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PID Autotuning Algorithm Based on Saturation Function Feedback

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Abstract

We use the slope bounded saturation nonlinear feedback element instead of relay to find ultimate gain and period of linear plant. Saturation nonlinear element reduces the high harmonics of plant output. The reduction of high harmonics improve the accuracy of describing function method used to find ultimate gain and period. We give a simple procedure to find ultimate gain and period with saturation nonlinear element. A PID controller design method with known time delay element is also given, which is very useful when oscillation is not occurred with nonlinear element.

요 약

릴레이 대신 포화함수를 이용하여 선형 플랜트의 극한 이득과 주기를 결정하였다. 포화함수를 사용함으로서 플랜트의 고주파 성분을 감소 시켜 묘사함수 사용에 의한 극한이득과 주기 결정의 정확도를 증가 시켰다. 또한 적절한 포화함수를 사용할 수 있는 간단한 절차를 기술하였으며 진동이 발생하지 않는 플랜트에 대해서 지연요소를 이용한 자동동조 방법을 제시하였다.

Key words : saturation function, autotuning, PID controller, relay feedback

I. Introduction

Even though modern control theory has been significantly developed, PID controllers have been dominantly used in the industries since field engineers are familiar with the structure of PID controllers. Many methods to find PID control parameters, i.e., proportional gain, integral time, and derivative time, have been suggested and commercialized[2]. Automatically setting the parameters called autotuning of PID controllers has been received much attention because of reducing a start-up time and easy to use; simply pushing a single button to tune a controller's parameters. One of autotuning methods is of the use of relay feedback suggested by [1]. The method used the relay feedback to generate a self-sustained oscillation in the closed-loop system. With the measurements of the period and amplitude of oscillation in the plant output, the ultimate gain and period of unknown plant can be calculated using describing function approximation, which corresponds to one point identification in the Nyquist plot of unknown plant. Based on one point information on the Nyquist plot, various PID controller design methods, e.g., Ziegler-Nichols tuning formula and phase margin and gain margin, can be applied to find controller parameters. Since the tuning method[1] using the relay feedback was introduced, there have been

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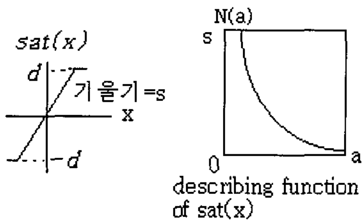

그림1. 포화함수 및 묘사함수
Fig. 1. Saturation nonlinear element and its describing function.

some efforts to develop the PID controller design method using ultimate gain and period [3, 4, 5]. Since the describing function approximation used to calculate ultimate gain and period is based on the assumption of which plant output contains only fundamental frequency component, it is desirable that the output of unknown plants does not contain the high frequency components. However relay feedback easily excites the high frequency component, since relay feedback acts like a step function, and results in containing the unwanted frequency components in the plant output. Consequently, it is possible that there is an error between the ultimate gain and period calculated from describing function and real one due to the presence of high harmonics of plant output. In this paper we use the slope bounded saturation function instead of relay feedback element to generate the self-sustained oscillation in the closed-loop system. Saturation functions reduce the excitation of high frequency component of plant, hence ultimate gain and period are more accurate than those obtained by the use of relay feedback. Moreover since most of actuators have nonlinear characteristics in the limit case, it is expected that plant output can be distorted, hence it is desirable that a slope of actuator input signal is less than that of actuator. This point can be achieved by setting a small enough slope of saturation function. However there are several technical problems to use saturation function as a feedback element: it is well known that whenever the Nyquist plot of unknown plant intersect the negative real axis, a self-sustained oscillation is possible in the relay feedback case, but it does not in the saturation function case [6]. We analyze the existence condition of self-sustained oscillation with saturation function feedback. Based on the analysis, we give a simple procedure to find a saturation function which guarantees the oscillation as well as the reduction of high harmonics in the plant output. The reduction of high harmonics in the plant output results in more accurate identification of ultimate gain and period using the describing function approximation. We illustrate the performance comparison between relay feedback and saturation function.

Ⅱ. The describing function analysis with saturation nonlinearity

The describing function method is a popular tool to analyze periodic solutions for linear time-invariant dynamic system with nonlinear feedback element. To have a self-oscillation in the plant output, the following harmonic balance equation should have a nontrivial solution[6]

$$1+G(jw)\cdot N(a)=0 \qquad (1)$$

where $G(jw)$ is the frequency response of plant, a is the amplitude of fundamental component of plant output, and $N(a)$ is the describing function of nonlinear element. When a relay is used for nonlinear feedback element, the describing function of relay is given by

$$N_{rly}=4d/(\pi a) \qquad (2)$$

where d is the amplitude of relay. Whenever G(jw) intersects negative real axis in the Nyquist plot, the harmonic balance equation (1) has a nontrivial solution and there is an oscillation in the plant output. This fact was used to justify the existence of periodic solution with relay feedback element[1]. In fact, the paper[1] used the equation (2) to find $N_{rly}(a)$ with a

taken from the observed amplitude of plant output. The observed period of plant output and $N_{rly}(a)$ are taken as a ultimate period and gain of unknown plant, respectively. Since we use the saturation function defined by

$$sat(x) = \begin{cases} -d & x < -d/s \\ sx & -d/s \le x \le d/s \\ d & x > d/s \end{cases}$$

as a nonlinear feedback element to reduce the high harmonics in the plant output, we need more analysis for the existence of periodic solution in the output. The describing function of the saturation nonlinear element in Fig. 1 is given by

$$N_{sat}(a) = \begin{cases} s & \text{if } d > sa \\ \frac{2s}{\pi}\left[\arcsin\left(\frac{d}{sa}\right) + \frac{d}{sa}\sqrt{1-\left(\frac{d}{sa}\right)^2}\right] & \text{if } d \le sa \end{cases} \quad (3)$$

From Fig. 1, one can observe that $0 < N_{sat} \le s$. Therefore it is possible that if the slope is not large enough, i.e., $s < |1/G(jw)|$ at negative real axis, then there is no periodic solution in the plant output, even if $G(jw)$ intersects the negative real axis in the Nyquist plot. Hence the larger slope of saturation nonlinearity is used, the more class of system, $G(jw)$, is guaranteed the periodic solution in the closed-loop system. So far we analyze the existence condition of periodic solution in the closed-loop system when the saturation function is used for nonlinear feedback element. The complete analysis of existence condition of periodic solution in the closed-loop system with general nonlinear element can be found in [7, Theorem 10.9]. Since we are interested in the reduction of high frequency component in the plant output to improve the accuracy of describing function approximation, we investigate the effect of high frequency component of plant output for finding the ultimate gain using the describing function method. Suppose that there is a periodic solution in the closed-loop system with saturation function nonlinear element. Then the system satisfies

$$y(t) = -gn(y(t)) \quad (4)$$

where $y(t)$ is the plant output, $n(\cdot)$ denotes saturation function nonlinear element, and g is linear operator in time domain corresponding to $G(jw)$. Since $y(t)$ is a periodic signal, we can represent $y(t) = y_1(t) + y_h(t)$ where $y_1(t)$ and $y_h(t)$ are fundamental frequency component and high harmonics of $y(t)$, respectively. Solving the equation (4) is equivalent to solve both (5) and (6)

$$y_h = -P_h gn(y_1 + y_h) \quad (5)$$

$$y_1 = -P_1 gn(y_1 + y_h) \quad (6)$$

where projection operator P_1 and P_h are defined by $P_1(y) = y_1$ and $P_h(y) = y - y_1 = y_h$, respectively. Without no loss of generality, we assume that $y_1(t) = a \cdot \sin wt$. By adding $P_1 gn((a+\delta a)\sin wt)$ to both sides of (6), we can rewrite it as

$$y_1 + P_1 gn((a+\delta a)\sin wt) =$$
$$-\{(P_1 gn(y_1 + y_h) - P_1 gn((a+\delta a)\sin wt)\} \quad (7)$$

We obtain the following phasor equation from (7)

$$1 + N(a+\delta a)G(jw) =$$
$$-(1/a)\{(\delta a \cdot N(a+\delta a) + ((P_1 n(y_1+y_h)$$
$$- P_1 n((a+\delta a))\}G(jw) \quad (8)$$

Note that since the measurement of plant output contains high harmonics in general, we add the term δa in (7). The right hand side of equation(8) represents the error term of describing function approximation and it causes the error in the calculation of ultimate gain. If $y(t)$ does not contain high harmonics, i.e., $y_h = 0$ and $\delta a = 0$, the equation (8) results in equation (1). Hence it is desirable reducing high harmonics of plant output. Since the input of plant is the output of nonlinear element, the amplitude of high harmonics is increased as the slope is increased when a saturation function is used as a nonlinear element. The relay can be considered as the worst case in the sense of

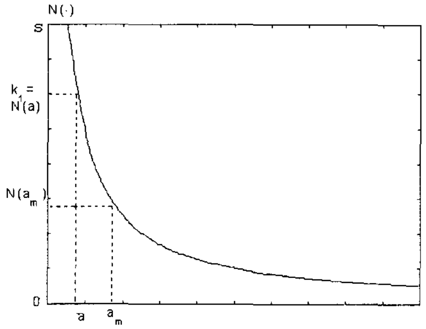

그림 2. 기울기가 s인 경우의 묘사함수
Fig. 2. The plot of describing function with slope=s.

reduction of high harmonics, since relay can be treated as the slope $\approx\infty$. To see the effect of high harmonics in the calculation of ultimate gain from observed plant output, suppose that we use the saturation function with slope s, $|1/G(jw)|$ = k_1 in the negative real axis, and the measured amplitude of oscillation is $a_m = a + \delta a$. If δa is a positive number, then $a_m > a$. Note that we describe a method to determine the sign of δa in the following section. From Fig. 2, we can figure out the describing function, $N(a_m)$, calculated from the equation(4) is smaller than the describing function with the amplitude of plant output in which does not contain high harmonics. Consequently, the calculated ultimate gain of plant, which is equal to $N(a_m)$, is smaller than the real ultimate gain of plant, k_1. The larger value of δa cause the larger error between ultimate gain calculated from describing function and real one. Therefore it is desirable to use the smallest slope saturation function as nonlinear element as long as oscillation is occurred in the closed-loop system. Next object is how do we find the smallest slope of saturation nonlinear element that guarantee oscillation. Since $0 < N(\cdot) \leq s$ for saturation function with the slope = s, the smallest slope such that the equation (1) has a nontrivial solution is equals to k_1.

III. Autotuning with saturation function

To set the lowest slope of saturation nonlinear element, we need the value of $|1/G(jw)|$ at negative real axis. However we do not know it in advance. We suggest a simple procedure to find a "good" slope of saturation function in this section; "good" slope means not necessary the lowest slope, but the slope that can have an oscillation and achieve a considerable reduction of high frequency component in the plant output to improve the accuracy of describing function approximation. Before testing the plant with nonlinear element, we know the actuator response curve which is supplied by actuator manufacture. The maximum slope of actuator, s_{max}, is a good starting point. If there is an oscillation in the output with the slope=s_{max}, we can measure the amplitude of oscillation, $a_m = a + \delta a$ and calculate the corresponding describing function, $N(a_m)$, of the saturation function with the slope=s_{max} using equation(3), The sign of δa can be either positive or negative. We describe how do we determine the sign of δa later on. For the purpose of easy explanation of idea, suppose that the sign of δa is positive. Even though the ideal slope equal to k_1, we do not know it in advance. To have an oscillation and to reduce the high harmonics, we only conjecture that a new candidate slope, s_{new}, should be $N(a_m) < s_{new} < s_{max}$. So we take a new slope as $s_{new} = \gamma N(a_m)$ where $\gamma > 1$ to be defined. The value of γ depends on high frequency component effect. Unfortunately we does not know the high frequency component. We rely on the "ad hoc" approach. We suggest that $\gamma = 1.3$, which means that the high frequency components effected approximate 30% to the value of describing function. If there is a reduction in the plant output, one can try another saturation nonlinear element with new slope and so on. If the calculated value of ultimate gain is changed little, the value of ultimate gain is taken as the best estimated one. By the same token, from Fig. 2, it can be also figured out that the calculated value $N(a_m) > k_1$ for the negative sign of δa. The new trial slope should be less than $N(a_m)$. We set a s_{max} = $\gamma_1 \cdot N(a_m)$ where $0 < \gamma_1 < 1$. One may use the $\gamma_1 = 0.7$. We can take the best ultimate gain by the similar fashion in case of the positive sign of δa. The remain problem is how do we find the sign of δa as we oppose it. The calculated describing function with the slope = s_{max}, $N(a_m) < k_1$ when $\delta a > 0$. There is no oscillation in the closed-loop system with the slope = $N(a_m)$ since $N(a_m) < k_1$. On the other hand, there is an oscillation in the closed-loop system with the slope = $N(a_m)$ when $\delta a < 0$ since $N(a_m) > k_1$. Hence we can determine the sign of δa by using the calculated describing function as the new slope. It can be happen that there is no oscillation in the plant output, when γ and γ_1 are too small. This can be quickly found out from the output of saturation nonlinear element by using the fact that whenever there is an oscillation, the output of saturation element is saturated. We summarize the procedure for finding the good slope:

A. start the test with maximum allowable slope and calculate describing function, $N(a_m)$, using (3) with observed plant output.

B. (determine the sign of δa)

set the slope= $N(a_m)$

- if there is an oscillation, $\delta a < 0$.
- otherwise, $\delta a > 0$.

C. • set the slope $= \gamma N(a_m)$ where $\gamma = 0.7$ when $\delta a < 0$ and $\gamma = 1.3$ when $\delta a > 0$

• if there is no oscillation with new slope, then slightly increase the slope.

D. calculate describing function, which is equal to the ultimate gain of unknown plant, from observed plant output with the slope found in step C.

Finally, we consider a case that there is no oscillation in the plant output with maximum slope of actuator. The delay element can be used as shown in Fig. 3. Insertion of delay element in the loop transfer function implies that the magnitude of $1/G(jw)$ does not change, but the phase is added by $-jwd_1$ for each frequency where d_1 represent amount of delay. Therefore the appropriate d_1 ensures that $(1/G(jw))e^{jwd_1}$ intersects [-s, 0]. It implies that the closed loop system has an oscillation. Similar method used finding the good slope can be applied to find the appropriate "d_1". In fact, started with small value of d_1, the value of d_1 is increased. Once we have measurement of the amplitude of plant output and period for $G(jw)e^{-jwd_1}$, we can design the PID controller for $G(jw)$ to satisfy the design specification such as phase margin or amplitude margin. For example, suppose that the critical frequency $w_c = w_1$ and ultimate gain $k_c = k_1$, which is determined from describing function approximation with saturation function feedback, for $G(jw)e^{-jwd_1}$ where d_1 is an inserted time delay. Suppose that the desired phase margin is ϕ_m. The structure of PID controller is given

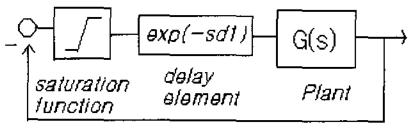

그림 3. 지연요소가 있는 폐회로
Fig. 3. The closed-loop system with delay element.

by $G_c(s) = k(1+sT_d+1/sT_d)$ where k, T_d, and T_i are proportional gain, differential time, and integral time, respectively, to be determined using w_1, d_1, k_1,and ϕ_m. Since the PID parameters of $G_c(\cdot)$ are to be chosen such that the following equation is satisfied [1]

$$\angle G_c(jw_1) + \angle G(jw_1) = -\pi + \phi_m \quad (9)$$

using $\angle G(jw_1) - w_1 d_1 = -\pi$, we can calculate $\angle G_c(jw_1) = \phi_m - w_1 d_1$ from equation (9). Following the design method of [1],

$$T_d = (\tan(\phi_m - w_1 d_1) + \sqrt{4\alpha + \tan^2(\phi_m - w_1 d_1)})/2w_1$$

$$T_i = T_d/\alpha$$

where a typical value of $\alpha = 0.25$. Since loop transfer function with the PID controller has unit gain at w_1, $|G_c(jw_1)G(jw_1)| = 1$.

$$\Rightarrow |G_c(jw_1)| = |1/G(jw_1)| = k_1$$

$$\Rightarrow k/\cos(\phi_m - w_1 d_1) = k_1$$

$$\Rightarrow k = k_1 \cos(\phi_m - w_1 d_1)$$

We can design similarly for the amplitude margin specification, and it can be shown

$$\begin{pmatrix} T_d & = & (\tan(-w_1 d_1) + \sqrt{4\alpha + \tan^2(-w_1 d_1)})/2w_1 \\ T_i & = & T_d/\alpha \\ k & = & (1/A_m)k_1 \cos(-w_1 d_1) \end{pmatrix} \quad (10)$$

where A_m is desired amplitude margin.

Ⅳ. Example

Consider the plant given by $G(s) = 1/(1+4s)e^{-2s}$. The Nyquist plot of $G(s)$ is shown in Fig. 4.

It intersects the real axis at (-0.26, 0) and w_c = 0.93 . Hence we can calculate the ultimate gain k_c = 3.84 and ultimate period $T_c = 2\pi/w_c = 6.75$. For

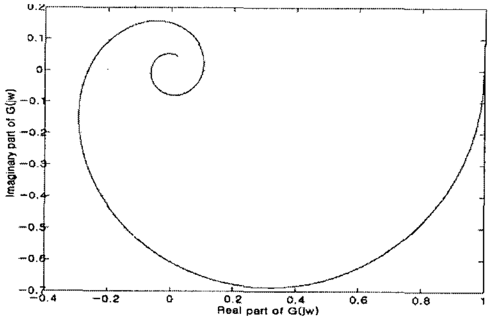

그림 4. $G(s) = 1/(1+4s)e^{-2s}$의 Nyquist 그림

Fig.4. The Nyquist plot of $G(s) = 1/(1+4s)e^{-2s}$.

the purpose of comparison, we use the two nonlinear feedback elements, relay and saturation, to find the ultimate gain and period. The second plot of Fig. 5 is the plot of the plant output using relay nonlinear

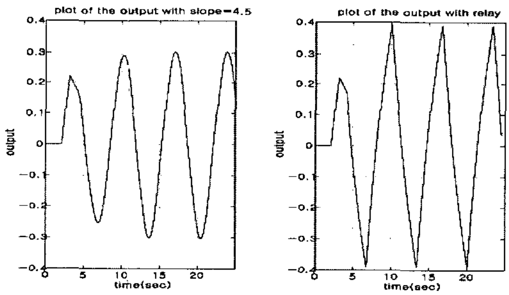

그림 5. 릴레이와 포화함수 사용시 출력

Fig. 5. The plot of plant output with relay and saturation nonlinear element.

element with an amplitude of relay=1. From Fig. 5, one can observe the amplitude of plant output, $a \approx 0.4$ and ultimate period, $T_c \approx 6.5(\text{sec})$. Using the equations (3) and (1), the ultimate gain can be found as k_c = 3.18. In the interest of simplicity to demonstrate determining the good slope of saturation nonlinearity, we assume that actuator is an ideal one; there is no limitation in the response curve of actuator. We start testing of plant with relay feedback. Since a_m = 0.4 for the relay feedback case, $N_m(\cdot)$ = 3.18, and we chose the γ = 1.4 . We set the new trail slope $s_{new} = \gamma N_m(\cdot)$ = 4.5 . The first plot of Fig. 5 is the output of the plant with the slope = 4.5 . The amplitude of output is 0.3, one can observe that the saturation nonlinearity reduces the high frequency component of the plant output. Using the equations (1) and (3), the ultimate gain k_c = 3.7 for the saturation nonlinear feedback case. The ultimate gain found with saturation nonlinear feedback is much closer to real one than that found with relay feedback. To discuss the effect of one point information from relay feedback and saturation function feedback in the controller design, we consider the design of PID controller for amplitude margin specification. Suppose that the desired amplitude margin is given by 2. Using the equation (10) with $d_1 = 0$, one can find controller parameters, $k = 1.85, T_d = 1.03, T_i = 4.13$ for saturation feedback case and $k = 1.59, T_d = 1.03, T_i = 4.13$ for relay feedback. One can verify that the amplitude margin is equal to 2.1 and 2.5 for saturation function feedback and relay feedback case, respectively. This results is expected since we already have seen that saturation function feedback can identify the ultimate gain more accurately than relay feedback.

IV. Conclusion

We propose the modified version of autotuning method based on relay feedback. We use the saturation nonlinear feedback element instead of relay feedback. Our method improves the accuracy of describing function approximation used to find ultimate gain by reduction of high harmonics of plant output in the expense of time, but not much of it. When a plant has a sharp low pass characteristics, high frequency component of could be filtered out. Hence the performance difference between relay feedback and saturation feedback could be negligible. However plant has not a sharp low pass characteristics, the proposed algorithm gives better performance over relay feedback. The proposed method can be used with other PID controller tuning method based on relay feedback.

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