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SUBMANIFOLDS WITH PARALLEL NORMAL MEAN CURVATURE VECTOR

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ABSTRACT. In this paper, we study submanifolds in the Euclidean space with parallel normal mean curvature vector and special quadric representation. Especially we give a complete classification result relative to surfaces satisfying these conditions.

1. Introduction

Let $x : M^n \to E^m$ be an isometric immersion of an n-dimensional Riemannian manifold into the m-dimensional Euclidean space, and $SM(m)$ be the space of the real symmetric matrices of order m. We define on $SM(m)$ the metric $g(P,Q) = \frac{1}{2}tr(PQ)$, for arbitrary P, Q in $SM(m)$. Then this space becomes the standard $\frac{1}{2}m(m+1)$-dimensional Euclidean space [2]. We regard x as a column matrix in E^m and denote by x^t the transpose of x. Let $\widetilde{x} = xx^t$. Then we obtain a smooth map $\widetilde{x} : M^n \to SM(m)$. Since the coordinates of $\widetilde{x}$ depend on the coordinates of x in a quadric manner, we call $\widetilde{x}$ the quadric representation of M^n [3]. It is well known that for the hypersphere centered at the origin which is embedded in the Euclidean space in the standard way, the quadric representation is just the second standard immersion of the sphere. Then a question arises naturally: To what extent does the quadric representation of a submanifold in E^m determine the submanifold? This question has attracted the interest of many mathematicians in this field and has been answered partly [1], [3], [4], [5]. In [3], I. Dimitric established some general results about the quadric representation, in particular those relative

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to the condition of $\widetilde{x}$ being of finite type. In [5], the author gave some results for hypersurfaces in E^m which satisfy $\Delta\widetilde{x} = B\widetilde{x} + C$ with B and C are two constant matrices. In this paper, we will study submanifolds in E^m with parallel normal mean curvature vectors which satisfy the condition $\Delta\widetilde{x} = B\widetilde{x} + C$. Especially we prove that a 2-dimensional surface with parallel mean curvature vector which satisfy $\Delta\widetilde{x} = B\widetilde{x} + C$ must be (a piece of) the 2-dimensional plane or (a piece of) the 2-dimensional sphere centered at the origin.

2. Preliminaries

Let us fix the notation first. Let $x : M^n \to E^m$ be an isometric immersion of an n-dimensional Riemannian manifold into the m-dimensional Euclidean space ($n < m$). We denote by H the mean curvature vector of M^n in E^m. Let $e_1, \dots, e_n, e_{n+1}, \dots, e_m$ be local orthonormal vector fields along M^n such that $e_1, \dots, e_n$ are tangent to M^n, $e_{n+1}, \dots, e_m$ are normal to M^n, and e_{n+1} is parallel to H. Then $H = \alpha e_{n+1}$, where α is the mean curvature of M^n in E^m. Let $\langle\ ,\rangle$ and $\overline{\nabla}$ be the Euclidean metric and the connection of E^m, and denote by ∇, h, D, A_r, $|A_r|$ respectively, the connection of M^n, the second fundamental form of M^n in E^m, the normal connection of M^n in E^m, the Weigarten endomorphism relative to the normal direction e_r, and the length of A_r, $r = n+1, \cdots, m$.

In this setting, the indices i, j, k always range from 1 to n, r, s, t from $n+1$ to m and β, γ from $n+2$ to m. At any point $x \in M^n$, for any column vector V in E^m, we denote by $V_T = \sum_i \langle V, e_i\rangle e_i$, $V_N = \sum_r \langle V, e_r\rangle e_r$, and $V_{\overline{N}} = \sum_\beta \langle V, e_\beta\rangle e_\beta$.

We define a map $*$ from $E^m \times E^m$ into $SM(m)$ by $V*W = VW^t + WV^t$, for column vectors V and W in E^m. Then $V*W = W*V$. Let $\widetilde{\nabla}$ denotes the Euclidean connection of $SM(m)$, then we have

$$\widetilde{\nabla}_V(W_1 * W_2) = (\overline{\nabla}_V W_1) * W_2 + W_1 * (\overline{\nabla}_V W_2), \tag{2.1}$$

$$\begin{aligned} & g(V_1 * V_2, W_1 * W_2) \\ = \ & \langle V_1, W_1\rangle\langle V_2, W_2\rangle + \langle V_1, W_2\rangle\langle V_2, W_1\rangle, \end{aligned} \tag{2.2}$$

and

(2.3) $$\Delta(V * W) = (\Delta V) * W + V * (\Delta W) - 2\sum_i (\overline{\nabla}_{e_i} V) * (\overline{\nabla}_{e_i} W),$$

where V, W, W_1, W_2, V_1 and V_2 are all vectors in E^m, and Δ is the Laplacian operator of M^n[3].

Using (2.3), by a direct computation, we have

(2.4) $$\Delta\widetilde{x} = -n\alpha e_{n+1} * x - \sum_i e_i * e_i,$$

and when $\alpha = 0$,

(2.5) $$\Delta^2\widetilde{x} = 2|A_{n+1}|^2 e_{n+1} * e_{n+1} - 2\sum_i (A_{n+1}e_i) * (A_{n+1}e_i).$$

Without noting, in this paper, we always denote by X, Y and Z the tangent vector of M^n, by ξ and η the normal vector of M^n in E^m, and V and W the column vector in E^m.

3. Submanifolds with parallel mean normal curvature vector

THEOREM 3.1. *Let $x : M^n \rightarrow E^m$ be an isometric immersion with parallel normal mean curvature vector. If its quadric representation satisfies $\Delta\widetilde{x} = B\widetilde{x} + C$, then $A_{n+1}x_T = \alpha x_T$.*

Proof. If $\alpha = 0$, then (2.4) becomes $\Delta\widetilde{x} = -\sum_i e_i * e_i$. Since $\Delta\widetilde{x} = B\widetilde{x} + C$, then $\Delta^2\widetilde{x} = B(\Delta\widetilde{x})$. Applying $g(\sim, e_r * e_r)$ to this relation and summing on r, we have $\sum_r |A_r|^2 = 0$. Then M^n is a totally geodesic submanifold of E^m, that is to say that M^n is (a piece of) the n-dimensional Euclidean space. Moreover, we can easily check that the n-dimensional Euclidean space does satisfy $\Delta\widetilde{x} = B\widetilde{x} + C$. In fact,

$$\Delta\widetilde{x} = -\begin{pmatrix} 2 & & & \\ & \ddots & & \\ & & 2 & \\ & & & 0 \end{pmatrix}$$

In this case, it is obvious that $A_{n+1}x_T = 0$. Then $A_{n+1}x_T = \alpha x_T$.

Now we suppose $\alpha \neq 0$. Since M^n has the parallel curvature vector in E^m, then $De_{n+1} = 0$. Differentiating $\Delta\widetilde{x} = B\widetilde{x} + C$ along X, an arbitrary tangent vector of M^n, we have

$$\begin{aligned} 0 = &(BX)x^t + (Bx)X^t - n\alpha(A_{n+1}X) * x \\ &+ n\alpha e_{n+1} * X + 2\sum_r (A_r X) * e_r + nX(\alpha)e_{n+1} * x. \end{aligned} \tag{3.1}$$

Now we find the $e_{n+1} * e_{n+1}$ component of (3.1), that is

$$(\langle BX, e_{n+1}\rangle + 2nX(\alpha))\langle x, e_{n+1}\rangle = 0. \tag{3.2}$$

Finding the $e_{n+1} * e_\alpha$ and $e_\alpha * e_\beta$ components of (3.1), respectively, we have

$$(\langle BX, e_{n+1}\rangle + 2nX(\alpha))\langle x, e_\alpha\rangle + \langle BX, e_\alpha\rangle\langle x, e_{n+1}\rangle = 0, \tag{3.3}$$

and

$$\langle BX, e_r\rangle\langle x, e_s\rangle + \langle BX, e_s\rangle\langle x, e_r\rangle = 0. \tag{3.4}$$

Also, we need to find the $e_{n+1} * Y$ and $Y * Z$ components of (3.1), these are

$$\begin{aligned} &(\langle BX, e_{n+1}\rangle + 2nX(\alpha))\langle x, Y\rangle \\ &+ (\langle BX, Y\rangle - 2n\alpha\langle A_{n+1}X, Y\rangle)\langle e_{n+1}, x\rangle \\ &+ (2n\alpha + \langle Bx, e_{n+1}\rangle)\langle X, Y\rangle + 4\langle A_{n+1}X, Y\rangle = 0, \end{aligned} \tag{3.5}$$

and

$$\begin{aligned} 0 = &\langle BX, Y\rangle\langle x, Z\rangle + \langle BX, Z\rangle\langle x, Y\rangle \\ &+ \langle Bx, Z\rangle\langle X, Y\rangle - 2n\alpha\langle A_{n+1}X, Y\rangle\langle x, Z\rangle \\ &+ \langle Bx, Y\rangle\langle X, Z\rangle - 2n\alpha\langle A_{n+1}X, Z\rangle\langle x, Y\rangle. \end{aligned} \tag{3.6}$$

In (3.5), let $X = Y = e_i$ and sum on i, we obtain

$$(3.7)\qquad \begin{aligned}&(\sum_i \langle Be_i, e_i\rangle - 2n^2\alpha^2)\langle e_{n+1}, x\rangle + 2nx_T(\alpha)\\ &+ 2n(n+2)\alpha + n\langle Bx, e_{n+1}\rangle + \langle Bx_T, e_{n+1}\rangle = 0.\end{aligned}$$

But in (3.6), let $Y = Z = e_i$ and sum on i, we have

$$(3.8)\qquad \langle BX, x_T\rangle + \langle Bx, X\rangle - 2n\alpha\langle A_{n+1}X, x_T\rangle = 0.$$

From (3.2), we know that at any point $x \in M^n$, $\langle e_{n+1,}x\rangle = 0$ or $\langle BX, e_{n+1}\rangle + 2nX(\alpha) = 0$ holds. But from (3.4) we know that $(BX)_{\overline{N}} = 0$ or $x_{\overline{N}} = 0$ holds. Thus, we discuss in the following three cases.

CASE 1. $\langle e_{n+1,}x\rangle = 0$ and $x_{\overline{N}} = 0$. Then $x = x_T$ and for any tangent vector Y of M^n,

$$0 = Y\langle e_{n+1}, x\rangle = -\langle A_{n+1}x, Y\rangle.$$

Thus $A_{n+1}x = 0$. In this case, (3.5) and (3.7)become

$$(3.9)\qquad \begin{aligned}0 &= (\langle BX, e_{n+1}\rangle + 2nX(\alpha))\langle x, Y\rangle\\ &+ (2n\alpha + \langle Bx, e_{n+1}\rangle)\langle X, Y\rangle + 4\langle A_{n+1}X, Y\rangle,\end{aligned}$$

and

$$(3.10)\qquad (n+1)\langle Bx, e_{n+1}\rangle + 2n(n+2)\alpha + 2nx(\alpha) = 0.$$

In (3.9), let $X = Y = x$, we have

$$(3.11)\qquad \langle Bx, e_{n+1}\rangle + n\alpha + nx(\alpha) = 0.$$

Combining (3.10) with (3.11), we obtain

$$(3.12)\qquad x(\alpha) = \frac{n+3}{n-1}\alpha.$$

Because of the arbitrariness of Y in (3.9), we have

$$0 = (\langle BX, e_{n+1}\rangle + 2nX(\alpha))x + (\langle Bx, e_{n+1}\rangle + 2n\alpha)X + 4A_{n+1}X. \tag{3.13}$$

But in (3.6), let $X = Y = x$, we obtain

$$\langle x, x\rangle\langle Bx, Z\rangle = -\langle Bx, x\rangle\langle x, Z\rangle.$$

Using the above relation and $A_{n+1}x = 0$, we have

$$\langle Bx, \widetilde{\nabla}_X e_{n+1}\rangle = -\langle Bx, A_{n+1}X\rangle = 0. \tag{3.14}$$

Then using (3.13) and (3.14), we can obtain

$$X\{(\langle Bx, e_{n+1}\rangle + 2n\alpha)x - 4e_{n+1}\} = 0,$$

that is, $(\langle Bx, e_{n+1}\rangle + 2n\alpha)x - 4e_{n+1}$ is a constant vector. Then

$$(\langle Bx, e_{n+1}\rangle + 2n\alpha)^2\langle x, x\rangle = C_0, \tag{3.15}$$

where C_0 is a constant. But from (3.10) and (3.12), we know

$$\langle Bx, e_{n+1}\rangle + 2n\alpha = -\frac{4n}{n-1}\alpha.$$

Thus, (3.15) becomes $\alpha^2\langle x, x\rangle = C_1$, where C_1 is also a constant. Differentiating this formula along the tangent vector field x, we have that $\alpha\langle x, x\rangle(x(\alpha) + \alpha) = 0$. But $\langle x, x\rangle \neq 0$ and $\alpha \neq 0$, so $x(\alpha) + \alpha = 0$. This is a contradiction with (3.12).

CASE 2. $\langle x, e_{n+1}\rangle = 0$ but $x_{\overline{N}} \neq 0$. In this case, $(BX)_{\overline{N}} = 0$ and from (3.3) we know that $\langle BX, e_{n+1}\rangle + 2nX(\alpha) = 0$. Then (3.5) and (3.7) become

$$(\langle Bx, e_{n+1}\rangle + 2n\alpha)\langle X, Y\rangle + 4\langle A_{n+1}X, Y\rangle = 0, \tag{3.16}$$

and

$$\langle Bx, e_{n+1}\rangle = -2(n+2)\alpha. \tag{3.17}$$

Combining (3.16) with (3.17), we know that $A_{n+1}X = \alpha X$. Obviously, we have $A_{n+1}x_T = \alpha x_T$.

CASE 3. $\langle e_{n+1}, x\rangle \neq 0$. In this case $\langle BX, e_{n+1}\rangle + 2nX(\alpha) = 0$ and from (3.3) we know that $(BX)_{\overline{N}} = 0$. Obviously $x_N \neq 0$. Then (3.5) and (3.7) become

$$\begin{aligned}&-2n\alpha\langle A_{n+1}X, Y\rangle\langle e_{n+1}, x\rangle\\&+(2n\alpha + \langle Bx, e_{n+1}\rangle)\langle X, Y\rangle\\&+4\langle A_{n+1}X, Y\rangle + \langle BX, Y\rangle\langle x, e_{n+1}\rangle = 0,\end{aligned} \tag{3.18}$$

and

$$\begin{aligned}&n\langle Bx, e_{n+1}\rangle + 2n^2\alpha + 4n\alpha\\&+(\sum_i\langle Be_i, e_i\rangle - 2n^2\alpha^2)\langle e_{n+1}, x\rangle = 0.\end{aligned} \tag{3.19}$$

From (3.18), we know that

$$\langle BX, Y\rangle = \langle X, BY\rangle. \tag{3.20}$$

Substituting $Y = x_T$ in (3.18), we have

$$\begin{aligned}0 = &-2n\alpha\langle A_{n+1}X, x\rangle\langle e_{n+1}, x\rangle\\&+ (2n\alpha + \langle Bx, e_{n+1}\rangle)\langle X, x\rangle\\&+ 4\langle A_{n+1}X, x\rangle + \langle BX, x_T\rangle\langle x, e_{n+1}\rangle.\end{aligned} \tag{3.21}$$

Combining (3.8) with (3.21), we obtain that

$$\begin{aligned}&(\langle Bx, e_{n+1}\rangle + 2n\alpha)\langle x, X\rangle\\&- \langle x, e_{n+1}\rangle\langle Bx, X\rangle + 4\langle A_{n+1}X, x\rangle = 0,\end{aligned}$$

that is

$$(\langle Bx, e_{n+1}\rangle + 2n\alpha)x_T - \langle x, e_{n+1}\rangle(Bx)_T + 4A_{n+1}x_T = 0. \tag{3.22}$$

In (3.6), let $X = Y = e_i$ and sum on i, we have

$$(\sum_i \langle Be_i, e_i\rangle - 2n^2\alpha^2)\langle x, Z\rangle + \langle Bx_T, Z\rangle + (n+1)\langle Bx, Z\rangle - 2n\alpha\langle A_{n+1}x_T, Z\rangle = 0.$$

that is

$$(\sum_i \langle Be_i, e_i\rangle - 2n^2\alpha^2)x_T + (Bx_T)_T + (n+1)(Bx)_T - 2n\alpha A_{n+1}x_T = 0. \tag{3.23}$$

But from (3.20) and (3.8), we have that

$$\langle BX, x_T\rangle + \langle Bx, X\rangle - 2n\alpha\langle A_{n+1}X, x_T\rangle = 0,$$

that is

$$(Bx_T)_T + (Bx)_T - 2n\alpha A_{n+1}x_T = 0. \tag{3.24}$$

Combining (3.22) with (3.23) and (3.24), we know that

$$4nA_{n+1}x_T + \{n\langle Bx, e_{n+1}\rangle + 2n^2\alpha + (\sum_i \langle Be_i, e_i\rangle - 2n^2\alpha^2)\langle e_{n+1}, x\rangle\}x_T = 0.$$

Using (3.19), the above relation becomes

$$A_{n+1}x_T = \alpha x_T.$$

Then the proof is complete. □

4. Surfaces with parallel mean curvature vector

In this section, we will give a complete classification result on surfaces with parallel mean curvature vector. To obtain the main result, we give some lemmas first.

LEMMA 4.1. *Let $x : M^2 \to E^m$ be a full surface. Then there does not exist a constant vector which is always normal to the tangent space $T_x(M^2)$ for any $x \in M^2$.*

We note that a surface M^2 of E^m is called full, if M^2 can not be contained in any lower dimensional linear subspace of E^m.

Proof. Suppose that there exists a constant vector X_0 which is normal to $T_x(M^2)$, for any $x \in M^2$. Then at any point x, differentiating $\langle X_0, x\rangle$ along any tangent vector X, we have $X\langle X_0, x\rangle = 0$. This means $\langle X_0, x\rangle$ is constant, and M^2 is contained in the linear subspace

$$\Lambda = \{W \in E^m; \langle W, X_0\rangle = 0\}.$$

This is a contradiction with that M^2 is full in E^m. □

LEMMA 4.2. *Let $x : M^2 \to E^m$ be a full surface, and B a constant matrix of order m. If $BX = \lambda(x)X$, for any point $x \in M^2$ and $X \in T_x(M^2)$, then $B = \lambda I_m$, where λ is a constant and I_m is the identity matrix of order m.*

Proof. Let x_1 be any point in M^2. From Lemma 4.1, we know that it is impossible that $T_x(M^2) = T_{x_1}(M^2)$ always holds for all points $x \in M^2$. On the other hand, if $T_x(M^2) \cap T_{x_1}(M^2) = \{0\}$ always holds for any other point $x(\neq x_1) \in M^2$, we choose a vector X_0 in $T_{x_1}(M^2)$. Then X_0 is normal to $T_x(M^2)$, for any other point x. This is a contradiction. Thus there exists a point x_2 in M^2, such that $T_{x_1}(M^2) \cap T_{x_2}(M^2) = \{X_1\}$, where X_1 is a non-zero vector in E^m. Since $BX_1 = \lambda(x_1)X_1 = \lambda(x_2)X_1$, we know that $\lambda(x_1) = \lambda(x_2)$. By induction, we can obtain $x_1, x_2, \dots, x_{m-1}$ in M^2, such that E^m can be spanned linearly by vectors in $\cup_{i=1}^{m-1} T_{x_i}(M^2)$ and $\lambda(x_1) = \cdots = \lambda(x_{m-1})$. Obviously, $BV = \lambda V$, for any $V \in E^m$, This means $B = \lambda I_m$. □

LEMMA 4.3. *Let $x : M^2 \to S^{m-1}(1) \subset E^m$ is an isometric immersion with parallel mean curvature vector. If its quadric representation satisfies $\Delta\widetilde{x} = B\widetilde{x} + C$, then M^2 is (a piece of) $S^2(1)$.*

Proof. Let $e_1, e_2, e_3, \dots, e_{m-1}$ be a local field of orthonormal frames of $S^{m-1}(1)$, such that restricted to M^2, e_1, e_2 are tangent to M^2 and e_3 is parallel to the mean curvature vector H' of M^2 in $S^{m-1}(1)$. Let $H' = \alpha' e_3$. we denote by A'_t and D' the Weigartan endomorphism relative to e_t, $t = 3, \dots, m-1$, and the normal connection of M^2 in $S^{m-1}(1)$. Then $2H = 2H' + x$ and (2.4) becomes

$$\Delta\widetilde{x} = -2\alpha' e_3 * x - 2x * x - \sum_i e_i * e_i.$$

Differentiating $\Delta\widetilde{x} = B\widetilde{x} + C$ along any tangent vector X and using the above relation, we obtain

$$0 = (BX)x^t + (Bx)X^t - 2\alpha(A_3'X) * x + 2\alpha' e_3 * X + 2\sum_{t=3}^{m-1}(A_t'X) * e_t + 6X * x.$$

Finding the $x * x$, $x * Y$, $e_3 * Y$ and $x * e_t$ components of the above relation respectively, we have

$$\langle BX, x\rangle = 0, \tag{4.1}$$

$$\langle BX, Y\rangle = (\langle Bx, x\rangle + 12)\langle X, Y\rangle - 6\alpha'\langle A_3'X, Y\rangle, \tag{4.2}$$

$$\langle Bx, e_3\rangle\langle X, Y\rangle + 4\alpha'\langle X, Y\rangle + 4\langle A_3X, Y\rangle = 0, \tag{4.3}$$

and

$$\langle BX, e_t\rangle = 0, \qquad t = 3, \cdots m-1. \tag{4.4}$$

From the above relations, we know that $BX = \lambda(x)X$. Then by Lemma 4.2, we have $B = \lambda I_m$. That is $\Delta\widetilde{x} = \lambda\widetilde{x} + C$ and we know that M^2 must be (a piece of) a sphere centered at the origin. The proof is complete. □

THEOREM 4.1. *Let $x : M^2 \to E^m$ be a surface with parallel mean curvature vector. If its quadric representation satisfies the condition $\Delta\widetilde{x} = B\widetilde{x} + C$, then M^2 is (a piece of) a plane or a sphere centered at the origin in E^3.*

Proof. From Theorem 3.1, we know that $A_3x_T = \alpha x_T$ holds.

If $\alpha = 0$, then M^2 is (a piece of) a plane.

If $\alpha \neq 0$ but $x_T = 0$, we know that M^2 is contained in the hypersphere $S^{m-1}(r)$. Since M^2 has parallel mean curvature vector, M^2 also has parallel mean curvature vector in $S^{m-1}(r)$. Then from Lemma 4.3, we know that M^2 is (a piece of) a sphere centered at the origin.

If $\alpha \neq 0$ and $x_T \neq 0$, then $A_3 = \alpha I_2$ and all formulas in Case 3 of Theorem 3.1 hold. Combining (3.18) with $A_3 = \alpha I_2$, we obtain

$$\langle BX, Y\rangle = \lambda(x)\langle X, Y\rangle,$$

where $\lambda(x) = \frac{-1}{\langle e_3, x\rangle}\{\langle Bx, e_3\rangle - 4\alpha^2\langle e_3, x\rangle + 8\alpha\}$. Moreover, we know that $(BX)_{\overline{N}} = 0$ and $\langle BX, e_3\rangle = 0$. Then $BX = \lambda(x)X$ holds for any $x \in M^2$ and $X \in T_x(M^2)$. By Lemma 4.3, we know that $B = \lambda I_m$, where λ is a constant. On the other hand,

$$\begin{aligned} \lambda X\langle x, x\rangle &= X\langle Bx, x\rangle \\ &= \langle BX, x\rangle + \langle Bx, X\rangle \\ &= 4\alpha^2\langle X, X\rangle = 2\alpha^2 X\langle x, x\rangle, \end{aligned}$$

here we used (3.24) and $A_{n+1}x_T = \alpha x_T$. Then we have $(\lambda - 2\alpha^2)\langle X, x\rangle = 0$. Since $x_T \neq 0$, we know that $\lambda = 2\alpha^2$. Substituting $B = 2\alpha^2 I_m$ and $X = x_T$ in (3.18), we obtain $\alpha = 0$. But we assume that $\alpha \neq 0$. This is a contradiction. The proof is complete. □

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