광스캐닝 홀로그래피의 해상도

Resolution in Optical Scanning Holography

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요 약

광학적 스캐닝 홀로그래피에 있어서, 물체의 3차원 홀로그래픽 정보는 2차원적 광스캐닝에 의해서 생성되며, 광스캐닝 광선은 time-dependent한 Gaussian 형태의 Fresnel 윤대판(zone plate)이다. 본 기술에서 홀로그래픽 정보는 그 자체로서 전기적인 신호로서 발생하기 때문에 전자광선 addressed - spatial light modulator을 사용하여 영상 재생이 가능하다. 이 기법의 응용분야로서 3-차원 원거리 광 센서로서 사용될 수 있으며, 특히 비행물체 확인에 응용될 수 있다. 본 논문에서, 우리는 먼저 광스캐닝 홀로그래피에 대해 간략한 기술과 본 기술 시스템에 있어서 광스캐닝 빔의 해상도를 먼저 유도하고, 그 다음으로 Gaussian 원리를 이용하여 홀로그래픽 image 재생을 위해 필요한 실상(real image) 및 허상(virtual image)에 대한 수학적 표현을 제시하고자 한다.

Abstract

In optical scanning holography, 3-D holographic information of an object is generated by 2-D active optical scanning. The optical scanning beam can be a time-dependent Gaussian apodized Fresnel zone plate. In this technique, the holographic information manifests itself as an electrical signal which can be sent to an electron-beam-addressed spatial light modulator for coherent image reconstruction. This technique can be applied to 3-D optical remote sensing especially for identifying flying objects. In this paper, we first briefly review optical scanning holography and analyze the resolution achievable with the system. We then present mathematical expression of real and virtual image which are responsible for holographic image reconstruction by using Gaussian beam profile.

I. Introduction

Holography has been an important tool for scientific and engineering studies, it has found a wide range of application[1]~[7]. Electronic holography is a technique in which electronic processing is used in the context of holography, allowing holographic recording to be performed in real time, bypassing the use of films for recording[4]~[7],[13]. Thus electronic holography lends itself to real-time applications. In this paper, we limit our study to one type of electronic holography in which an active optical heterodyne scanning technique is used to achieve 3-D holographic recording. Two main features of this tech-

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technique known as optical scanning holography (OSH) are that the recording is incoherent (phase insensitive) and that a large area detector can be used[8].

Ⅱ. Fresnel Zone Plate and Resolution

Consider a spherical wave of wavelength λ, traveling from left to right as shown in Fig. 1. The spherical wave is produced by a point source and is assumed to be incident on the plate. The wavefront at a given instant of time may be divided into a number of concentric zones in such a way that each zone is one-half of a wavelength farther away from point O. If the distance from the wave front to O is S, the distances from successive boundaries between zones to O are given by

$$S+\frac{\lambda}{2},S+\frac{2\lambda}{2},S+\frac{3\lambda}{2},\ldots S+\frac{m\lambda}{2} \quad (1)$$

Let R_m be the radius of the m-th boundary. Then R_m is given by

$$R_m=\sqrt{\left(S+m\frac{\lambda}{2}\right)^2-S^2} \quad (2)$$

These are either opaque or transparent, that is, Fresnel zone plates. This result was obtained by

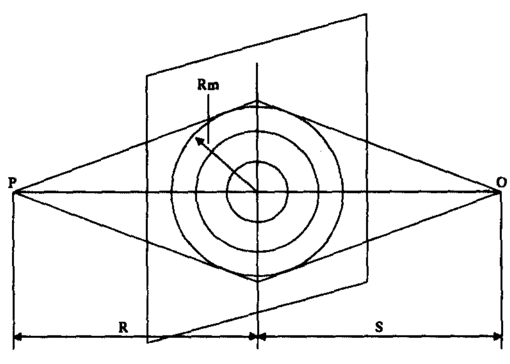

Fig. 1. Fresnel zone plate on a spherical wave front.

integration over all waves passing through the circular opening. The boundaries between zones have radii that are proportional to the square root of the integer number,

$$R_m=R_1\sqrt{m}, \quad m=1,2,3,\ldots \quad (3)$$

where R_1 is the radius of the innermost boundary. From Eq. (2)

$$S^2=\left(S+m\frac{\lambda}{2}\right)^2-R_m^{\ 2}, \quad R_m^{\ 2}\approx mS\lambda \quad (4)$$

Since λ is small compared with S, the term λ/S may be neglected. However S is also the focal length of the zone plate because of its focusing properties. Thus

$$f=\frac{R_m^{\ 2}}{m\lambda} \quad (5)$$

A zone lens does act both as a positive and a negative lens. Its focal length is $\pm f$. That is because light diffracted at the boundaries will be diffracted not only toward the optic axis, but away from it as well. Now consider Fig. 2 In which Fresnel zone plates have R_m and R_n as radii of the boundary and f_l and f_s are their focal lengths respectively. According to Eq. (5) we can have

$$f_l=\frac{R_m^{\ 2}}{M\lambda}, \quad f_s=\frac{R_n^{\ 2}}{N\lambda} \quad (6)$$

where M, N are the integer. Resolution limit is defined as $R\approx\frac{\lambda}{\theta}$. In order to make the two zone lens to give the same resolution on image reconstruction, we need to have

$$(R)_l=(R)_s, \quad \frac{f_l}{D_l}\lambda=\frac{f_s}{D_s}\lambda, \quad \frac{M}{N}=\frac{R_m}{R_n} \quad (7)$$

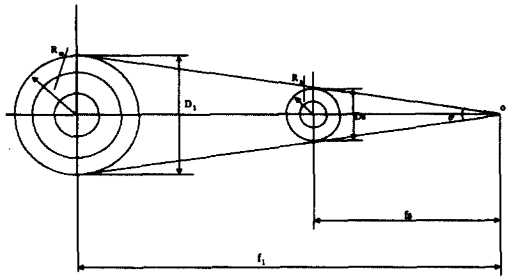

Fig. 2. Fresnel zone plate of two different size.

which shows that the ratio of the number of zones is inversely proportional to that of the radius of the zone lens plate.

Ⅲ. Theory of Optical Scanning Holography

In optical scanning holography, an object is two-dimensionally scanned by a time-dependent Fresnel zone plate(TDFZP) created by the superposition of a plane wave and a spherical wave of different temporal frequencies to generate a hologram of the object, as shown in Fig. 3. We model the scanning plane wave and the spherical wave by two Gaussian beams of a broad and a narrow waist, respectively. In practice, a broad and a narrow Gaussian beam are combined with a beamsplitter [9],[10]. One of the Gaussian beams is frequency-shifted so that the two beams interfere to form a temporally modulated Gaussian apodized Fresnel zone plate $I(x, y, z, t)$ at the location of the object $P(x, y, z)$:

$$I(x, y, z, t) = b(x, y, z) + \frac{2}{\pi w_u(0)\, w_u(z)\, w_v(z)} \cdot \exp\left(-\left(\frac{x^2+y^2}{w(z)^2}\right)\right) \sin\left(\frac{k_0}{2R(z)}(x^2+y^2)+\Omega t\right), \quad (8)$$

where k_0 is the wave number of the light and z is the distance measured away from the waist of the narrow beam to the location of the object $P(x, y; z)$, as shown in Fig. 3. $b(x, y; z)$ is the DC term and the AC term, modulated at temporal frequency Ω, is the temporally modulated Gaussian apodized Fresnel zone plate. Ω denotes the temporal frequency shift between the narrow Gaussian beam and the board Gaussian beam. The photodetector PD collects the scattered light after the modulated zone plate interacts with the object, $P(x, y; z)$, to give a current

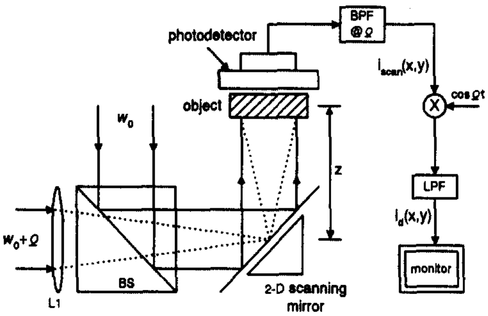

Fig. 3. Real-time optical heterodyne scanning holographic system.

$$\begin{aligned} i(x, y; z; t) &\sim \int\int_A P(x', y'; z) I(x(t) - x', y(t) - y'; z; t)\, dx'\, dy' \\ &= I(x, y; z; t) * P(x, y; z) \\ &= b(x, y; z) * P(x, y; z) \\ &\quad + P(x, y; z) * C(z) \exp\left(-\left(\frac{x^2+y^2}{w(z)^2}\right)\right) \\ &\quad \cdot \sin\left(\frac{k_0}{2R(z)}(x^2+y^2)+\Omega t\right) \end{aligned} \quad (9)$$

Where C(z) is $\frac{2}{\pi w_u(0)\, w_u(z)\, w_v(0)\, w_v(z)}$

Note that the integration is over the photodetector area A; $x(t)$ and $y(t)$ are determined by the xy-scanner's motion. Therefore, the convolution operation * is effected by the physical scanning. The AC term can be separated by a bandpass filter (BPF) tuned at frequency to give the signal which can be demodulated by mixing it with $\cos(\Omega t)$ and lowpass filtered to give a demodulated current i_d :

$$i_d(x,y;z) \sim P(x,y;z) * C(z)\exp\left(-\frac{(x^2+y^2)}{w(z)^2}\right) \cdot \sin\left(\frac{k_0}{2R(z)}(x^2+y^2)\right) \quad (10)$$

$i_d(x,y;z)$ contains the holographic information of the object being scanned. In fact it is a zone plate coded information of the object. $i_d(x,y;z)$ can be stored on a computer for digital reconstruction. If this holographic information needs to be displayed or stored on films, a constant bias can be added to it to give a hologram of an amplitude transmittance $t(x,y;z)$

$$t(x,y;z) = b + i_d = b + P(x,y;z) * C(z)\exp\left(-\frac{(x^2+y^2)}{w(z)^2}\right) \sin\left(\frac{k_0}{2R(z)}(x^2+y^2)\right) \quad (11)$$

Note that the space-variant term $b(x,y;z)$ in $i(x,y;z;t)$ has been filtered out by the bandpass filter. Therefore, in computer simulations to be performed later, we will plot the scanning beam profile as

$$f_s(x,y;z) = I(x,y;z;t=0) - b(x,y;z) = C(z)\exp\left(-\frac{(x^2+y^2)}{w(z)^2}\right) \cdot \sin\left(\frac{k_0}{2R(z)}(x^2+y^2)\right) \quad (12)$$

To reconstruct the hologram $t(x,y;z)$ optically, we can illuminate it with a plane wave.

Let us now concentrate on a point object reconstruction. We, therefore, let $P(x,y;z) = \delta(x,y)$, located at $z = z_0$ away from the waist of the narrow beam, the point-object hologram, according to (11), is

$$t_\delta(x,y;z_0) = b + \exp\left(-\frac{(x^2+y^2)}{w(z_0)^2}\right) \cdot \sin\left(\frac{k_0}{2R(z_0)}(x^2+y^2)\right) \quad (13)$$

Upon plane wave illumination, the complex field distribution at a distance z away from the point-object hologram is given by

$$\psi(x,y;z) = t_\delta(x,y;z_0) * h(x,y;z) \quad (14)$$

where

$h(x,y;z) = \exp(-jk_0(x^2+y^2)/2z)$ is the free space impulse response [11,12], neglecting some constants. By expanding (14), we have

$$\begin{aligned}\psi(x,y;z) \sim\ & b * \exp\left(-j\frac{k_0}{2z}(x^2+y^2)\right) \\ & + \frac{1}{2j}\exp\left(-\frac{(x^2+y^2)}{w(z_0)^2}\right)\exp \\ & \cdot \left(j\frac{k_0}{2R(z_0)}(x^2+y^2)\right)\exp\left(-j\frac{k_0}{2z}(x^2+y^2)\right) \\ & - \frac{1}{2j}\exp\left(-\frac{(x^2+y^2)}{w(z_0)^2}\right) \\ & \exp\left(-j\cdot\frac{k_0}{2R(z_0)}(x^2+y^2)\right) \\ & \exp\left(-j\frac{k_0}{2z}(x^2+y^2)\right)\end{aligned} \quad (15)$$

At a distance $z = z_0$, the second term can be evaluated and gives rise to a real image reconstruction:

$$\psi_r = \pi\frac{w(z_0)^2}{4}\exp(-a^2 w(z_0)^2(x^2+y^2)) \cdot \exp(-ja(x^2+y^2)) \quad (16)$$

where $a = k_0/2R(z_0)$. Hence, a point-object gives rise to a Gaussian distribution with its waist given by $1/aw(z_0)$, i.e., the resolution of the reconstructed point depends on the size of the scanning Gaussian beam, as well as the number of zones within the beam on the scanned object, i.e., $\sim a$. The third term in (15) gives a twin-image ψ_t in the $z = z_0$ plane:

$$\psi_t = \frac{\pi\, w(z_0)^2}{4(1+4a^2w(z_0)^4)^{1/2}} \cdot \exp\left(-\frac{a^2w(z_0)^2(x^2+y^2)}{1+4a^2w(z_0)^4}\right) \cdot \exp\left(-ja(x^2+y^2)\left(\frac{1+2a^2w(z_0)^4}{1+4a^2w(z_0)^4}\right)\right) \quad (17)$$

The first term in (15) gives a constant background π/a. To find the intensity distribution on the real-image reconstruction plane, we calculate

$$P(x, y; z) = \psi(x, y; z)\psi^*(x, y; z) = |\pi/a + \psi_r + \psi_t|^2 \quad (18)$$

We now want to investigate the effect of the parameters $k_0/2R(z_0)$ and $w(z_0)$ on reconstruction based on the above equation. It is evident that the resolution is given by the waist of the reconstructed point and is equal to $1/\left\{\left(\frac{k_o}{2R(z_0)}\right)w(z_0)\right\}$. Referring to eq. (7), the larger the scanning beam waist, $w(z_0)$, the better is the resolution if $k_0/2R(z_0)$ remains the same. Also, a larger number of zones within the Gaussian beam, i.e., the larger the $k_0/2R(z_0)$, will give a better resolution if the beam waist is the same. For a given $\{k_0/2R(z_0)\} \times w(z_0)$, the resolution of the reconstructed image remains the same.

Ⅳ. Concluding Remarks

We have discussed Optical Scanning Holography(OSH). The principle of OSH is based on optical heterodyneing and scanning and therefore is an electro-optical hybrid sysem that is real-time in nature. It has been shown that by scanning an object with a temporally modulated Gaussian apodized Fresnel zone plate to acquire holographic information, the ratio of the number of zones is inversely proportional to that of the radius of the zone lens plate. We also discussed the resolution achievable is directly proportional to the size of the beam and the number of zones within the beam. Note that the two scanning beams will give the same resolution upon image reconstruction on the condition that the product of $nw(z_0)$ and $\left(\frac{k_0}{2R(z_0)}\right)\left(\frac{1}{n}\right)$ gives $w(z_0) \times \left(\frac{k_0}{2R(z_0)}\right)$.

One may argue that scanning with a larger beam causes twin-image noise to interact with a broader part of the image. Further investigation into the effect of the parameter $k_0/2R(z_0)$ and $w(z_0)$ on reconstruction remains for the future work. This effect may prove to be more important if the object is contaminated with noise. It may be that when the size of the scanning zone plate is larger than that of the object so that the scanning beam always illuminates the entire object, we have a situation reminiscent of coherent holographic recording since the hologram is the simultaneous superposition of the individual holograms of each point within the object. On the contrary, if the zone plate size is much smaller than that of the object, the zone plate overlaps only a small part of the object at any instant of the scanning process, a situation reminiscent of a partially coherent recording [14].

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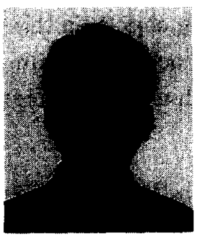

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