Journal of the Korean Institute of Industrial Engineers
Vol.24, N0.4, December, 1998

A Physical Ring Design Problem of Synchronous Optical Networks (SONET) for Mass Market Multimedia Telecommunication Services*

멀티미디어 서비스를 제공하는 소넷링 물리구조 설계문제

Youngho Lee** · Junghee Han** · Seong-In Kim**

이영호 · 한정희 · 김성인

Abstract

In this paper, we deal with a node weighted Steiner Ring Problem (SRP) arising from the deployment of Synchronous Optical Networks (SONET), a standard of transmission using optical fiber technology. The problem is to find a minimum weight cycle (ring) covering a subset of nodes in the network considering node and link weights. We have developed two mathematical models, one of which is stronger than the other in terms of LP bounds, whereas the number of constraints of the weaker one is polynomially bounded. In order to solve the problem optimally, we have developed some preprocessing rules and valid inequalities. We have also prescribed an effective heuristic procedure for providing tight upper bounds. Computational results show that the stronger model is better in terms of computation time, and valid inequalities and preprocessing rules are effective for solving the problem optimally.

1. Introduction

In this paper, we consider a (node weighted) Steiner Ring Problem (SRP) arising from the deployment of Synchronous Optical Networks (SONET). The SONET is a standard of transmission technology over optical fiber networks. For example, a typical capacity of SONET technology permits the transmission of 2.4Gbps on a single fiber, which is equivalent to over 38,000 voice circuits (see the technical details in Wu, 1992). Thus, any failure even in a single link may result in a tremendous loss of customer service. Accordingly, telecommunications companies are adopting SONET ring architecture in order to enhance the survivability of networks. In this context, an important issue for telecommunications companies is to design cost-effective SONET ring that guarantees network survivability in the event of link failure. In order to provide the link survivability, we need to connect all the nodes in a single ring by using SONET add-drop multiplexer (ADM), which is capable of adding and

* This research was sponsored by Korea Science and Engineering Foundation Grant 981-1-1015-084-2.

** Department of Industrial Engineering, Korea University.

dropping the traffic, at each node. However, due to the capacity limit of SONET equipment such as SONET ADM and optical fiber, we need to partition the network into several sub-networks (clusters), and connect all nodes in each cluster. The problem of our concern is to physically connect all nodes in a cluster in order to form a ring with minimum cost. The logical clustering problem itself is also a difficult combinatorial optimization problem (see Lee et al., 1998).

Now, we describe our problem as follows. When we have a set of ADM sites, a set of non-ADM sites, and a set of existing links in the network, we would like to construct a minimum cost SONET ring covering all ADM sites. However, if it is not feasible to build a physical ring only with ADM sites and their direct links, we need to use non-ADM sites for connectivity although adding non-ADM sites incurs additional fiber and repeater cost. This problem can be formally expressed as follows. For a given undirected graph $G = (V, E)$, a node set $N \subset V$ corresponding to the ADM sites, a set of link weights $c_{ij} \geq 0$ for $(i,j) \in E$, and a set of node weights $w_k \geq 0$ for $k \in \overline{N} \equiv (V - N)$, find a minimum weighted ring that covers all the nodes of set N. The nodes in $\overline{N}$ are called *Steiner nodes*, and the resulting ring is called a *Steiner ring* of G. Accordingly, we call this problem as a *Steiner Ring Problem* (SRP). Note that SRP reduces to the Traveling Salesman Problem (TSP) by letting $\overline{N} = \Phi$.

Padberg and Rinaldi (1991) has solved symmetric TSP optimally up to 2392 cities by means of problem reduction technique and branch-and-cut procedure. On the other hand, Gendreau, Laporte and Semet (1997) has presented polyhedral approach to the Covering Tour Problem (CTP) on $G = (V \cup W, E)$ that determines the minimum length Hamiltonian cycle on a subset of V such that every node in W is within a prespecified distance from the cycle. Here, note that CTP has additional constraints compared to SRP such as every node in W being within prespecified distance from the cycle. However, the fact that node set V in CTP corresponds to the set of Steiner nodes in SRP implies that CTP does not reduce to SRP, nor does the reverse apply. Balas (1989) has investigated the polyhedral structure of the Prize Collecting Traveling Salesman Problem (PTSP), where a salesman wishes to minimize his travel cost and net penalties while visiting enough cities to collect a prescribed amount of prize money. Note that PTSP has constraints of total prize collected from the tour being greater than or equal to a prescribed amount of money. However, the set of nodes in PTSP corresponds to the set of Steiner nodes in SRP. Although there is an extensive body of literature for the TSP, this paper is the first effort to solve the SRP.

This paper is organized as follows. In the next section, we present two alternate formulations for the problem and compare the strength of these formulations in terms of the lower bound obtained by solving the LP relaxation of the formulation. In Section 3, we describe several classes of valid inequalities, which hopefully tighten the initial formulation. In Section 4, we develop heuristic and exact procedures. We also present computational results of the solution procedures in Section 5. Section 6 concludes this paper.

2. Problem Formulations

In this section, we develop two alternate formulations for solving the problem. Toward this end, let us define some notations. Let $x_{ij} = 1$ if node $i \in V$ is connected to node $j(> i) \in V$ and 0 otherwise, and let $u_k = 1$ if node $k \in \overline{N} (\equiv V - N)$ is in the ring and 0 otherwise. We assume that triangle inequalities are satisfied for the set of link weights. That is, $c_{ij} + c_{ik} \geq c_{jk}$ for all (i,j), (i,k), $(j,k) \in E$. Then, the problem SRP can be formulated as follows.

SRP: Minimize $\sum_{i \in V} \sum_{j \in V: i < j} c_{ij} x_{ij} + \sum_{k \in \overline{N}} w_k u_k$

Subject to

$$\sum_{j\in V} x_{(ij)} = 2, \quad i \in N, \tag{1}$$

$$\sum_{i\in V} x_{(ik)} = 2u_k, \quad k \in \overline{N}, \tag{2}$$

$$x_{(ik)} \le u_k, \quad i \in V, k \in \overline{N}, \tag{3}$$

$$\sum_{i\in S}\sum_{j\in \overline{S}} x_{(ij)} \ge 2, \tag{4}$$

$$S \subset V,\ S\cap N \ne \Phi,\ \overline{S}\cap N \ne \Phi,$$

$$x_{(ij)} \in \{0,1\} \quad i < j, \quad i,j \in V,$$

$$u_k \in \{0,1\} \quad k \in \overline{N},$$

where $x_{(ij)} = x_{ij}$ if $i < j$ and x_{ji} otherwise.

Constraints (1) indicate that two links incident to node $i \in N$ should be on the ring, and constraints (2) indicate that two links incident to node $k \in \overline{N}$ should be on the ring if node $k \in \overline{N}$ is on the ring. Also, in order to increase the lower bound of LP relaxation, we consider constraints (3). Constraints (4) prevent subtour.

Remark 1. Observe that for a given (binary) feasible solution of x, the u variables are automatically binary at optimality, even if treated as continuous. Also, note that SRP is NP-hard since it reduces to TSP by letting $\overline{N} = \Phi$. Moreover, since the number of constraints (4) goes up to $2^{|N|}$-2, generating all constraints of type (4) is impractical, though not impossible. Motivated by this, we develop another formulation in the following.

Now, we develop an alternative formulation, denoted by SRPF, based on the flow concept. We conceive of a flow that enters a *single* node from an artificial node of supply $|N| + \sum_{k\in \overline{N}} u_k$, denoted by node 0, and we seek to send the flow to other appropriate nodes to satisfy the connectivity conditions. Toward this end, define f_{ij} to be a directed flow from node i to node $j, i \ne j$. With these flow variables, we obtain the following formulation SRPF for the problem SRP.

SRPF: Minimize $\sum_{i\in V}\sum_{j\in V: i<j} c_{ij}x_{ij} + \sum_{k\in \overline{N}} w_k u_k$

Subject to

$$\sum_{j\in V} x_{(ik)} = 2, \quad i \in N, \tag{5}$$

$$\sum_{i\in V} x_{(ik)} = 2u_k, \quad k \in \overline{N}, \tag{6}$$

$$x_{(ik)} \le u_k, \quad i \in V, k \in \overline{N} \tag{7}$$

$$\sum_{i\in V} f_{0i} = |N| + \sum_{k\in \overline{N}} u_k, \tag{8}$$

$$f_{0i} + \sum_{j\ne i: j\in V} (f_{ji} - f_{ij}) = 1, \quad i \in N, \tag{9}$$

$$f_{0i} + \sum_{j\ne i: j\in V} (f_{ji} - f_{ij}) = u_i, \quad i \in \overline{N}, \tag{10}$$

$$f_{ij} \le |V|\, x_{(ij)}, \ i \in V \cup \{0\}, j \in V, \tag{11}$$

$$\sum_{i\in V} x_{0i} = 1, \tag{12}$$

$$f_{ij} \ge 0, \quad i \ne j,\ i \in V\cup\{0\},\ j \in V,$$

$$x_{(ij)} \in \{0,1\}, \quad i < j, \quad i,j \in V,$$

$$u_k \in \{0,1\}, \quad k \in \overline{N}.$$

Constraints (8) determine the amount of flows from an artificial node to all nodes in V. Constraints (9) and (10) ensure the connectivity of the ring. Constraints (11) indicate that node i should be connected to node j, if positive flow is sent from node i to node j, $i \ne j$. Constraints (12) force the flow from an artificial node to enter a single node in V. Note that, contrary to the formulation SRP, the number of constraints in the formulation **SRPF** is polynomially bounded.

Next, we compare the strength of the two formulations.

Let $\overline{P}$ denote the LP relaxation of formulation P, and let $v(P)$ denote the optimal objective function value of a given formulation P. Then, the following result shows that the formulation SRPF is weaker than the formulation SRP in terms of LP bound.

Proposition 1. $v(\overline{SRP}) \geq v(\overline{SRPF})$.

Proof. Let $(\bar{x}, \bar{u})$ be a feasible solution to $\overline{SRP}$. Then, we see that there exists a feasible solution $(\bar{x}, \bar{u}, \bar{f})$ to $\overline{SRPF}$ of the form $\bar{f}_{Oi} = 1$ for $i \in N$, $\bar{f}_{Oi} = \bar{u}_i$ for $i \in \overline{N}$, $\bar{x}_{Oi} = 1/|V|$ for $i \in V$, and $\bar{f}_{ij} = 0$ for $i, j(\neq i) \in V$. This completes the proof.

Now, we need to see if $v(\overline{SRP}) \leq V(\overline{SRPF})$, which implies that $v(\overline{SRP}) = V(\overline{SRPF})$ due to Proposition 1. However, we can easily disprove $v(\overline{SRP}) \leq V(\overline{SRPF})$ in the following example by showing that there exists a problem instance such that $v(\overline{SRP}) > V(\overline{SRPF})$.

Example 1. Consider a graph with $N = \{1, 2, 4, 5\}$ $\overline{N} = \{3, 6\}$, $E = \{(1,2), (1,3), (1,6), (2,3), (3,4), (4,5), (4,6), (5,6)\}$, and $c_{12} = c_{46} = 3$, $c_{23} = c_{56} = 4$, $c_{13} = c_{45} = 5$, $c_{16} = c_{34} = M$, $w_3 = w_6 = 2$, where M is sufficiently large number. This example is illustrated in Figure 1. Then, we may have $(\bar{x}, \bar{u}, \bar{f})$ of $\overline{SRPF}$ as follows withv $(\overline{SRPF}) = 28$, $\bar{x}_{01} = \bar{x}_{06} = 0.5$, $\bar{x}_{12} = \bar{x}_{13} = \bar{x}_{23} = \bar{x}_{45} = \bar{x}_{46} = \bar{x}_{56} = 1$, $\bar{u}_3 = \bar{u}_6 = 1$, $\bar{f}_{01} = \bar{f}_{06} = 3$, $\bar{f}_{12} = \bar{f}_{56} = 2$, $\bar{f}_{23} = \bar{f}_{45} = 1$, and 0 for all other variables. Solutions $\bar{x}, \bar{u}$ but $\bar{f}$ are shown on links of Figure 1 of the form $(\bar{x}, \bar{f})$. Here, we see that there are two subtours $\{(1, 2), (1, 3), (2, 3)\}$ and $\{(4, 5), (4, 6), (5, 6)\}$ in $(\bar{x}, \bar{u}, \bar{f})$ of $\overline{SRPF}$, which violates the constraint (4) of $\overline{SRP}$. Moreover, we have that $v(\overline{SRP}) = M + 24$. This example shows that $v(\overline{SRP}) > v(\overline{SRPF})$ when $M > 4$. This implies that formulation SRPF can be arbitrarily bad in terms of LP bound.

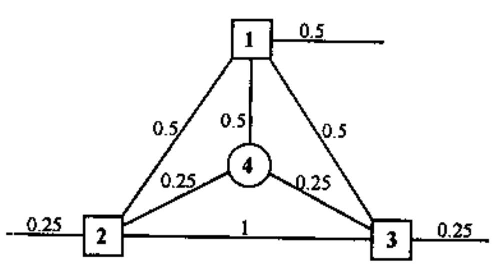

Figure 1. An example of $v(\overline{SRP}) > v(\overline{SRPF})$.

3. Valid Inequalities

In this section, we develop some valid inequalities that tighten the formulation SRP. Motivated by the strength of lifted cycle inequality for asymmetric TSP (see Lawler, 1985), we strengthen the cycle inequality in consideration of node set $\overline{N}$.

Proposition 2. Suppose that there exists a triangle (cycle) in the subgraph induced by $S = \{i, j, k\} \subset V$ along with its incident links. If $j, k \in \overline{N}$, then the optimal solution of SRP satisfies the following inequality for $l \in V - S$

$$x_{(ij)} + x_{(ik)} + x_{(jk)} + x_{(il)} \leq 2. \tag{13}$$

Proof. If $x_{(il)} = 0$ in an optimal solution, (13) is valid since $\{(i,j), (i,k), (j,k)\}$ is a subtour. And, if $x_{(il)} = 1$, (13) is also valid since $j, k \in \overline{N}$ and the triangle inequalities are satisfied for all i, j and $k \in S$. This completes the proof.

Also, we strengthen the subtour elimination constraint introduced by Dantzig, Fulkerson and Johnson (1954).

Proposition 3. Define $L(S)$ as the set of links induced by node set S. Let $S \subset V$ be such that $S \cap \overline{N} \neq \Phi$. Then, the following inequality is valid for SRP.

$$\sum_{(i,j) \in L(S)} x_{(ij)} \leq |S| - 2 + u_k, \quad k \in S \cap \overline{N}. \tag{14}$$

Proof. If $u_k = 1$, (14) becomes subtour elimination constraint. And if $u_k = 0$, we see that $\sum_{(i,j)\in L(S)} x_{(ij)} = \sum_{(i,j)\in L(S-k)} x_{(ij)}$. Then, (14) also becomes subtour elimination constraint. This completes the proof.

Example 2. Consider an example of $S = \{1, 2, 3, 4\}$ and $S \cap \overline{N} = \{4\}$, which is illustrated in Figure 2. Here we see that the left hand side of (14) is $\bar{x}_{12} + \bar{x}_{13} + \bar{x}_{14} + \bar{x}_{23} + \bar{x}_{24} + \bar{x}_{34} = 0.5 + 0.5 + 0.5 + 1 + 0.25 + 0.25 = 3$, and the right hand side of (14) is $|S| - 2 + \bar{u}_4 = 4 - 2 + 0.5 = 2.5$, which violates the inequality of type (14).

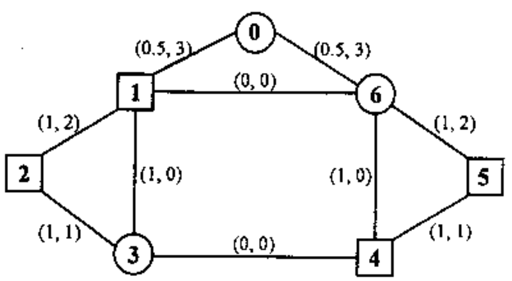

Figure 2. An example of valid inequality of type (14).

4. Solution Procedures

First, we develop a heuristic procedure for SRP. If there exists an Hamiltonian tour in the subgraph G' induced by node set $N \subseteq V$ along with its incident links, then we can apply the *spanning-tree perfect-matching* heuristic (see Christofides, 1976) to obtain an initial tour. However, we may not know in advance if there exists an Hamiltonian tour in G'. Hence, we begin our heuristic with $G = (V, E)$ to find an initial tour. Then, we improve the initial tour. Now, we define some notations to describe the heuristic procedure. Let $\delta(i, S)$ be the set of links incident to node i in the subgraph $S \subseteq G$. Also, let $N(S)$ be the set of nodes in $S \subseteq G$, and let $E(S)$ be the set of links in $S \subseteq G$. Then, the heuristic procedure can be described as follows.

Initial Tour Finding Heuristic

Step 1: Construct a spanning tree T of $G = (V, E)$.

Step 2: Define $L = \{i \in N(T) : \delta(i, T) = 1\}$. If $L = \Phi$, then stop.

Step 3: Remove node set $L \cap \overline{N}$ from T, and update L.

Step 4: Find the shortest path P between any pair of nodes $s, t \in L$ such that $E(P) \cap E(T) = \Phi$. If path does not exist for any pair of $s, t \in L$, then go to Step 7.

Step 5: Add P to T, and let C be the cycle containing nodes s and t. If T is a tour, then stop.

Step 6: Define $U = \{i \in N(C) : |\delta(i, T)| > 2\}$. Remove link $(s, t) = \text{arg}max_{(i,j)\in E(C), i\in U, j\in N(C-P)} c_{ij}$ from T, and go to Step 2.

Step 7: Find the shortest path P between nodes $s \in L$, and $t \in N(T) - L$, and then go to Step 5.

Remark 2. In order to find the shortest path considering node weights as well as link weights, we modify the *Dijkstra algorithm* (see Nemhauser and Wolsey, 1988), which runs in Step 4 and 7 of the initial heuristic procedure. The idea is to transform a node to a link. That is, split a node into two artificial nodes, and then introduce an auxiliary link connecting the two artificial nodes, where the weight of auxiliary link is set equal to the weight of the original node. Then, we can apply the *Dijkstra algorithm* to our problem without any consideration of node weights. However, for practical implementation, we add node weights when computing the tentative minimum distance for the unlabeled nodes in the *Dijkstra algorithm* as follows: $d_j = \min(d_j, d_i + c_{ij} + w_j)$, where d_i is the tentative minimum distance from the source node to node $i \in V$, c_{ij} is the distance from node i to node j, and w_i is the weight of node $i \in V$. The node weights for ADM-sites are set equal to 0. That is, $w_i = 0$ for $i \in N$.

Now, we describe a heuristic procedure for improving the initial tour. Here, we denote the current tour by T. The first step of the heuristic procedure for improving the initial tour is to remove currently unfavorable Steiner nodes from the current tour T, and then perform *two-opt* heuristic (see Nemhauser and Wolsey, 1988).

Local Improvement Heuristic

Repeat Step 1 and Step 2 for some fixed computation times, i.e., 10 seconds.

Step 1 (Removing Steiner nodes from the tour):

- Step 1.1: Pick an arbitrary node $s \in N$ such that $|\delta(s, G)| \geq 3$.
- Step 1.2: Pick an arbitrary node $t \in \overline{N} - N(T)$ such that $(s, t) \in E$ and $|\delta(t, G)| \geq 2$.
- Step 1.3: Define L the set of nodes $\{i \in N(T) - s$: (i, t), $(t, i) \in E$ such that any node in L is not preceded by node $j \in N$ following the T from s in either clockwise or counter-clockwise direction.
- Step 1.4: Compute $\eta(i) = c_{st} + w_t + c_{ti} - \Sigma_{j \in N(P_i) - \{s,i\}} w_j - \Sigma_{e \in P_i} c_e$ for all $i \in L$, where P_i is the path from s to node $i \in L$ following the T such that $N(P_i) \subseteq L$.
- Step 1.5: If $\min_{i \in L} \eta(i) < 0$, then replace the existing path P_j with $\{(s, t), (t, j)\}$, where $j = \text{argmin}_{i \in L} \eta(i)$.

Step 2 (Two-opt): If $N(T) \cap \overline{N} \neq \emptyset$, then perform two-opt for some iterations ω, a predetermined parameter. Then, go to Step 1.1.

In the following, we consider preprocessing rules in order to reduce the problem size assuming that triangle inequalities are satisfied.

Preprocessing Rules

Define $\delta(i)$ as a set of links incident to node $i \in V$ in the graph G. Then, we have the following results.

- Rule 1. Let $\overline{N} = \overline{N} - i$, if $|\delta(i)| = 1$ for $i \in \overline{N}$. That is, a leaf node can be deleted.
- Rule 2. Let $\overline{N} = \overline{N} - i$, if $|\delta(i)| = 2$ for $i \in \overline{N}$, and $\{(i, j), (i, k), (j, k)\} \subset E$.
- Rule 3. Let $E = E - (j, k)$, if $|\delta(i)| = 2$ for $i \in N$, and $\{(i, j), (i, k), (j, k)\} \subset E$.

Using the heuristic procedure and preprocessing rules, we now describe an exact solution procedure as follows.

Exact Procedure

- Step 1 (Preprocessing): Reduce the size of SRP instance according to the preprocessing rules. Define $\bar{z}$ the lower bound of SRP instance, and let $\bar{z} = 0$.
- Step 2 (Computing upper bound): Compute an upper bound of SRP using the heuristic procedure.
- Step 3 (LP relaxation): Solve LP relaxation of SRP. If LP optimal solution (x, u) is integral, and has no subtour, then stop.
- Step 4 (Generating constraints): Generate violated constraints of type (4), (13) and (14) as many as we can. If any violated constraint is identified, then add these inequalities and go to Step 3.
- Step 5 (Branch and bound): Run the branch and bound procedure, and obtain an integer optimal solution $z = (x, u)$, and let $\bar{z} = z$. If (x, u) has subtour, then go to Step 4.

For the generation of subtour elimination constraints of type (4), we follow a depth-first search procedure (see Nemhauser and Wolsey, 1988), which identifies subtour even if LP solution is fractional. That is, starting from a node $i \in N$, we search for a cycle such that every link in the cycle has non-zero value. Then, we examine if the cycle does not contain all the nodes of N and violates the constraints of type (4). The identification of violated constraints of type (13) is straightforward. For the constraints of type (14), the choice of S is restricted to $|S| = 4$ in order to reduce the computational effort.

5. Computational Results

We have tested heuristic and exact procedures for various problem sizes. For each problem size, we have generated 10 random instances. However, we report a few hard problem instances. Here, note that the maximum number of ADM sites allowed in a ring is 16 due to the SONET protocol using only 4 bits in order to identify ADM sites in the ring. We obtained optimal solutions for such small problem instances within a few seconds. Also, in order to demonstrate the computational efficiency of the proposed heuristic and exact procedures, we present computational results on the problems with more than 16 ADM sites. For this, we have randomly generated the locations of nodes in Euclidean space having coordinates ranging from 0 to 100. Also, we randomly generated the link set over an initial tour, where the node sequence of the initial tour is determined arbitrarily. The link weights were generated according to the distance between nodes on the link set. The node weights for Steiner nodes were generated uniformly between 1 to 10, rather small value compared to the weights of links, in order to make the problems more difficult. The developed procedures were coded in C coupled with CPLEX 5.0 on Pentium PC 100 MHz. Test results using valid inequalities (13) and (14) are reported in Table 1. The first column indicates the problem size of the form, $|N|$ - $|\overline{N}|$ - $|E|$, where N is the set of ADM sites, $\overline{N}$ is the set of Steiner nodes, and E is the set of existing links in the network. The column heading "LP" indicates the initial LP lower bound of SRP before we do the branch and bound, the Step 5 of our proposed exact procedure. Since we showed that the LP bound of SRPF can be arbitrarily bad compared to that of SRP in terms of LP bound in the Proposition 1 and Example 1, we do not present the LP bound of SRPF. The column heading "OPT" indicates the optimal objective function value obtained by the cutting plane and

Table 1. Computational results of some hard problem instances.

SIZE	LP	OPT	UB	GAP/OPT (%)	SRP (seconds)	SRPF (seconds)	CPLEX (seconds)
20, 10, 200	499.0	544	544	0.0	2	200	5,600
30, 20, 500	489.0	501	511	2.0	8	1,411	7,659
30, 20, 500	521.0	544	544	0.0	131	6,215	3,277
30, 20, 500	556.5	600	601	0.2	203	14,443	*
20, 40, 600	363.0	427	435	1.9	9,029	45,852	*
20, 40, 600	435.0	456	458	0.4	1,048	2,169	*
30, 30, 700	531.0	558	560	0.4	44	17,598	*
30, 30, 700	489.0	535	545	1.9	1,081	93,661	*
30, 30, 700	438.5	469	474	1.1	3,235	65,056	*
30, 30, 700	446.0	492	502	2.0	1,072	54,084	*

the branch and bound methods, where computation times are recorded in the sixth column labeled as "SRP" based on SRP. In order to compare the two formulations, the computation time of SRPF is also presented in the seventh column, labeled as "SRPF". All computation time is measured in seconds. The upper bounds obtained by the heuristic procedure are recorded in the fourth column, labeled as "UB". In our computational experiments, we have run *two-opt* heuristic for = 100000 iterations. Total computation time to obtain an upper bound does not exceed 10 seconds for all test problems. Also, note that the ratio "GAP/OPT" does not exceed 2 %, where GAP = UB OPT. Computation times of the CPLEX based on SRP with subtour elimination constraints are recorded in the eighth column, labeled as "CPLEX". Asterisk mark (*) in the eighth column indicates that CPLEX failed to find any feasible solution within 24 hours for the corresponding problem instance.

6. Conclusion

We have presented the problem SRP arising from the deployment of SONET in practice. In order to solve the SRP optimally, we have developed two mathematical models and some valid inequalities for the problem. Also, we have developed an efficient heuristic procedure to provide tight upper bounds.

Computational results using the cutting plane method show that SRP is better than SRPF in terms of computation time. We have observed that LP bound of SRP is much stronger than that of SRPF, and that the valid inequalities are effective in tightening LP bounds. Also, computational results demonstrate that the heuristic procedure provides very tight upper bounds for the problem. For on-going research, we are looking into the problems of bidirectional rings and multiple types of SONET rings.

References

[1] Balas, E., "The Prize Collecting Traveling Salesman Problem," Networks, Vol. 19, pp. 621-636, 1989.

[2] Christofides, N., "Worst-case Analysis of a New Heuristic for the Traveling Salesman Problem," Report 388, Graduate School of Industrial Administration, Carnegie-Mellon University, Pittsburgh, PA, 1976.

[3] CPLEX, *Using the CPLEX Callable Library Version 5.0*, ILOG, 1997.

[4] Dantzig, G. B., D. R. Fulkerson, and S. M. Johnson, "Solution of a Large-scale Traveling-salesman Problem," *Operations Research*, Vol. 2, pp. 393-410, 1954.

[5] Gendreau, M., G. Laporte, and F. Semet, "The Covering Tour Problem," *Operations Research*, Vol. 45, pp. 568-576, 1997.

[6] Lawler, E. L., J. K. Lenstra, A. H. G. Rinnooy Kan, and D. B. Shmoys, *The Traveling Salesman Problem: A Guided Tour of Combinatorial Optimization*, John Wiley & Sons, New York, 1985.

[7] Lee, Y., J. Han, and S. Kim, "A Branch and Cut Algorithm for Solving an Intra-Ring Design Problem of Synchronous Optical Networks," *Working Paper*, KU-IE-98-005, Department of Industrial Engineering, Korea University, March, 1998.

[8] Nemhauser, G. and L. A. Wolsey, *Integer and Combinatorial Optimization*, John Wiley & Sons, New York, 1988.

[9] Padberg, M. and G. Rinaldi, "A Branch-and-Cut Algorithm for the Resolution of Large-scale Symmetric Traveling Salesman Problem," *SIAM Review*, Vol. 33, pp. 60-100, 1991.

[10] Wu, T-H, *Fiber Network Service Survivability: Architecture, Technologies, and Design*, Artech House, 1992.

98년 6월 최초 접수, 98년 9월 최종 수정