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Bayesian Reliability Estimation of Two-Unit Hot Standby System

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Abstract

We shall propose several Bayes estimators for reliability of a two-unit hot standby system with the imperfect switch based upon a complete sample of failure times observed from an exponential distribution, and the proposed reliability Bayesian estimators are compared numerically each other in sense of mean squared error.

Key Words and Phrases: Reliability, Standby System, Bayes estimator

1. Introduction

In the context of the lifetime of industrial equipments and components, the probability that a given system will function for a specified period of "mission" time t_0 under the certain conditions is known as a system reliability. The estimation problem for a system reliability plays an important role in many practical reliability analysis. Here we consider the problem of estimation for a two-unit hot standby system reliability with imperfect switch by using the Bayesian approach.

The two-unit standby redundant system configuration is a form of paralleling where only one component is in operation ; if the operating component fails, then another component is brought into operation, and the redundant configuration continues to function. Depending on failure characteristics, standby redundancy is classified into three types. Hot standby system, where each component has the same failure rate regardless of whether it is standby or in operation ; Cold standby system, where components do not fail when they are in standby ; Warm standby

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system, where a standby component can fail but has a smaller failure rate than the principal component. Osaki & Nakagawa(1971 & 1975) computed the reliability for a two-unit standby redundant system with constant failure rate and studied the two-unit standby redundant system with imperfect switch. Subramanian & Ravichandran(1978), Goel & Gupta(1984) and Veklerov(1987) studied the two-unit hot standby redundant system with an imperfect switch. Fujii & Sandoh(1984) considered the Bayesian reliability estimation for a two-unit hot standby redundant system with an imperfect switch.

In this paper, we shall propose some Bayes estimators for reliability of a two-unit hot standby system with the imperfect switch based upon a complete sample of failure times observed from an exponential distribution, and proposed Bayes estimators for hot standby system reliability are numerically compared each other in the sense of mean squared error by the Monte Carlo method.

2. Bayesian Reliability Estimation

We consider an exponential distribution of lifetime governed by the density function

$$f(t|\lambda) = \begin{cases} \lambda e^{-\lambda t}, & 0 < t < \infty \\ 0 \quad , & otherwise. \end{cases} \tag{1}$$

Here we shall consider the Bayes estimators for the two-unit hot standby system reliability with imperfect switch, which can be considered as analogous to an on-line banking system. In the two-unit hot standby system, we shall assume the following:

1. The system consists of two independent and identically distributed units and a switch.
2. One unit serves as a hot standby when the other is in use.
3. The times to failure of both units in use and standby are independent and exponentially distributed with the failure rate λ.
4. The switch is instantaneous when the one in use fails.
5. The unit and the switch are independent.

In the standby system with the imperfect switch, there are several ways which failure can occur. The possible modes of switch failure depend on particular switching mechansim and system. Here we shall consider that the switch simply can fail to operate when called upon. Let ρ be the probability that the switch performs when required. Then the reliability for a two-unit hot standby system with imperfect switch is given by

$$R(t_0) = e^{-\lambda t_0}(1 + \rho\lambda t_o), \quad t_0 > 0 \tag{2}$$

Most Bayes estimators of reliability derived so far were based upon the priors on the unknown parameters which are related to the reliability than the reliability itself. Hence we shall consider uniform and gamma prior distributions on the failure rate λ, and uniform and beta prior distributions on the probability ρ. Also we shall use the squared error loss function to estimate the system reliability. Let $T_1, \cdots, T_n$ be a simple random sample from an exponential distribution with a failure rate λ and $T = \sum_{k=1}^{n} T_k$ be the total test time. And let $X_1, \cdots, X_m$ be a simple random sample from a binomial distribution with success rate ρ and $X = \sum_{k=1}^{m} X_k$ be the number of successes. Assume that prior distributions on the failure rate Λ and the probability ρ are independently and unformly distributed so that

$$g(\lambda;\beta) = \frac{1}{\beta}, \quad 0 < \lambda < \beta \text{ and } g(\rho) = 1, \quad 0 < \rho < 1 \tag{3}$$

denoted by $\Lambda \sim UNIF(0, \beta)$ and $\rho \sim UNIF(0,1)$, respectively. Then according to the Bayes theorem, the joint posterior distribution of Λ and ρ given $T = t$ and $X = x$ is

$$g(\lambda, \rho|t, x; \beta) = \frac{\Gamma(m+2)\rho^x(1-\rho)^{m-x}t^{n+1}\lambda^n e^{-\lambda t}}{\Gamma(x+1)\Gamma(m-x+1)\Gamma(n+1,\beta t)}, \quad 0 < \lambda < \beta, \ 0 < \rho < 1 \tag{4}$$

where $\Gamma(a)$ is the gamma function and $\Gamma(a, z)$ is the incomplete gamma function.

Since the Bayes estimator of the system reliability is found by taking the expectation of reliability function (2) with respect to the posterior distribution (4), under the squared error loss function, the Bayes estimator $\widehat{R}_{UU}(t_0)$ for the two-unit hot standby system reliability $R(t_0)$ with imperfect switch is

$$\widehat{R}_{UU}(t_0) = \frac{(T/(T+t_0))^{n+1}}{\Gamma(n+1,\beta T)} \cdot \Big\{\Gamma(n+1, \beta(T+t_0)) + \frac{t_0(X+1)}{(m+2)(T+T_0)}\Gamma(n+2, \beta(T+t_0))\Big\}. \tag{5}$$

Now we suppose that the failure rate Λ has an UNIF(0, β) prior distribution and the probability ρ has independently a beta prior distribution given by

$$g(\rho; \beta, x_0, m_0) = \frac{\Gamma(m_0)}{\Gamma(x_0)\Gamma(m_0 - x_0)}\rho^{x_0-1}(1-\rho)^{m_0-x_0-1}, \quad 0 < \rho < 1 \tag{6}$$

denoted by $BETA(x_0, m_0)$.

According to the Bayes theorem, the joint posterior distribution of Λ and ρ given $T = t$ and $X = x$ is

$$g(\lambda, \rho | t, x; \beta, x_0, m_0) = \frac{\Gamma(m+m_0)}{\Gamma(x+x_0)\Gamma(m+m_0-x-x_0)\Gamma(n+1,\beta t)} \times t^{n+1}\rho^{x+x_0-1}(1-\rho)^{m+m_0-x-x_0}\lambda^n e^{-\lambda t}, \quad 0<\lambda<\beta,\ 0<\rho<1. \tag{7}$$

Therefore, under the squared error loss function, the Bayes estimator $\widehat{R}_{UB}(t_0)$ for the two-unit hot standby system reliability $R(t_0)$ with imperfect switch is

$$\widehat{R}_{UB}(t_0) = \frac{(T/(T+t_0))^{n+1}}{\Gamma(n+1,\beta T)} \cdot \Big\{\Gamma(n+1,\beta(T+t_0)) + \frac{t_0(X+x_0)}{(m+m_0)(T+t_0)}\Gamma(n+2,\beta(T+t_0))\Big\}. \tag{8}$$

Next, we consider that the failure rate Λ has a gamma prior distribution given by

$$g(\lambda;\alpha,\beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha}\lambda^{\alpha-1}e^{-\lambda/\beta}, \quad 0<\lambda<\infty \tag{9}$$

denoted by GAM(α,β) and the probability ρ that the switch performs when required has independently UNIF(0,1) prior distribution. Then the joint posterior distribution of Λ and ρ given $T=t$ and $X=x$ is

$$g(\lambda;\rho|t,x;\alpha,\beta) = \frac{\Gamma(m+2)(\beta t+1)^{\alpha+n}}{\Gamma(x+1)\Gamma(m-x+1)\Gamma(\alpha+n)\beta^{\alpha+n}} \times \rho^x(1-\rho)^{m-x}\lambda^{\alpha+n-1}e^{-\lambda(\frac{\beta t+1}{\beta})}, \quad 0<\lambda<\infty,\ 0<\rho<1 \tag{10}$$

Therefore, under the squared error loss function, the Bayes estimator $\widehat{R}_{GU}(t_0)$ for the two-unit hot standby system reliability $R(t_0)$ with imperfect switch is

$$\widehat{R}_{GU}(t_0) = \Big(\frac{\beta T+1}{\beta T+\beta t_0+1}\Big)^{\alpha+n} \times \Big\{1+\frac{t_0\beta(\alpha+n)(X+1)}{(m+2)(\beta T+\beta t_0+1)}\Big\} \tag{11}$$

Finally, we consider that the failure rate Λ has a GAM(α,β) prior distribution, while the probability ρ that the switch performs when required has independently $BETA(x_0,m_0)$ prior distribution. Then the joint posterior distribution of Λ and ρ given $T=t$ and $X=x$ is

$$g(\lambda, \rho|t, x; \alpha, \beta, x_0, m_0) = \frac{\Gamma(m+m_0)(\beta t+1)^{\alpha+n}}{\Gamma(x+x_0)\Gamma(m+m_0-x-x_0)\Gamma(\alpha+n)\beta^{\alpha}} \cdot \rho^{x+x_0-1}(1-\rho)^{m+m_0-x-x_0}\lambda^{\alpha+n-1}e^{-\lambda(t+\frac{1}{\beta})}, \quad (12)$$
$$0<\lambda<\infty,\ 0<\rho<1.$$

Therefore, under the squared error loss function, the Bayes estimator $\widehat{R}_{GB}(t_0)$ for the two-unit hot standby system reliability $R(t_0)$ with imperfect switch is

$$\widehat{R}_{GU}(t_0) = \Big(\frac{\beta T+1}{\beta T+\beta t_0+1}\Big)^{\alpha+n}\Big\{1+\frac{t_0\beta(\alpha+n)(X+x_0)}{(m+m_0)(\beta T+\beta t_0+1)}\Big\}. \quad (13)$$

Tables 1 through 4 show the simulated values for the mean squared errors(MSE) of the proposed Bayes estimators of reliability for the two-unit hot standby system with imperfect switch when $\lambda = 1.0, \rho = 0.9, 0.95, 0.99, n = 25$ and various values of specified mission time t_o. From Tables, Bayes estimator $\widehat{R}_{GB}(t_0)$ for reliabllity of the two-unit hot standby system with imperfect switch under GAM(α, β) prior on Λ and BETA(x_0, m_0) prior on ρ is more efficient than other Bayes estimator for the large values of system reliability $R(t_0)$. Bayes estimator $\widehat{R}_{UU}(t_0)$ for reliabllity of the two-unit hot standby system with imperfect switch under UNIF(0, β) prior on Λ and UNIF(0,1) prior on ρ is more efficient than other Bayes estimator for the small values of system reliability $R(t_0)$.

Table 1. Simulated MSE's for the Bayes estimator for the two-unit hot standby system with imperfect switch under UNIF(0, β) prior on Λ and UNIF(0,1) prior on ρ.

	$\rho \backslash t_o$	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$\Lambda \sim$ UNIF(0,3)	0.90	0.0026	0.0085	0.0112	0.0103	0.0079	0.0056	0.0037	0.0024
&	0.95	0.0035	0.0102	0.0127	0.0112	0.0084	0.0057	0.0037	0.0024
$\rho \sim$ UNIF(0,1)	0.99	0.0046	0.0121	0.0143	0.0122	0.0089	0.0059	0.0038	0.0024
$\Lambda \sim$ UNIF(0,4)	0.90	0.0026	0.0085	0.0112	0.0103	0.0079	0.0056	0.0037	0.0024
&	0.95	0.0035	0.0102	0.0127	0.0112	0.0084	0.0056	0.0037	0.0024
$\rho \sim$ UNIF(0,1)	0.99	0.0046	0.0121	0.0143	0.0122	0.0089	0.0059	0.0038	0.0024

where simulations were repeated 5000 times for $\lambda = 1.0$.

Table 2. Simulated MSE's for the Bayes estimator for the two-unit hot standby system with imperfect switch under UNIF(0, β) prior on Λ and BETA(x_0, m_0) prior on P

	$\rho \backslash t_o$	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$\Lambda \sim$ UNIF(0,3)	0.90	0.0025	0.0084	0.0112	0.0103	0.0080	0.0056	0.0038	0.0025
&	0.95	0.0034	0.0101	0.0126	0.0112	0.0084	0.0058	0.0038	0.0024
$\rho \sim$ BETA(2,3)	0.99	0.0045	0.0119	0.0142	0.0122	0.0089	0.0060	0.0038	0.0024
$\Lambda \sim$ UNIF(0,4)	0.90	0.0025	0.0083	0.0111	0.0103	0.0080	0.0057	0.0038	0.0025
&	0.95	0.0034	0.0100	0.0126	0.0112	0.0084	0.0058	0.0038	0.0025
$\rho \sim$ BETA(3,4)	0.99	0.0044	0.0118	0.0141	0.0122	0.0089	0.0060	0.0039	0.0024

Table 3. Simulated MSE's for the Bayes estimator for the two-unit hot standby system with imperfect switch under GAM(α,β) prior on Λ and UNIF(0,1) prior on P

	$\rho \backslash t_o$	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$\Lambda \sim$ GAM(1,1.5)	0.90	0.0019	0.0065	0.0091	0.0090	0.0075	0.0057	0.0042	0.0029
&	0.95	0.0027	0.0077	0.0100	0.0095	0.0076	0.0057	0.0040	0.0028
$\rho \sim$ UNIF(0,1)	0.99	0.0036	0.0092	0.0112	0.0101	0.0079	0.0057	0.0040	0.0027
$\Lambda \sim$ GAM(1,2)	0.90	0.0020	0.0068	0.0094	0.0092	0.0076	0.0057	0.0041	0.0028
&	0.95	0.0028	0.0081	0.0104	0.0097	0.0077	0.0057	0.0040	0.0027
$\rho \sim$ UNIF(0,1)	0.99	0.0038	0.0097	0.?117	0.0104	0.0080	0.0058	0.0040	0.0027

Table 4. Simulated MSE's for the Bayes estimator for the two-unit hot standby system with imperfect switch under GAM(α,β) prior on Λ and BETA(x_0, m_0) prior on P

	$\rho \backslash t_o$	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$\Lambda \sim$ GAM(1,1.5)	0.90	0.0019	0.0065	0.0091	0.0090	0.0076	0.0058	0.0042	0.0030
&	0.95	0.0027	0.0077	0.0100	0.0095	0.0077	0.0058	0.0041	0.0029
$\rho \sim$ BETA(2,3)	0.99	0.0036	0.0091	0.0112	0.0101	0.0079	0.0058	0.0041	0.0028
$\Lambda \sim$ GAM(1,2)	0.90	0.0019	0.0067	0.0093	0.0092	0.0077	0.0058	0.0042	0.0029
&	0.95	0.0027	0.0079	0.0103	0.0097	0.0078	0.0058	0.0041	0.0028
$\rho \sim$ BETA(3,4)	0.99	0.0036	0.0094	0.0115	0.0104	0.0081	0.0058	0.0041	0.0028

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