JOURNAL OF THE
CHUNGCHEONG MATHEMATICAL SOCIETY
Volume 11, August 1998

DYNAMICAL STABILITY AND SHADOWING PROPERTY OF CONTINUOUS MAPS

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ABSTRACT. This paper deals with the topological stability of continuous maps. First, the notion of local expansion is given and we show that local expansions of compact metric spaces have the shadowing property. Also, we prove that if a continuous surjective map f is a local homeomorphism and local expansion, then f is topologically stable in the class of continuous surjective maps. Finally, we find homeomorphisms which are not topologically stable.

1. Introduction and Preliminaries

This paper deals with the topological stability and shadowing property of continuous surjective maps in dynamical systems. We guess that one who studies the dynamical systems theory knows the importance of the topological stability in the stability theory of dynamical systems. It is known that every expansive homeomorphism with the shadowing property is topologically stable in the class of homeomorphisms[8].

But unfortunately, it is impossible that a homeomorphism is topologically stable in the class of continuous surjective maps[6]. Thus it is natural to ask the following question:

Question : Which continuous maps are topologically stable in the class of continuous surjective maps?

This work was supported by the Korea Science and Engineering Foundation, 1997; Porj. No. 971-0103-015-1.

Received by the editors on June 25, 1998.

1991 *Mathematics Subject Classifications*: Primary 58F10, 54H20.

Key words and phrases: local expansion, positively expansive, shadowing property, topological stability.

For example, we can see that every expanding map satisfies the topological stability in the class of continuous surjective maps[6]. In this paper, the notion of local expansion is given and we see that this concept can be used usefully in studying the shadowing property and topological stability.

First, we show that local expansions have the shadowing property in Proposition 2.5. Next, as answers to the above question, we will show that if a continuous surjective map f is a local homeomorphism and local expansion, then f is topologically stable in the class of continuous surjective maps in Theorem 3.3. Finally, we consider homeomorphisms which are not topologically stable in Theorem 4.3.

Throughout this paper, we assume that X is a compact metric space with a metric function d and $f : X \to X$ denotes a continuous surjective map. f is called a *positively expansive map with positively expansive constant* c provided that if $d(f^n(x), f^n(y)) \leq c$ for every nonnegative integer n then $x = y$ holds.

A sequence of points $\{x_i\}_{i=a}^{b}(-\infty \leq a < b \leq \infty)$ is called *f-orbit* if $f(x_i) = x_{i+1}$ and is called *δ-pseudo-orbit* of f if $d(f(x_i), x_{i+1}) < \delta$ for $a \leq i < b$, respectively. A sequence of points $\{x_i\}_{i=a}^{b}, (a \leq i \leq b)$ is called to be *ε-shadowed* by a f-orbit $\{y_i\}_{i=a}^{b}$ if $d(x_i, y_i) < \varepsilon$ holds for $a \leq i \leq b$. We say that f has the *shadowing property* if for every $\varepsilon > 0$ there is $\delta > 0$ such that every δ-pseudo-orbit of f is ε-shadowed by some f-orbit of X.

We say that f is *topologically stable in the class of continuous surjective maps (homeomorphisms)* if for every $\varepsilon > 0$ there is $\delta > 0$ with the property that for every continuous surjective map (homeomorphism) g of X with $d(f(x), g(x)) < \delta$ for every $x \in X$, there is a continuous map $h : X \to X$ such that for every x in X

$$\text{(i)} \;\; h \circ g(x) = f \circ h(x) \qquad \text{and} \qquad \text{(ii)} \;\; d(h(x), x) < \varepsilon$$

hold.

Let $B(x,\varepsilon)$ denote $\{y \in X : d(x,y) < \varepsilon\}$ and $\overline{M}$ denote the closure of $M \subset X$.

2. Local expansions

Here we give the notion of local expansions and find some examples and the dynamical properties of local expansions.

The family of all nonempty subsets of X is denoted by $K(X)$. If $A \subset X$ and $\varepsilon > 0$, then $B(A,\varepsilon)$ denotes $\{x \in X : d(x,a) < \varepsilon \text{ for some } a \in A\}$, and for A, B in $K(X)$, the Hausdorff metric h_d induced by d is defined as

$$h_d(A,B) = \inf\{\varepsilon > 0 : A \subset B(B,\varepsilon) \quad \text{and} \quad B \subset B(A,\varepsilon)\}.$$

Equivalently,

$$h_d(A,B) = \max\{\sup_{x\in A} d(x,B),\ \sup_{y\in B} d(A,y)\},$$

where $d(x,S)$ is defined by $\inf_{y\in S} d(x,y)$. Here we can regard f^{-1} as a set-valued map from X to $K(X)$. Now we give the notion of local expansions as follows.

DEFINITION. A continuous surjective map $f : X \to X$ is called a *local expansion* if there exist positive numbers ε and $k < 1$ such that $d(x,y) < \varepsilon$ implies $h_d(f^{-1}(x), f^{-1}(y)) \le kd(x,y)$.

To illuminate the above notion, we give a few examples of local expansions.

EXAMPLE 2.1. Let X be a subspace of the real line $\mathbb{R}$ given by $X = \{0\} \cup \{a_n\}_{n=0}^{\infty}$, where $a_n = \frac{1}{2^n}$ for $n > 0$ and $a_0 = 1$. Define a continuous map f of X onto itself as follows; 0 and a_0 are fixed points of f and $f(a_n) = a_{n-1}$ for $n > 0$. Then $d(x,y) < \frac{1}{2}$ implies $h_d(f^{-1}(x), f^{-1}(y)) = \frac{1}{2}d(x,y)$. Hence, f is a local expansion.

EXAMPLE 2.2. Let $Z = \prod_{i=0}^{\infty} X_i$, where each X_i is a copy of X. The metric ρ for Z is defined by

$$\rho(x,y) = \sum_{i=0}^{\infty} d(x_i, y_i)/2^i, \qquad (x = (x_i), \quad y = (y_i) \in Z).$$

The general shift map $\sigma_k : Z \to Z$, $(k > 0)$, defined by $\sigma_k(x_0, x_1, x_2, \cdots) = (x_k, x_{k+1}, x_{k+2}, \cdots)$ is a continuous surjective map and a local expansion since $h_\rho(\sigma_k^{-1}(x), \sigma_k^{-1}(y)) = \frac{1}{2^k}\rho(x,y)$.

EXAMPLE 2.3. Let f_s, $(0 < s < 1)$, be a continuous surjective map of the interval $\{0 \leq x \leq 1\}$ onto itself defined by

$$f_s(x) = \begin{cases} \frac{1}{s}x & \text{if } 0 \leq x \leq s, \\ -\frac{1}{1-s}x + \frac{1}{1-s} & ifs \leq x \leq 1. \end{cases}$$

Then f is a local expansion. However, f is not positively expansive. It is known that interval $\{0 \leq x \leq 1\}$ does not admit any positively expansive map (cf. [1]).

LEMMA 2.4. ([4, p 657]) *Let A, B in $K(X)$ be such that $h_d(A,B) < \delta$. Then for any element a in A there exists some b in B such that $d(a,b) < \delta$.*

In [4] we can see that shift maps given in Example 2.2 have the shadowing property. The following result shows that the shadowing property is a general property which every local expansion can have.

PROPOSITION 2.5. *Every local expansion has the shadowing property.*

Proof. Let f be a local expansion and let $\varepsilon > 0$ and $0 < k < 1$ be numbers such that

$$(1) \qquad d(a,b) < \varepsilon \quad \text{implies} \quad h_d(f^{-1}(a), f^{-1}(b)) \leq kd(a,b).$$

Let $\gamma > 0$ be given. Take $\delta > 0$ so that $\delta < \min\{\varepsilon, \frac{\gamma(1-k)}{2k}\}$. Let $\{x_n\}_{n=-\infty}^{\infty}$ is a δ-pseudo-orbit of f. First, by using the mathematical induction, we claim that for every $n > 0$ there exists a family of sequences of points $\{A_i\}_{i=-n}^{n}$, such that $A_i = \{y_j^i\}_{j=-n-i}^{0}$ satisfies the following;

$$d(y_j^{i-1}, y_{j-1}^i) < k^{-j+1} \cdot \delta \quad \text{and} \quad f(y_{j-1}^i) = y_j^i. \tag{2}$$

Let y_0^n denote x_n for every integer n. Let $n = 1$. Since $d(f(y_0^0), y_0^1) < \delta$ and $d(f(y_0^{-1}), y_0^0) < \delta$, by (1), there are points y_{-1}^1 in $f^{-1}(y_0^1)$ and y_{-1}^0 in $f^{-1}(y_0^0)$ such that

$$d(y_0^0, y_{-1}^1) < k\delta \quad \text{and} \quad d(y_0^{-1}, y_{-1}^0) < k\delta. \tag{3}$$

Also, by (1) and (2), there is y_{-2}^1 in $f^{-1}(y_{-1}^1)$ such that $d(y_0^{-1}, y_{-2}^1) < k^2\delta$. Then

$$A_1 = \{y_0^{-1}\},\ A_0 = \{y_0^0, y_{-1}^0\}, \text{ and } A_1 = \{y_0^1, y_{-1}^1, y_{-2}^1\} \tag{4}$$

are sequences of points that satisfy (2). Next, for $n > 0$, assume that a family of sequences $\{A_i\}_{i=-n}^{n}$, satisfying (2) exists. Consider the sequence of points

$$A_n = \{y_{-2n}^n, y_{-2n+1}^n, \cdots, y_{-1}^n, y_0^n\}.$$

By (1) and from the fact that $d(f(y_0^n), y_0^{n+1}) < \delta$, there is a point y_{-1}^{n+1} in $f^{-1}(y_0^{n+1})$ such that $d(y_0^n, y_{-1}^{n+1}) < k\delta$. Also, since

$$h_d(f^{-1}(y_0^n), f^{-1}(y_{-1}^{n+1})) \leq kd(y_0^n, y_{-1}^{n+1}) < k^2\delta,$$

we can take a point y_{-2}^{n+1} in $f^{-1}(y_{-1}^{n+1})$ such that $d(y_{-1}^n, y_{-2}^{n+1}) < k^2\delta$. Continuing this process, we get a sequence of points

$$\tilde{A}_{n+1} = \{y_{-2n-1}^{n+1}, y_{-2n}^{n+1}, \cdots, y_{-1}^{n+1}, y_0^{n+1}\} \tag{5}$$

such that $d(y_j^n, y_{j-1}^{n+1}) < k^{1-j}\delta$, where $-2n-1 \leq j \leq -1$.

On the other hand, let us consider the sequence of points

$$\{y_0^{-n}, y_{-2}^{-n+2}, \cdots, y_{-2n-1}^{n+1}\}.$$

By the induction hypothesis, $d(y_{-i-n}^i, y_{-i-n-1}^{i+1}) < k^{i+n+1}\delta$, $(-n \leq i \leq n)$ and by the fact that $d(f(y_0^{-n-1}), y_0^{-n}) < \delta$ and (1), we can take a sequence of points

$$\{y_0^{-n-1}, y_{-1}^n, y_{-2}^{-n+1}, \cdots, y_{-2n-2}^{n+1}\}$$

satisfying that

$$d(y_{-i-n-1}^i, y_{-i-n-2}^{i+1}) < k^{i+n+2}\delta \quad \text{and} \quad f(y_{-i-n-1}^i) = y_{-i-n}^i,$$

where $-n-1 \leq i \leq n$. Define a family of sequences of points $\{B_i\}_{i=-n-1}^{n+1}$ by putting

$$\begin{aligned} B_{-n-1} &= \{y_0^{-n-1}\}, \\ B_i &= A_i \cup \{y_{-n-1-i}^i\}, \quad (-n \leq i \leq n) \\ B_{n+1} &= \tilde{A}_{n+1} \cup \{y_{-2n-2}^{n+1}\} \end{aligned}$$

Then we can see that $\{B_i\}_{i=-n-1}^{n+1}$ is a family of sequences of points satisfying (2). Therefore, we conclude that for every n there is a family of sequence of points $\{A_i\}_{i=-n}^n$ which satisfies (2).

Let us fix an integer m and consider the sequence of points $\Theta_m = \{y_{-i}^{m+i}\}_{i=0}^\infty$. Since $d(y_i^m, y_{i-1}^{m+1}) < k^{1-i}\delta$, the sequence Θ_m is a Cauchy sequence and, moreover, we get

$$\begin{aligned} d(x_m, y_{-i}^{m+1}) \leq & d(x_m, y_{-1}^{m+1}) + d(y_{-1}^{m+1}, y_{-2}^{m+2}) \\ & + \cdots + d(y_{-i+1}^{m+i-1}, y_{-i}^{m+i}) \\ < & k\delta + k^2\delta + \cdots + k^i\delta < \frac{k\delta}{1-k} < \frac{\gamma}{2} \end{aligned}$$

Let, for each n, Θ_n approach to z_n as $i \to \infty$. Then it is not hard to show that

$$d(x_n, z_n) < \gamma \quad \text{and} \quad f(z_n) = z_{n+1}.$$

This shows that f has the shadowing property and this completes the proof of this result. □

3. Topologically stable maps

In this section we find several continuous surjective maps which are topologically stable in the class of continuous surjective maps.

PROPOSITION 3.1. *Every local expansion is an open map*

Proof. Let f be a local expansion and let $\varepsilon > 0$ and $0 < k < 1$ be numbers with the property of local expansion. Let U be an open subset of X and $y \in U$. Let $B(y, \gamma) \subset U$ for some $\gamma > 0$. Choose a positive number δ with $\delta < \min\{\varepsilon, \gamma\}$. Let $z \in B(f(y), \delta)$. Since f is a local expansion we get that

$$h_d(f^{-1}(f(y)), f^{-1}(z)) \leq kd(f(y), z) < k\delta < \delta.$$

Hence, there is z_{-1} in $f^{-1}(z)$ such that $d(y, z_{-1}) < \delta$. Therefore, we have

$$z = f(z_{-1}) \in f(B(y, \delta)) \subset f(B(y, \gamma)) \subset f(U).$$

This shows that f is an open map. □

f is said to be *expanding* if f is positively expansive open map.

LEMMA 3.2. ([6, p 505]) *every expanding map is topologically stable in the class of continuous surjective maps.*

THEOREM 3.3. *If f is a local homeomorphism and local expansion, then f is topologically stable in the class of continuous surjective maps.*

Proof. In view of the above lemma, it is sufficient to show that f is a positively expansive map. Let $\varepsilon > 0$ and $0 < k < 1$ be numbers with the property of local expansion. Also, since X is compact and f is a local homeomorphism there is $\alpha > 0$ such that if $x \neq y$, $(x, y \in X)$, and $f(x) = f(y)$, then $d(x,y) \geq \alpha$. Take $\bar{k}$ with $k < \bar{k} < 1$ and let us choose a positive number e so that $e < \min\{\varepsilon, \frac{\alpha}{1+\bar{k}}\}$. We claim that e is a positively expansive constant for f. To see this, assume that

$$d(f^i(x), f^i(y)) \leq e \qquad \text{for every integer} \quad i \geq 0. \tag{1}$$

Let $n > 0$ be fixed. Since $d(f^n(x), f^n(y)) < \varepsilon$ we have

$$h_d(f^{-1}(f^n(x)), f^{-1}(f^n(y))) \leq kd(f^n(x), f^n(y)) < \bar{k}d(f^n(x), f^n(y)).$$

Hence, there is z in $f^{-1}(f^n(y))$ such that

$$d(f^{n-1}(x), z) < \bar{k}d(f^n(x), f^n(y)). \tag{2}$$

Now, we claim that z must be $f^{n-1}(y)$. For if not otherwise; by (2), we get that

$$\begin{aligned} d(f^{n-1}(x), z) &\geq d(z, f^{n-1}(y)) - d(f^{n-1}(y), f^{n-1}(x)) \\ &\geq \alpha - e > \bar{k}e \geq \bar{k}d(f^n(x), f^n(y)). \end{aligned}$$

This contradicts (2). Thus we have

$$d(f^{n-1}(x), f^{n-1}(y)) < \bar{k}d(f^n(x), f^n(y)). \tag{3}$$

This shows that for every $n > 0$

$$d(x,y) < \bar{k}^n d(f^n(x), f^n(y)) < \bar{k}^n e$$

holds. Consequently, x must be y and, thus, f is a positively expansive map as required. Thus the proof of this theorem is complete. □

EXAMPLE 3.4. Let f be a map given in Example 2.1. Then f is a local homeomorphism and local expansion. Thus f is topologically stable in the class of continuous surjective maps.

REMARK 3.5. It is known that if f is expanding then f is a local homeomorphism [1, p.50]. However, the converse of Theorem 3.3 does not hold. To see this, consider the following example; Let X be a subspace of the real line $\mathbb{R}$ given by $X = \{0\} \cup \{a_n\}_{n=0}^{\infty} \cup \{b_n\}_{n=0}^{\infty}$, where

$$a_0 = 1,\ a_n = \frac{1}{2^n} \text{ if } n > 0, \text{ and } b_0 = 2,\ b_n = 1 + \frac{1}{2^n} \text{ if } n > 0.$$

Let f be a continuous surjective map defined as follows; $0, a_0$ and b_0 are fixed points of f, and $f(a_n) = a_{n-1}, f(b_n) = b_{n-1}$ for $n > 0$. It is easily checked that f is expanding, but not a local expansion.

The example of continuous maps given in Remark 3.5 shows that openness of f does not imply the continuity of f^{-1}. But it is easily checked that the continuity of f^{-1} means the openness of f. Hence we can derive the following result.

PROPOSITION 3.6. *Let f be positively expansive and f^{-1} be continuous. Then f is topologically stable in the class of continuous surjective maps.*

Let $\Omega(f) = \{x \in X :$ for every neighbourhood U of x and integer $n_0 > 0$, there exists an integer $n \geq n_0$ such that $f^n(U) \cap U \neq \emptyset\}$ denote the set of nonwandering points of a continuous map f. A sequence of finite points $\{z_0, z_1, \cdots, z_m\}$ is called a *δ-chain* from z_0 to z_m if $d(z_{i-1}, z_i) < \delta$ for $i = 1, 2, \cdots, m$. In connected spaces for each positive number δ and each pair of points p and q, there is a δ-chain from p to q.

REMARK 3.7. (i) Let f be a local contraction of a compact connected metric space X into itself, ie., there are positive numbers

$\varepsilon, k < 1$ such that $0 < d(x,y) < \varepsilon$ implies $d(f(x), f(y)) \leq kd(x,y)$ holds. Then $\Omega(f)$ is a singleton. In fact, since f is a local contraction f has a unique fixed point x. In fact, let $z \in \Omega(f)$ such that $z \neq x$. Let us choose a positive number ε with $\varepsilon < \frac{\lambda}{2}$ and $B(x,\varepsilon) \cap B(z,\varepsilon) = \emptyset$. Since X is connected there is $\lambda/2$-chain $\{w_0, w_1, \cdots, w_m\}$ from x to z. Then for an arbitrary $a \in B(z,\varepsilon)$, $\{w_0, w_1, \cdots, w_{m-1}, a\}$ is a λ-chain from x to a. Since f is a local contraction, we have

$$\begin{aligned} d(x, f^n(a)) &\leq k^n(d(w_0, w_1) + d(w_1, w_2) + \cdots + d(w_{m-1}, a)) \\ &\leq m\lambda k^n. \end{aligned}$$

Let N be an integer such that for all $n \geq N$, $d(x, f^n(a)) < \varepsilon$. As $z \in \Omega(f)$ there is some point b in $B(z,\varepsilon)$ such that $f^L(b) \in B(z,\varepsilon)$ for some $L > N$. This contradicts to our choice of N and ε. Therefore $\Omega(f) = \{x\}$.

(ii) Let f be a homeomorphism of a compact connected space X and supposes that f is a local expansion. Then $\Omega(f) = \Omega(f^{-1})$ is a singleton by (i). It was shown that if f has the shadowing property and $f|_{\Omega(f)}$ is topologically transitive, then $\Omega(f) = X$ holds [2]. Therefore X must be one point set $\Omega(f)$. This shows that a compact connected space which is not one point does not admit any local expansion homeomorphisms. Note that, for a homeomorphism f, f is a local expansion if and only if f^{-1} is a local contraction.

4. Homeomorphisms which are not topologically stable

In this section we consider homeomorphisms which are not topologically stable. A homeomorphism f of a compact space is called *topologically transitive* if there is a dense orbit of f in X. A homeomorphism f is called *positively recurrent (negatively recurrent)* if $x \in \omega_f(x) (x \in \alpha_f(x))$ for every x in X, where $\omega_f(x)$ and $\alpha_f(x)$ denotes the positive and negative limit set of x for f, respectively.

LEMMA 4.1. *Let f be a homeomorphism of X. If f is topologically transitive and positively or negatively recurrent and X contains a proper invariant closed subset of X then f does not have the shadowing property.*

Proof. First, suppose that f is positively recurrent and has the shadowing property. Let M be a proper closed invariant subset of X and $x_0 \in X \setminus M$ and $d(x_0, M) = 2\varepsilon$. Let $z_0 \in M$ and $\delta = \delta(\varepsilon)$ with $0 < \delta < \varepsilon$ be a number with the property of the shadowing property for f. Since f is topologically transitive there is y in X such that $\overline{O_f(y)} = X$. Hence we can choose positive integers n_1, n_2 satisfying that

$$d(f^{n_1}(y), f(x_0)) < \delta \quad \text{and} \quad d(f^{n_2}(y), z_0) < \delta.$$

Assume $n_1 \geq n_2$. Since f is positively recurrent we have

$$f^{n_2}(y) \in \omega_f(f^{n_2}(y)) = \omega_f(y)$$

and so we can choose an integer n' with $n' > n_1$ such that $d(f^{n'}(y), z_0) < \delta$. So we may assume that $n_2 > n_1$. Then we can construct a δ-pseudo-orbit $\{a_i\}_{i=0}^{\infty}$ of f by putting

$$\begin{aligned} a_0 &= x_0 \\ a_i &= f^{n_1+i}(y), \qquad 1 \leq i \leq n_2 - n_1, \\ a_i &= f^{i-n_2+n_1-1}(z_0), \quad i > n_2 - n_1. \end{aligned}$$

Since f satisfies the shadowing property there is a in X such that $d(f^i(a), a_i) < \varepsilon$ for all $i \geq 0$. Then

$$d(a, a_0) = d(a, x_0) < \varepsilon \qquad \text{and}$$
$$d(f^i(a), f^i(z_0)) < \varepsilon \qquad \text{for} \quad i > n_2 - n_1$$

hold. So we have

$$O_f^+(f^{n_2-n_1+1}(a)) \subset B(O_f^+(z_0), \varepsilon) \subset B(M, \varepsilon)$$

and hence

$$\omega_f(a) = \omega_f(f^{n_2-n_1+1}(a)) \subset \overline{O_f^+(f^{n_2-n_1+1}(a))} \subset \overline{B(M,\varepsilon)}.$$

However, by the positive recurrence of f, $a \in \omega_f(a)$ and therefore $d(M,a) \le \varepsilon$. So we have

$$d(x_0, M) \le d(x_0, a) + d(a, M) < 2\varepsilon.$$

Thus contradicting that $2\varepsilon = d(x_0, M)$.

Next, assume that f is negatively recurrent. Then we can choose negative integers m_1, m_2 with $m_1 > m_2$ such that

$$d(f^{m_1}(y), f(x_0)) < \delta \quad \text{and} \quad d(f^{m_2}(y), z_0) < \delta.$$

Let us define a sequence of points $\{b_{-i}\}_{i=0}^{\infty}$ by putting

$$\begin{aligned} b_0 &= x_0, \\ b_{-i} &= f^{m_1-i+1}(y), \qquad 1 \le i \le m_1 - m_2, \\ b_{-i} &= f^{-i-m_1+m_2-1}(z_0), \qquad i > m_1 - m_2. \end{aligned}$$

Obviously, $\{b_{-i}\}_{i=0}^{\infty}$ is a δ-pseudo-orbit of f^{-1}. Since f^{-1} also has the shadowing property there is b in X such that $d(f^i(b), b_i) < \varepsilon$ for all $i \le 0$. In particular, we have

$$\begin{gathered} d(b, b_0) = d(b, x_0) < \varepsilon \quad \text{and,} \\ d(f^{i-m_1+m_2-2}(b), f^i(z_0)) < \varepsilon \quad \text{for} \quad i \ge 0. \end{gathered}$$

Similarly, we can obtain the following fact that

$$b \in \alpha_f(b) \subset \overline{B(M,\varepsilon)}.$$

This shows that $d(M,b) \le \varepsilon$ and we have

$$d(x_0, M) \le d(x_0, b) + d(b, M) < 2\varepsilon.$$

Hence we can derive the same contradiction, and so completes the proof. □

LEMMA 4.2. ([1,p 93]) *Every homeomorphism of a compact manifold which is topologically stable in the class of homeomorphisms has the shadowing property.*

Lemma 4.1 and Lemma 4.2 yield the following;

THEOREM 4.3. *Let f be a homeomorphism of a compact manifold X. If f is topologically transitive and positively or negatively recurrent, and X is not minimal, then f is not topologically stable in the class of homeomorphisms.*

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