression, life and meaning.

Six months after administration of the tests we exposed these drawings again to the children (and their authors in some cases) along with the question “What is drawn in the picture?” The results are remarkable. A large majority of children perceive this plain drawings as faces with some expressions, and are able to describe them relatively freely. For example, dealing with Figure 3a children answered the question “Who/What is in the picture?” in words:

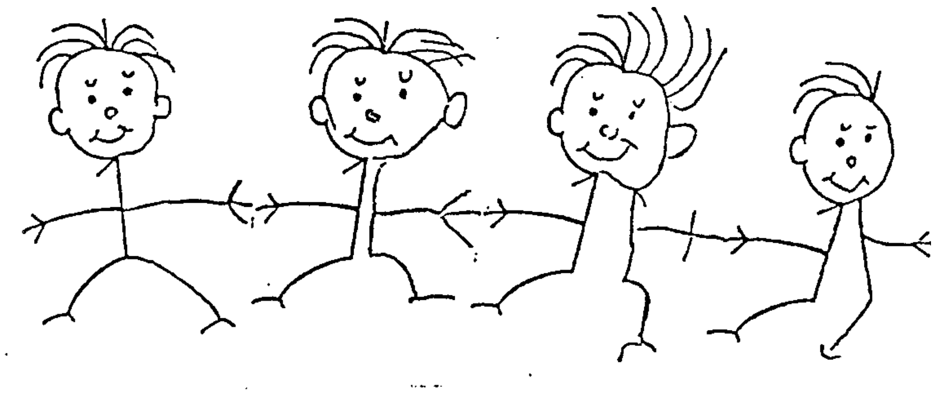
“The boss. He looks thoughtful.”
“The headmaster. He is strict.”
“A cyclist who wants to win a race.”
“He is above himself and thinks, that he has everything in the world.”
“There is something that he does not want to do and he has to do it. He is angry.”
“He is angry and thinks: I spoiled it again!”
“He is evil. He wants to do something bad; he hates somebody.”
“An angry gentleman. His car ran over his finger. He had to go to the hospital.”
“A sad gentleman. His Mummy forgot him. He has nothing to eat.”
“A kind lady. She likes her children and her husband, she is comfortable. She is a shop assistant.”
“Daddy’s friend looks that way. They are quarrelling. They work together.”
“A lady, somebody harmed her and she is so angry that she wants to kick out.”
“It’s a gentleman, he has the blues. No, it’s a lady.”
“He perhaps is angry with his mother or his dog. He drives a car.”
“In his eyes you can see that he is bad. He looks like daddy.”

"He is offended."
"Something worried him a bit."

From Problem 1 (face) we think we can deduce the following findings. Pupils are able to perceive rather complicated shapes as a whole and in the course of their reproduction respect their topological attributes (number of regions, connectivity) as well as the "local metrical" attributes (e. g., straight lines reproduced as straight). They also respect an arrangement of fragments of a drawing, but the location of particular parts of a drawing change conspicuously within the rectangle. Elementary-school teachers will agree with these findings in virtue of their knowledge of the difficulties that pupils have in practising writing. Correspondingly are problems that are connected with perception. Here the consequences of pure development of children's motor skills at the beginning of school attendance naturally comes up. That children themselves are conscious of their lack of motor skills, is obvious from frequent comments on this topic:

"I did not do it well."
"I can not do a pretty eight."
"I do not like drawing, I prefer arithmetic."
"I put a dot here that should be a cross."

With Problem 4 (boy) we looked especially at whether "a half" of a figure calls up in pupils an idea of symmetry. Our conviction of a positive response was fully justified. Our school beginners have a developed sense of primary symmetry. Naturally we are not concerned here with symmetry in the exact geometrical meaning. Likewise in real life it does not occur in a pure form. Besides, small changes in symmetry, it is said, give the whole a certain aesthetic value. In the pictures we can observe a scale of aberrations in symmetry, from almost perfect symmetry to small aberrations to asymmetry. We can illustrate this with children's drawings in Figure 4.



F*igure* 4. Symmetry in children's drawings

Asymmetry in a drawn picture indeed can not be considered as a mistake, because the text was “Finish the picture”. It consequently allows only drawing in the left part, as well as asymmetry for the whole. Remember, that only a few pupils coloured the given left part of the picture (6% of the children).

With symmetry another impulse, which in our examination appeared non-planned, showed itself, namely a grade of schematization of the picture. Only 28% of pupils regarded a vertical segment not only as an axis of symmetry, but also as a body for the figure (Figure 5a). The majority of pupils (60%) represented the body with two parallel segments or a closed region (Figure 5b).

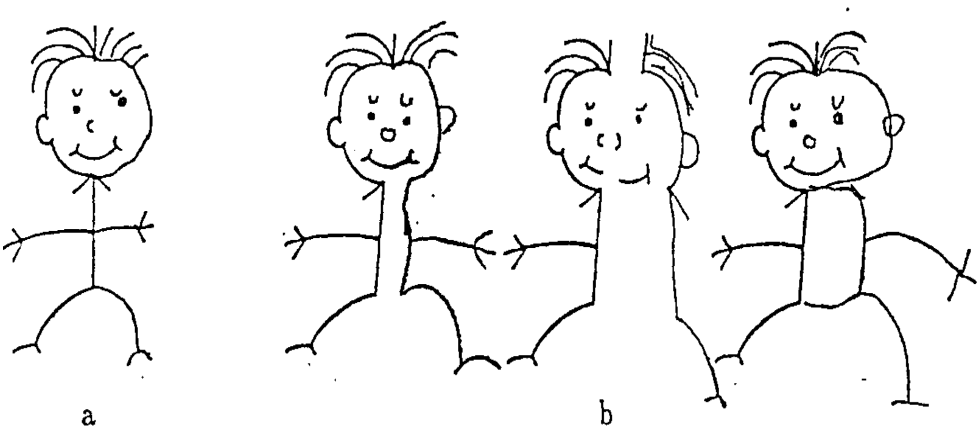


Figure 5. An axis of symmetry

These results are understandable: Axes on symmetrical shapes in nature very often do not occur, and a representation of a three-dimensional or plane shape with a one-dimensional one asks for a high grade of schematization.

In a series of children’s results we can document that animation of the drawn figure is gotten by breaking either the symmetry or the asymmetry of a drawing.

With Problem 6 (umbrella) we wanted to find out, what number of pupils not only understand a described orientation in space, but also can display two views of an umbrella. It was difficult to understand the task as we can see from the comments:

“How can I do it from the front?”

“Somebody is walking and I am looking down from the sky.”

In spite of this, the results were surprisingly good. 42% of the children in our research correctly differentiated these views of an umbrella, and with a certain extent of tolerance of evaluation, drew apposite pictures. 32% of the children drew two very similar pictures.

18% drew virtually the same picture. Some of them knew, however, that there is some difference, but they were not able to draw it. The following expressions show this:

"I know what it looks like, but I do not know how to draw it."
"It does not work. We have not done it yet."
"A circle drawn down. It is difficult."
"There is one difference."
"From above and from the front are the same, only it is hollow."
"It is difficult. We put it in the bathroom this way up."
8% of the children drew only one picture.

It is notable that a range of children drew the object from the aspect of its normal use, rather than how they would actually see it. We can document that some pupils tend to visualise in the shape of a chart or engineering tracing. In Figure 6 we reproduce views only from the front.

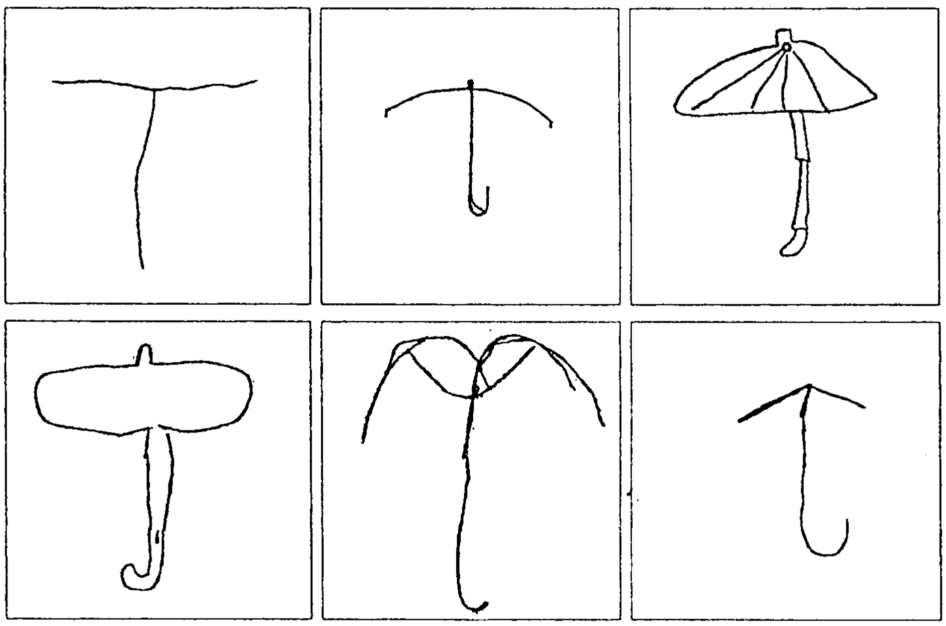


Figure 6. The umbrella (1)

We show examples of good insight with a view from the front and with an aerial view of the top of the umbrella in Figure 7.

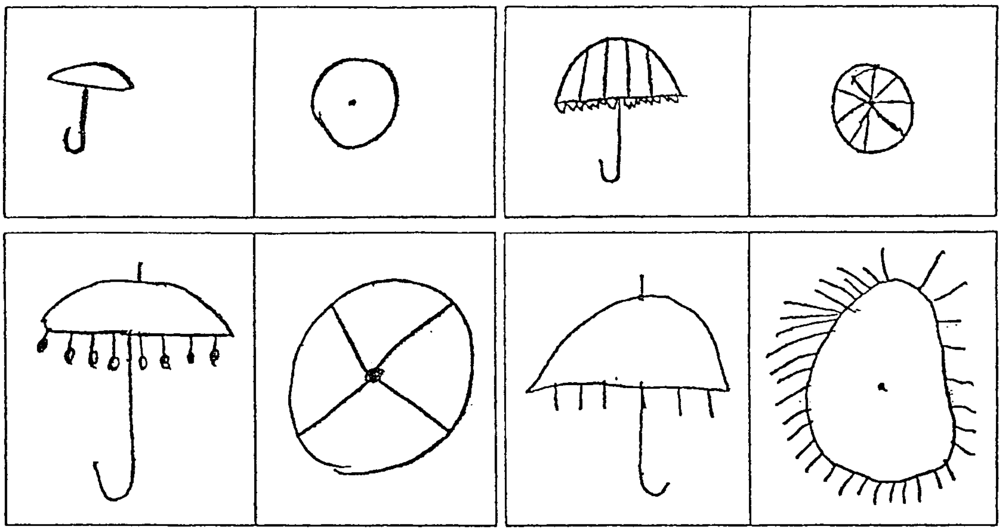


Figure 7. The umbrella (2)

In Figure 8 it is evidently the view of the front of an umbrella as carried by the observer.

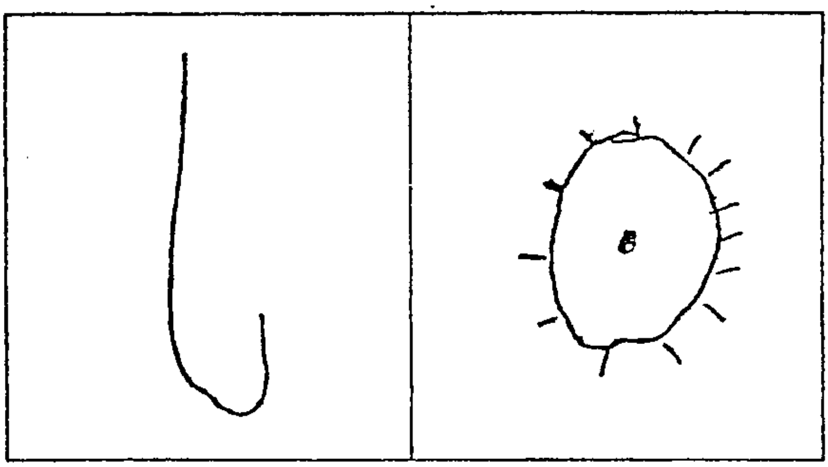


Figure 8. The view of the front

In Figure 9 we remember that some children do not realise the difference between both views. In the course of analysis it was apparent that older children were more successful in this distinction and drawing of both of these views.

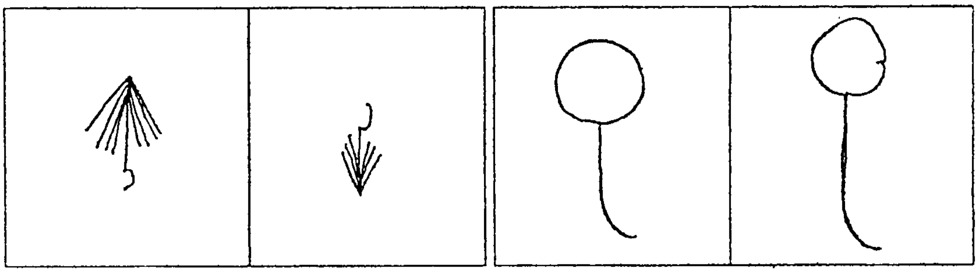


Figure 9. Distinction of views

3. THE RESULTS OF THE OTHER TASKS

Let us now look at the tasks in which the number of correct and false solutions can be exactly determined, i. e., tasks 2, 3 and 5.

The children were very successful with the solution of Problem 2 (cubes). Although children in this age have scarcely any experience with the representation of three-dimensional shapes, 72% of the pupils solved this task correctly. Even though the children mostly were not counting at school, when the experiment ran, 2/3 of the correct answers were explained with the counting of cubes. Most often the children discovered that they needed 5 and 6 cubes; or 2 + 3 and 3 + 3; or 2 + 2 + 1 and 3 + 3. Some explanations were surprisingly exact:

"The lying (building) is smaller, but I need more cubes."

Some children solved this problem with the visualisation of the appropriate activity, evidenced in explanations such as:

"It seems to me that it would be higher if we constructed it to this height."
"To do it higher, I wanted to rebuild it as a tower."

Dealing with other solutions, the question arises whether the correct result always means an understanding of the problem. Some comments of the children show that a pupil came to the correct solution through false reasoning, for example:

- the pupil counted cubes in the picture of the tower and counted squares in the picture of the house and commented in these words: "Because here, there is 12 and, here, there is 5."
- the pupil supposed that he had to build a house the same height as the tower and he

said: “I need some more, because it lies and it will be built up high.”
- the pupil estimated the number of cubes wrongly and said:

“Because it is flat.”
“This is more fat.”
“More, because it's smaller.”

Some explanation could be interpreted as well as a failure of short term memory. Some children estimated the solution:

“I saw it.”
“I guessed it.”
“I found it out. Here it would be higher.”

Incorrect solutions were naturally explained in that the tower is higher, e.g.:

“It is long and small.”
“I would make a chimney. I need more cubes for it.”
“Because of the cubes. Here they are built on each other.”
“When it lies, it is more than while it stands.”
“· · · the building has floors.”
“It is bigger, because these (cubes) are stacked up and those are laid down.”

In many cases the pupil solved the problem in a wrong way, but it's again apparent that this is due to faulty memory or a misunderstanding of the instructions.

With Problem 3 (chocolate) 73% of the children solved it by drawing a straight line according to the border of the chocolate squares. This procedure evidently reflects their experience.

Some children even intended to express a breaking of the chocolate with graphical means (Figure 10). Experience perhaps played a role in some incorrect solu-tions, e.g. a boy who has 3 siblings divided it into 4. Other children did not give out in the same portions.

Children’s comments again show that children mostly counted squares of chocolate:

“It can be 2 and 2, 3 and 3, 4 and 4.”
“I divided it into 3 and 3.”
“There are 12 squares, 6 + 6 = 12.”

Solutions dividing the chocolate in the middle of indicated squares (8%) or across the bare of chocolate (1%) appeared less often (Figure 11).

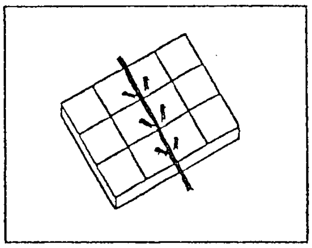


Figure 10. Graphical means

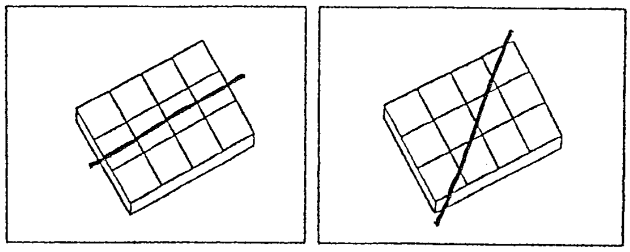


Figure 11. Dividing the choolate

In solving Problem 5 (triangles) children were very successful, although the task demanded coloring a shape according to its place in the picture. 72% of the pupils correctly solved the first part of the problem (marking a red triangle). The second part (marking a blue triangle) was solved correctly by 58%.

Most of the errors were caused by children's difficulty between right and left. Rarely did children colour other shapes or both of the top triangles.

4. Conclusions

In our opinion this experiment confirms that our children have, at the beginning of school attendance, a good level of visual appreciation of their surrounding world. Our schools do not systematically develop further this skill.

An examination of these children's answers was the stimulus for the following questions:

Q1. Does animation (picture, geometric scheme, $\cdots$) reflect incapability and dislike of coping in what is given, or does it rather show the spontaneous creativity in a child?

Q2. How can we evolve more creativity in mathematics (and in geometry especially)?

Q3. How can we make use of the relatively good geometric experience of children in mathematical instruction?

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