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A Dynamic Production and Transportation Model with Multiple Freight Container Types*

다수의 화물컨테이너를 고려한 동적 생산-수송 모형에 관한 연구*

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Abstract

This paper considers the single-product production and transportation problem with discrete time, dynamic demand and finite time horizon, an extension of classical dynamic lot-sizing model. In the model, multiple freight container types are allowed as the transportation mode and each order (product) placed in a period is shipped immediately by containers in the period. Moreover, each container has type-dependent carrying capacity restriction and at most one container type is allowed in each shipping period. The unit freight cost for each container type depends on the size of its carrying capacity. The total freight cost is proportional to the number of each container type employed. Such a freight cost is considered as another set-up cost. Also, it is assumed in the model that production and inventory cost functions are dynamically concave and backlogging is not allowed. The objective of this study is to determine the optimal production production policy and the optimal transportation policy simultaneously that minimizes the total system cost (including production cost, inventory holding cost, and freight cost) to satisfy dynamic demands over a finite time horizon. In the analysis, the optimal solution properties are characterized, based on which a dynamic programming algorithm is derived. The solution algorithm is then illustrated with a numerical example.

1. Introduction

The single-product deterministic lot-sizing models stemming from the work of Wagner and Whitin [15] have divided time horizon into discrete-periods, and allowed for demand to occur dynamically over discrete periods and for inventory levels to be observed only at the end of each period. They have often considered dynamically-variant concave cost functions for the objective of finding the optimal lot size that minimizes the total production and inventory cost to satisfy dynamic demands over a finite time horizon. These models have been called

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Dynamic Lot-Sizing Model (DLSM) in the literature. Zangwill [16, 17] studied a problem with backlogging allowed. Sobel [13] considered a problem with start-up cost allowed. Baker *et al.* [1], Florian and Klein [3], Lambert and Luss [6], Love [9], and Swoveland [14] analyzed various problems with a variety of different limited production capacities. Bitran and Yanasse [2] and Florian *et al.* [4] studied the problem complexity and the algorithm complexity for DLSMs. Sung and Park [13] investigated rolling schedules for a single-facility multi-product problem. Sung and Lee [12, 13] studied rolling schedules for a problem with start-up cost allowed and also investigated the effect of setup cost reduction in a problem with multiple finite production rates incorporated.

As seen from the literature, DLSMs have not considered any production-inventory problem incorporating transportation activities. These days, the issue of transportation scheduling for shipping products (or delivering orders) by proper transportation modes at right time becomes significantly important in production (or distribution) management, or in import and export activities. Each company uses a freight container as a transportation unit to ship its manufactured products (or ordered products) to customers, which may lead to a managerial decision problem to select (rent or ask for services with) appropriate types from among a variety of different container types and to determine about how many of them may be needed. This provides us with a motivation to consider the optimal production (lot-sizing) and transportation problem incorporating production-inventory functions and transportation functions together.

Several articles have studied the extended works of the classical DLSM. Lippman [8] studied two deterministic multi-period production planning models; monotone cost model and concave model. Hwang and Sohn [5] dealt with a DLSM in which the transportation mode and the order size for a deteriorating product were simultaneously considered. However, they considered no capacity restriction on the transportation mode. Lee [7] considered a DLSM allowing set-up cost including a fixed charge cost and a freight cost, where a fixed single container type with limited carrying capacity is considered and the freight cost is proportional to the number of containers used.

This paper considers the single-product production and transportation model with discrete time, dynamic demand and finite time horizon, an extension of classical dynamic lot-sizing model. In the model, M container types each having type-dependent carrying capacity are allowed as the transportation mode but at most one of them can be used (rented) in each shipping period. Also, each order (product) placed in a period is shipped immediately by containers in the period. The unit freight cost for each container type depends on the size of its carrying capacity and the total freight cost is proportional to the number of each container type employed. Thus, such a freight cost can be considered as the additional multiple set-up cost. It is assumed in the model that production and inventory cost functions are dynamically concave and backlogging is not allowed. The objective of the model is then to determine the optimal production (lot-sizing) and transportation policy that minimizes the total system cost (including production cost, inventory holding cost, and freight cost) to satisfy dynamic demands over finite time horizon.

The proposed problem is described and expressed in a mathematical model in Section 2. In Section 3, the optimal solution properties are characterized. Section 4 presents a dynamic programming algorithm for finding the optimal solution. A numerical example is solved to illustrate the algorithm in Section 5. Section 6 gives some concluding remarks.

2. Model Formulation

Without loss of generality, it is assumed that for any t, M container types have their unit freight costs f_{t1}, f_{t2}, $\cdots$, f_{tM} with $f_{t1} > 0$, $f_{tM} = max_j\{f_{tj}\}$ and $f_{tj} < f_{t(j+1)}$, $j = 1, 2, \cdots, M-1$, which are related to their carrying capacities W_1, W_2, $\cdots$,

W_M with $W_j>0$, $W_M=\max_j\{W_j\}$, and $W_j<W_{j+1}$, $j=1, 2, \cdots, M-1$.

Some other notations are introduced as follows:

- T = length of the time horizon,
- t = time index ($t = 1, 2, \cdots, T$),
- j = container type index ($j = 1, 2, \cdots, M$),
- d_t = amount demanded in period t,
- x_{tj} = amount produced (ordered) in period t and shipped by container type j,
- y_{tj} = number of container type j employed in period t (nonnegative integer),
- I_t = amount of inventory at the end of period t,
- S_t = setup cost in period t,
- p_t = unit production (ordering) cost in period t,
- h_t = unit inventory holding cost from period t to period $t+1$, and
- $\delta(x)$ = 1 if $x > 0$, and 0 otherwise.

The proposed problem has the objective of determining a policy (x_{tj}, y_{tj}) for $t=1, 2, \cdots, T$ and $j=1, 2, \cdots, M$ such that all the demands over the given horizon are satisfied at the minimum total cost. Therefore, the T-period problem can be formulated in a mathematical programming as follows:

$$\underset{x_{tj},y_{tj}}{Minimize}\sum_{t=1}^{T}\left\{S_t\,\delta\left[\sum_{j=1}^{M}x_{tj}\right]+p_t\left[\sum_{j=1}^{M}x_{tj}\right]+h_tI_t+\sum_{j=1}^{M}f_{tj}y_{tj}\right\} \quad (1)$$

$$s.t. \quad I_t = I_{t-1} + \sum_{j=1}^{M}x_{tj} - d_t, \quad \forall t, \quad (2)$$

$$x_{tj} \le W_j \cdot y_{tj}, \quad \forall t, j, \quad (3)$$

$$\sum_{j=1}^{M}\delta(x_{tj}) \le 1, \quad \forall t, \quad (4)$$

$$I_0 = I_T = 0, \quad (5)$$

$$x_{tj} \ge 0, \; I_t \ge 0, \quad \forall t, j, \quad (6)$$

$$y_{tj} : \text{nonnegative integer}, \quad \forall t, j. \quad (7)$$

Note that if $M=1$ and all the relevant cost functions are time-invariably concave, the model reduces to that of Lee [7], and that if $M=1$ and $f_{tj}=0$ for all t, the model reduces to the Wagner-Whitin model [15]. The constraints (2)-(7) define a closed bounded convex set and the objective function is concave, so that it attains its minimum at an extreme point of the convex set. In the next section, the extreme points shall be characterized further in association with the optimal solution.

Firstly, the following property is derived, which plays a central role in our approach.

Theorem 1 (Inventory Decomposition Property). Suppose that the constraint

$$I_t=0, \text{ for some } t\in\{1,2,\cdots,T-1\}, \quad (8)$$

is added to those described in (2)-(7), then the optimal solution to the T-period problem can be found by independently finding solutions to the subproblems over intervals (1, t) and (t, T).

Proof. The production costs depend only on the amount produced in a particular period and the transportation costs depend only on the number of each container type employed in such a production period. Moreover, the inventory costs associated with the subproblem over the interval (t, T) also depend only on production decisions over the interval by virtue of (8). This completes the proof. Q.E.D.

Theorem 1 leads to a dynamic programming recursion using the periods $0,1,\cdots,T$ as states. We define the (u,v)-*problem* as the problem of finding the optimal solution over periods $u+1$, $u+2$, $\cdots$, v with the constraints

$$I_u=I_v=0 \text{ and } I_t>0 \text{ for } u<t<v. \quad (9)$$

Then, let D_{uv} be the corresponding minimum cost of the (u,v)-problem. Also, let F_v be the cost associated with the optimal solution over periods $0, 1, \cdots, v$, given that $I_v = 0$. Then we can solve the T-period problem through the following recursive equations;

$$F_0 = 0,$$

$$F_v = \min_{0 \le u \le v} \{ F_u + D_{uv} \},$$
$$\text{for } v = 1, 2, \cdots, T. \qquad (10)$$

Unfortunately, this recursion may not be too useful, since, in general, finding the values of D_{uv} can be almost as difficult as solving the original problem. Moreover, the optimal solution properties that were characterized by Lee [7] may not hold in our problem. Hence, to seek a better way of finding D_{uv}, we will characterize the optimal solution properties of the (u,v)-problem in the next section.

3. Optimal Solution Properties

Let period t as a *production point* if $\sum_{j=1}^{M} x_{tj} > 0$, and period s as a *regeneration point* if $I_s = 0$. Also let period k as an *inventory point* if $I_k < d_{k+1}$. Then, the following property may be obtained.

Theorem 2. In the optimal solution of the T-period problem, period t must be a production point if period $(t\text{-}1)$ is an inventory point.

Proof. Consider any feasible solution $(x_{11}, x_{12}, \cdots, x_{TM})$ and the corresponding $(I_1, I_2, \cdots, I_T)$ determined by (2)-(7). Since period $(t\text{-}1)$ is an inventory point, $I_{t-1} < d_t$ and no shortage is allowed, we must have $\sum_{j=1}^{M} x_{tj} > 0$. This completes the proof. Q.E.D.

Let container type m be a *fraction container* at period t if $nW_m < x_{tm} < (n+1)W_m$ for some $n \in \{0, 1, 2, \cdots\}$. Also, let period l be a *fraction point* if that period includes a fraction container. Then, the following property is derived.

Theorem 3. In the (u,v)-problem, there exists the optimal solution that includes at most one fraction point.

Proof. Suppose that there exists the optimal solution $(x_{(u+1)1}, x_{(u+1)2}, \cdots, x_{vM})$ with two fraction points s and t $(u+1 \le s < t \le v)$ in which container types m_s and m_t are fraction containers at fraction points s and t, respectively. Hence, there exists x_{sm_s} and x_{tm_t} such that $n_s W_{m_s} < x_{sm_s} < (n_s+1) W_{m_s}$ for some $n_s (= 0, 1, 2, \cdots)$ and $n_t W_{m_t} < x_{tm_t} < (n_t+1) W_{m_t}$ for some $n_t (= 0, 1, 2, \cdots)$. Let $\epsilon = \frac{1}{2} \min \{ x_{sm_s} - n_s W_{m_s}, (n_s+1) W_{m_s} - x_{sm_s}, x_{tm_t} - n_t W_{m_t}, (n_t+1) W_{m_t} - x_{tm_t}, \min_{s \le k \le t} I_k \}$. Then we can construct a new production plan $(x^1_{(u+1)1}, x^1_{(u+1)2}, \cdots, x^1_{vM})$ such that $x^1_{sm_s} = x_{sm_s} - \epsilon$, $x^1_{sj} = x_{sj}$ for all $j \neq m_s$, $x^1_{tm_t} = x_{tm_t} + \epsilon$, $x^1_{tj} = x_{tj}$ for all $j \neq m_t$, and $x^1_{kj} = x_{kj}$ for all j and $k \neq s, t$ $(u < k \le v)$. Also, we can construct another production plan $(x^2_{(u+1)1}, x^2_{(u+1)2}, \cdots, x^2_{vM})$ such that $x^2_{sm_s} = x_{sm_s} + \epsilon$, $x^2_{sj} = x_{sj}$, for all $j \neq m_s$, $x^2_{tm_t} = x_{tm_t} - \epsilon$, $x^2_{tj} = x_{tj}$ for all $j \neq m_t$, and $x^2_{kj} = x_{kj}$ for all j and $k \neq s, t$ $(u < k \le v)$. For any $\epsilon > 0$, $(x_{(u+1)1}, x_{(u+1)2}, \cdots, x_{vM}) \neq (x^1_{(u+1)1}, x^1_{(u+1)2}, \cdots, x^1_{vM}) \neq (x^2_{(u+1)1}, x^2_{(u+1)2}, \cdots, x^2_{vM})$ and $(x_{(u+1)1}, x_{(u+1)2}, \cdots, x_{vM}) = \frac{1}{2}(x^1_{(u+1)1}, x^1_{(u+1)2}, \cdots, x^1_{vM}) + \frac{1}{2}(x^2_{(u+1)1}, x^2_{(u+1)2}, \cdots, x^2_{vM})$. Also, two new production plans $(x^1_{(u+1)1}, x^1_{(u+1)2}, \cdots, x^1_{vM})$ and $(x^2_{(u+1)1}, x^2_{(u+1)2}, \cdots, x^2_{vM})$ satisfy the constraint (2). It implies that $(x_{(u+1)1}, x_{(u+1)2}, \cdots, x_{vM})$ is not an extreme point. Hence $(x_{(u+1)1}, x_{(u+1)2}, \cdots, x_{vM})$ is not the optimal solution. This completes the proof. Q.E.D.

Before a solution procedure to find D_{uv} is presented. the following property is further derived.

Corollary 1. For the (u,v)-problem, there exists the optimal production plan such that for any production point but not a fraction point t $(u+1 < t \le v)$, $I_{t-1} > 0$ and $x_{tj} = n_j \cdot W_j$ for some j and some n_j, and $x_{ti} = 0$ for all $i \neq j$, where n_j is a nonnegative integer.

Proof. From the result of Theorem 3, we know that except a fraction point, any other production point t cannot become a fraction point. This completes the proof. Q.E.D.

4. A Solution Procedure for Computing D_{uv}

The optimal solution properties characterized in Section 3 provide some insight for generating all possible production plans to compute D_{uv}. Let W denote the greatest common divisor of $W_1, W_2, \cdots, W_M$. If there is no common divisor greater than one, let $W=1$. Let us define ε_W as $\varepsilon_W = \sum_{i=u+1}^{v} d_i - n \cdot W$, where n is some nonnegative integer and $0 \le \varepsilon_W < W$. Then, the following property can be easily derived from the theorems in Section 3.

Corollary 2. In the (u,v)-problem, there exists the optimal solution such that

i) for fraction point s $(u < s \le v)$, $x_{sj} = k \cdot W + \varepsilon_W$ for some j and some $k \in \{0,1,\cdots,n\}$, and $x_{si}=0$ for all $i \neq j$,

ii) for any production point t and $t \neq s$ $(u < s \le v)$, $x_{tl} = m \cdot W$ for some l and some $m \in \{1,2,\cdots,n\}$, and $x_{ti}=0$ for all $i \neq l$.

Based on Corollary 2, a solution procedure for computing D_{uv} is presented. Let

D_t = cumulative demand for periods from $u+1$ through t (i.e., $D_t = \sum_{i=u+1}^{t} d_i$)

R_t = set of feasible cumulative production levels made by period t since period $u+1$,

X_t = element of R_t.

$$N(X) = \begin{cases} 1 & \text{if X includes a fraction point,} \\ 0 & \text{otherwise.} \end{cases}$$

For $t=u$, R_u has only one element, $X_u=0$.

For $t=u+1, u+2, \cdots, v-1$, R_t can be generated from R_{t-1} by considering the following three cases derived from Theorem 3.

i) When $N(X_{t-1})=0$ and $N(X_t)=0$, X_t equals $X_{t-1} + k \cdot W$, $k=0,1,\cdots,n$, and $D_t < X_t \le D_v$.

ii) When $N(X_{t-1})=0$ and $N(X_t)=1$, X_t equals $X_{t-1}+k \cdot W+ \varepsilon_W$, $k=0,1,\cdots,n$ and $D_t < X_t \le D_v$.

iii) When $N(X_{t-1})=1$ and $N(X_t)=1$, X_t equals $X_{t-1}+k \cdot W$, $k=0,1,\cdots,n$, and $D_t < X_t \le D_v$.

For $t=v$, R_v has only one element, $X_v=D_v$.

Two variables $B(t,x)$ and $C(t,X,z)$ are introduced additionally as follows:

$B(t,x)$ = minimum freight cost to ship the production amount x in period t, and

$C(t,X,z)$ = minimum system cost associated with a feasible cumulative production level X in period t, $u+1 \le t \le v$, where the production level includes z fraction points and z is a nonnegative integer.

Note that from the result of Theorem 3, the number z can have either 0 or 1.

Then $B(t,x)$ satisfies the following equation;

$$B(t,x) = \min_{j=1,2,\cdots,M} \{ f_{tj} \cdot \lceil x/W_j \rceil \}, \tag{11}$$

where $\lceil y \rceil$ denotes the smallest integer that is greater than or equal to y. Hence, $C(t,X,z)$ satisfies the following recursive equations.

$$C(u, X_u, 0) = 0, \tag{12}$$

$$C(t,X_t,0) = \min_{\substack{X_{t-1} \in R_{t-1} \\ X_{t-1} \le X_t}} \begin{cases} C(t\text{-}1, X_{t-1}, 0) + S_t + h_t(X_t - D_t) + B(t, X_t - X_{t-1}), & \text{if } X_t = X_{t-1} + kW \text{ and } k \in \{1,2,\cdots,n\}, \\ C(t\text{-}1, X_{t-1}, 0) + h_t(X_t - D_t), & \text{if } X_t = X_{t-1}, \text{ for } u+1 \le t \le v \text{ and } X_t \in R_t. \end{cases} \tag{13}$$

$$C(t,X_t,1) = \min_{\substack{X_{t-1} \in R_{t-1} \\ X_{t-1} \le X_t}} \begin{cases} C(t\text{-}1, X_{t-1}, 0) + S_t + h_t(X_t - D_t) + B(t, X_t - X_{t-1}), & \text{if } X_t = X_{t-1} + kW + \varepsilon_W \text{ and } k \in \{0,1,\cdots,n\} \\ C(t\text{-}1, X_{t-1}, 1) + S_t + h_t(X_t - D_t) + B(t, X_t - X_{t-1}), & \text{if } X_t = X_{t-1} + mW \text{ and } m \in \{1,2,\cdots,n\}, \\ C(t\text{-}1, X_{t-1}, 1) + h_t(X_t - D_t), & \end{cases}$$

if $X_t = X_{t-1}$, *for* $t = u+1, u+2, \cdots, v$ *and* $X_t \in R_t$. (14)

Then, $D_w = min\ \{C(v, D_v, 0), C(v, D_v, 1)\}$.

From Theorem 3 and Corollaries 1 and 2, the following observations about computing the equations (11)-(14) can be obtained.

i) For $u+1 \leq t \leq v$, the production levels $D_{u+1}, D_{u+2}, \cdots, D_{v-1}$ *cannot be the element of* R_t because such production levels lead the inventory level to be zero in a period.

ii) If $mod(X_t, W) = 0$, let $C(t, X_t, 1) = \infty$, where $mod(p, q)$ denotes the remainder of p divided by q. Otherwise, let $C(t, X_t, 0) = \infty$.

iii) If $\varepsilon_w = 0$, let $C(t, X_t, 1) = \infty$ for $u+1 \leq t \leq v$ and $X_t \in R_t$.

iv) Consider the calculation of the recursive equation (13). For $u+1 < t \leq v$, if there does not exist some type j such that $mod(X_t - X_{t-1}, W_j) = 0$ for $j = 1, 2, \cdots, M$, then it implies that period t is a fraction point. Hence, let. $C(t, X_t, 0) = \infty$.

v) Consider the case satisfying $X_t = X_{t-1} + mW$ in the recursive equation (14). For $u+1 \leq t \leq v$, if there does not exist some type j such that $mod(X_t - X_{t-1}, W_j) = 0$ for $j = 1, 2, \cdots, M$, then it implies that period t is a fraction point. Hence, let $B(t, X_t - X_{t-1}) = \infty$.

vi) If there exists more than one production sequence which has the equal cumulative production levels in a period, then the sequence having a higher cost can not be optimal.

It is noticed that the efficiency of the algorithm is strongly dependent on the relative magnitude of the greatest common divisor W.

5. A Numerical Example

Consider a simple 5-period problem with two freight container types, where their carrying capacities $W_1 = 100$ and $W_2 = 150$ are considered. The relevant data including cost parameters are given in Table 1.

Table 1. The relevant data including cost parameters.

T	1	2	3	4	5
d_t	90	150	220	40	50
S_t	70	50	50	80	70
p_t	7	6	6	8	7
h_t	1	1	1	1	1
f_{t1}	100	90	90	100	100
f_{t2}	150	135	135	150	150

D_{02} is selected to show the procedure of the recursive equations (12)-(14).

Because the greatest common divisor of W_1 and W_2 is 50, it holds that $W = 50$.

$\{D_1, D_2\} = \{90, 240\}$ and so $\varepsilon_w = mod(D_2, W)$ = mode $(240, 50) = 40$.

For $t=0$, $R_0 = \{0\}$ and $C(0,0,0) = 0$.

For $t=1$, $R_1 = \{100, 140, 150, 190, 200, 240\}$,

$B(100) = \min\{100 \times 1, 150 \times 1\} = 100$,

$C(1,100,0) = 70 + 7 \times 100 + 1 \times (100-90) + 100 = 880$,

$C(1,100,1) = \infty$,

$B(140) = \min\{100 \times 2, 150 \times 1\} = 150$,

$C(1,140,0) = \infty$,

$C(1,140,1) = 70 + 7 \times 140 + 1 \times (140-90) + 150 = 1250$,

$B(150) = \min\{100 \times 2, 150 \times 1\} = 150$,

$C(1,150,0) = 70 + 7 \times 150 + 1 \times (150-90) + 150 = 1330$,

$C(1,150,1) = \infty$,

$B(190) = \min\{100 \times 2, 150 \times 2\} = 200$,

$C(1,190,0) = \infty$,

$C(1,190,1) = 70 + 7 \times 190 + 1 \times (190-90) + 200 = 1700$,

$B(200) = \min\{100 \times 2, 150 \times 2\} = 200$,

$C(1,200,0) = 70 + 7 \times 200 + 1 \times (200-90) + 200 = 1780$,

$C(1,200,1) = \infty$,

$B(240) = \min\{100 \times 3, 150 \times 2\} = 300$,

$C(1,240,0) = \infty$,

$C(1,240,1) = 70 + 7 \times 240 + 1 \times (240-90) + 400 = 2200$.

For $t=2$, $R_2 = \{240\}$,

For X_i=100,

B(2,240-100)=min{90×2, 135×1}=135,

For X_i=140,

B(2,240-140)=min{90×1, 135×1}=90,

For X_i=150,

B(2,240-150)=min{90×1, 135×1}=90,

For X_i=190, B(2,240-190)=∞,

For X_i=200,

B(2,240-200)=min{90×1, 135×1}=90,

For X_i=240, B(2,0)=0,

C(2,240,0)=∞,

C(2,240,1)=

min{C(1,100,0)+50+6×(240-100)+B(2,240-100)=1905,

C(1,140,1)+50+6×(240-140)+B(2,240-140)=1990,

C(1,150,0)+50+6×(240-150)+B(2,240-150)=2010,

C(1,190,1)+50+6×(240-190)+B(2,240-190)=∞,

C(1,200,0)+50+6×(240-200)+B(2,240-200)=2160,

C(1,240,1)+B(2,0)=2200}=1905.

Hence,D_{02}=min {C(2,240,0), C(2,240,1)}=min{∞, 1905}= 1905.

Table 2 shows the computation summary of D_{uv} and F_v for $0 \le u \langle v \le 5$. The optimal production policy gives the solution which produces 100 units in period 1, 150 units in period 2, 300 units in period 3. Also, the optimal transportation policy gives the solution that uses one unit of the container type 1 in period 1, one unit of the container type 2 in period 2, and three units of the container type 1 or two units of the container type 2 in period 3. Then the associated optimal total cost is at $ 4235.

6. Concluding Remarks

This paper has analyzed a dynamic production and transportation model with multiple freight container types. In the model, each container has type-dependent carrying capacity restriction and at most one container type is allowed in each shipping period. The unit freight shipping

Table 2. Computaional Results

v	u	D_{uv}	Production Amount Associated with D_{uv}	Container Type Associated with D_{uv}	F_u+D_{uv}	F_v	Optimal Solution	
							Production Amount	Container Type
1	0	800	(90)	(1)	800	800	(90)	(1)
2	0	1905	(100,140)	(1,2)	1905			
	1	1085	(150)	(2)	1885	1885	(90,150)	(1,2)
3	0	3520	(100,160,200)	(1, 2, 1)	3520			
	1	2700	(170,200)	(1, 1)	3500			
	2	1640	(220)	(1 or 2)	3485	3485	(90,150,220)	(1,2,1 or 2)
4	0	3835	(100,150,250,0)	(1, 2,1 or 2, -)	3835			
	1	3100	(200,210,0)	(1,1 or 2, -)	3900			
	2	1920	(260,0)	(1 or 2, -)	3805	3805	(90,150,260,0)	(1,2,1 or 2,-)
	3	500	(40)	(1)	3985			
5	0	4235	(100,150,300,0,0)	(1,2,1 or 2,-,-)	4235	4235	(100,150,300,0,0)	(1,2,1 or 2,-,-)
	1	3460	(160,300,0,0)	(1,1 or 2,-,-)	4260			
	2	2410	(310,0,0)	(1, -,-)	4290			
	3	950	(90,0)	(1,-)	4435			
	4	520	(50)	(1)	4325			

cost for each container type depends on the size of its carrying capacity.

For the model, some important solution properties are characterized, based upon which a forward dynamic programming algorithm (10) was derived by partitioning the given problem into smaller subproblems. The solution procedure including the recursive equations (11)-(14) was also developed to find the optimal value for each of such subproblems (D_{uv}).

The algorithm has the complexity at the order of $O((v-u)(R_{uv}/W)^2M)$ in finding D_{uv} for given u and v, where $R_{uv} = \sum_{t=u+1}^{v} d_t$, so in total at the complexity order of $O(T^3 (R_{1T}/W)^2M)$ in finding F_T. Thus, the efficiency of the proposed algorithm is strongly dependent on the ratio of R_{1T} to W.

Our further research will consider an extension of the problem where a general concave cost function is incorporated and various container types are allowed to employ together in each period.

7. References

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