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APPROXIMATE FIBRATIONS ON PL MANIFOLDS

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ABSTRACT. If N is any cartesian product of a closed simply connected n-manifold N_1 and a closed aspherical m-manifold N_2, then N is a codimension 2 fibrator. Moreover, if N is any closed hopfian PL n-manifold with $\pi_i N = 0$ for $2 \leq i < m$, which is a codimension 2 fibrator, and $\pi_1 N$ is normally cohopfian and has no proper normal subgroup isomorphic to $\pi_1 N/A$ where A is an abelian normal subgroup of $\pi_1 N$, then N is a codimension m PL fibrator.

1. Introduction

In studying proper maps between manifolds, approximate fibrations, introduced and studied by Coram and Duvall[1], form an important class of mappings, nearly as effective as Hurewicz fibrations.

A proper map $p : M \to B$ between locally compact ANRs is called an approximate fibration if it has the following homotopy property: Given an open cover ϵ of B, an arbitrary space X and two maps $g : X \to M$ and $F : X \times I \to B$ such that $p \circ g = F_0$, there exists a map $G : X \times I \to M$ such that $G_0 = g$ and $p \circ G$ is ϵ-close to F.

When $p : M \to B$ is an approximate fibration, there is a homotopy exact sequence developed by Coram and Duvall[1];

$$\cdots \to \pi_{i+1}(B) \to \pi_i(p^{-1}b) \to \pi_i(M) \to \pi_i(B) \to \cdots$$

just like the one for Hurewicz fibrations, relating homotopy data of the total space, base space, and typical fiber.

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Throughout this paper, we fix once and for all the setting and notation to be used throughout this paper: M is an orientable $(n+k)$-manifold, B is a k-dimensional ANR, and $p : M \to B$ is a proper map such that each point preimage $p^{-1}b$ has the homotopy type of a closed, connected, orientable manifold.

Daverman[4] introduced the following definitions. When N is a closed, orientable manifold, such a proper map $p : M \to B$ is called N-like if each $p^{-1}b$ is homotopy equivalent to N. We call N a codimension k fibrator if, for all orientable $(n+k)$-manifolds M and N-like proper maps $p : M \to B$, p is an approximate fibration. If N has this property for all $k > 0$, call N simply a fibrator.

Then we have the following natural question.

MAIN QUESTION. *Which manifolds are codimension k fibrators?*

Most closed manifolds are known to be codimension 1 fibrators[3]. For codimension 2 fibrators, we have fairly rich data[4, 5, 6, 10, 12, 13]. In particular, every closed surface except torus is a codimension 2 fibrator[4]. Also manifolds that satisfy a certain hopfian property are codimension 2 fibrators if they have either non-zero Euler characteristic or hyperhopfian fundamental groups[6].

In section 3, we restrict objects to the PL category. Restriction to the PL category offers some advantages. The target spaces are standard geometric objects, obviously finite dimensional and locally contractible, features which a priori dispel potentially troublesome issues lurking in the background of the general (non-PL) setting[4]. The chief benefit is not the simplicial structure of the image, however, but rather the potential for inductive arguments, as in the classical PL topology, which apply to the restriction of p over certain links in the target and bring about the lowering of fiber codimension without changing fiber character.

Also, surprisingly many manifolds are known to be codimension k PL fibrators[8]. If N is a closed, aspherical manifold which is a codimension 2 fibrator, then N is a codimension 3 PL fibrator. Moreover, if N^n is a closed aspherical manifold with certain fundamental group, then N^n is a PL fibrator.

So far, the matter of closure with respect to cartesian products of codimension k fibrators is still open. On this front, Im[13] has shown that all cartesian products of surfaces of non-negative Euler characteristic are codimension 2 fibrators. For this direction, we give some results about the cartesian products of manifolds.

In this paper, we show that if N is any cartesian product of a closed simply connected n-manifold N_1 and a closed aspherical m-manifold N_2, then N is a codimension 2 fibrator. Moreover, if N is any closed hopfian PL n-manifold with $\pi_i N = 0$ for $2 \leq i < m$, which is a codimension 2 fibrator, and $\pi_1 N$ is normally cohopfian and has no proper normal subgroup isomorphic to $\pi_1 N/A$ where A is an abelian normal subgroup of $\pi_1 N$, then N is a codimension m PL fibrator. A s a result, a product of a closed aspherical manifold with hyperhopfian fundamental group and a sphere S^m is a codimension m PL fibrator.

A group G is hopfian if every epimorphism $\Theta : G \to G$ is necessarily an isomorphism, while a finitely presented group G is hyperhopfian if every homomorphism $\Psi : G \to G$ with $\Psi(G)$ normal and $G/\Psi(G)$ cyclic is an automorphism. A group G is normally cohopfian if every monomorphism $\Phi : G \to G$ with normal image is an automorphism.

A closed manifold N is hopfian if it is orientable and every degree one map $N \to N$ is a homotopy equivalence. This term plays an important role in determining approximate fibrations. Swarup[15] has shown this hopfian feature for closed orientable n-manifolds N with $\pi_i(N) = 0$ for $1 < i < n-1$, and Hausmann has done the same for all closed orientable 4-manifolds[11].

The (absolute) degree of a map $f : N \to N$, where N is a closed, connected, orientable n-manifold, is the non-negative integer such that the induced endomorphism of $H_n(N : Z) \cong Z$ amounts to multiplication by d, up to sign.

Homology is computed with integer coefficients unless the coefficient module is mentioned.

A PL map $p : M \to B$ has Property $R \cong$ ($R_* \cong$) if, for each $b \in B$, a retraction $R : U \to p^{-1}b$ defined on some open set $U \supset p^{-1}b$ induces $\pi_1(H_1)$-isomorphisms $(R|)_\# : \pi_1(p^{-1}b') \to \pi_1(p^{-1}b)((R|)_* : H_1(p^{-1}b') \to H_1(p^{-1}b))$ for all $b' \in B$ sufficiently close to b.

2. Codimension 2 fibrators

In this section, we show that if N is any cartesian product of a closed simply connected n-manifold N_1 and a closed aspherical m-manifold N_2, then N is a codimension 2 fibrator.

LEMMA 2.1 [6]. *All closed, hopfian manifolds with hyperhopfian fundamental group is a codimension 2 fibrator.*

LEMMA 2.2. *Let N_1^n be a closed, simply connected manifold and N_2^m be a closed manifold with $\pi_i(N_2) = 0$ for $1 < i \leq n$. If $h : N_1 \times N_2 \to N_1 \times N_2$ is a degree one map, then so is $h_1 = pr \circ h \circ i : N_1 \to N_1$, where $pr : N_1 \times N_2 \to N_1$ is the projection and $i : N_1 \to N_1 \times N_2$ is the inclusion map.*

Proof. Assume that $h : N_1 \times N_2 \to N_1 \times N_2$ is a degree one map. By taking a universal covering space $(\tilde{N}_2, \theta)$ of N_2, $(N_1 \times \tilde{N}_2, id \times \theta)$ is a covering space of $N_1 \times N_2$. Let $\tilde{i} : N_1 \to N_1 \times \tilde{N}_2$ be a continuous map for which $(id \times \theta) \circ \tilde{i} = i$ and $\tilde{h} :\to N_1 \times \tilde{N}_2 \to N_1 \times \tilde{N}_2$ be a continuous map such that $h \circ (id \times \theta) = (id \times \theta) \circ \tilde{h}$ by the lifting property. Consider the following commutative diagram

$$\begin{array}{ccccccc} N_1 & \xrightarrow{\tilde{i}} & N_1 \times \tilde{N}_2 & \xrightarrow{\tilde{h}} & N_1 \times \tilde{N}_2 & \xrightarrow{q} & N_1 \\ \downarrow{\scriptstyle id} & & \downarrow{\scriptstyle id\times\theta} & & \downarrow{\scriptstyle id\times\theta} & & \downarrow{\scriptstyle id} \\ N_1 & \xrightarrow{i} & N_1 \times N_2 & \xrightarrow{h} & N_1 \times N_2 & \xrightarrow{pr} & N_1 \end{array}$$

where q is the projection from $N_1 \times \tilde{N}_2$ onto N_1. Because of $\pi_i(N_2) = 0$ for $1 < i \leq n$, we obtain that $\pi_i(\tilde{N}_2) = 0$ for $1 \leq i \leq n$, and then $H_i(\tilde{N}_2) = 0$ for $1 \leq i \leq n$. According to the Künneth Theorem, we have $H_n(N_1 \times \tilde{N}_2) \cong H_n(N_1)$ and $H_n(N_1 \times N_2)$ is isomorphic to the direct sum of $\bigoplus_{i=0}^{n} H_{n-i}(N_1) \otimes H_i(N_2)$ and $\bigoplus_{i=0}^{n} H_{n-i-1}(N_1) * H_i(N_2)$. Thus, by the diagram chasing, it is easily checked that $h_*(H_n(N_1)) \subset H_n(N_1)$ when we restrict h_* to $H_n(N_1)$ and equate $H_n(N_1)$ with $H_n(N_1) \otimes H_0(N_2) \subset H_n(N_1 \times N_2)$. Rewrite $H_n(N_1 \times N_2)$ in a form {torsion-free}$\oplus$ {torsion}. Note that $H_n(N_1 \times N_2)$ is finitely generated. Since h_* is an isomorphism, the restriction h_* to {torsion-free} of $H_n(N_1 \times N_2)$ is an isomorphism and induces an invertible $(k \times k)$-matrix of the following form

$$A = \begin{pmatrix} A_{11} & A_{12} & \dots & A_{1k} \\ A_{21} & A_{22} & \dots & A_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ A_{k1} & A_{k2} & \dots & A_{kk} \end{pmatrix}.$$

Here A_{11} is the matrix corresponding to map $h_*| : H_n(N_1) \to H_n(N_1)$ and A_{ij} is the matrix induced by the homomorphism from the i-th direct summand to the j-th direct summand of torsion-free part of $H_n(N_1 \times N_2)$. Because of $h_*(H_n(N_1)) \subset H_n(N_1)$, the restriction $h_*|\{\text{torsion-free}\}$ of h_* doesn't send the first factor $H_n(N_1)$ to any direct summand except itself and thus $A_{1j} = 0$ for $j = 2, \dots, k$. Since the isomorphism $h_*|\{\text{torsion-free}\}$ induces $detA = \pm 1$, we obtain $detA_{11} = \pm 1$. This implies $h_1 = pr \circ h \circ i$ is a degree one map. □

COROLLARY 2.3. *Let N_1^n be a closed, simply connected manifold and N_2^m be a closed, aspherical manifold . If $h : N_1 \times N_2 \to N_1 \times N_2$ is a degree one map, then so is $h_1 = pr \circ h \circ i : N_1 \to N_1$, where $pr : N_1 \times N_2 \to N_1$ is the projection and $i : N_1 \to N_1 \times N_2$ is the inclusion map.*

Now, we state the main result in this section.

THEOREM 2.4. *Let N_1^n be a closed, simply connected manifold and N_2^m be a closed aspherical manifold with hyperhopfian fundamental group. Then $N_1 \times N_2$ is a codimension 2 fibrator.*

Proof. Since the fundamental group of $N_1 \times N_2$ is isomorphic to the hyperhopfian group $\pi_1(N_2)$, it is sufficient to show that $N_1 \times N_2$ is a hopfian manifold by Lemma 2.1. Assume that $h : N_1 \times N_2 \to N_1 \times N_2$ is a degree one map. Because of the hyperhopfian property of $\pi_1(N_1 \times N_2)$, $h_\# : \pi_1(N_1 \times N_2) \to \pi_1(N_1 \times N_2)$ is an isomorphism.

To show that h is a homotopy equivalence, we consider homomorphisms

$$h_\# : \pi_i(N_1 \times N_2) \to \pi_i(N_1 \times N_2) \text{ for } i \geq 2.$$

Applying Corollary 2.3, the degree of $h_1 = pr \circ h \circ i : N_1 \to N_1$ is one, and then $(h_1)_* : H_i(N_1) \to H_i(N_1)$ is an isomorphism for each $i \geq 1$.

Since N_1 is simply connected, we obtain that $(h_1)_\# : \pi_i(N_1) \to \pi_i(N_1)$ is an isomorphism for each $i \geq 1$ by the Whitehead theorem. Implying the fact that $\pi_i(N_1 \times N_2) \cong \pi_i(N_1) \times \pi_i(N_2) \cong \pi_i(N_1)$ for each $i \geq 2$, $h_\# : \pi_i(N_1 \times N_2) \to \pi_i(N_1 \times N_2)$ is an isomorphism for each $i \geq 2$. Therefore, we conclude that h is a homotopy equivalence, and then $N_1 \times N_2$ is a codimension 2 fibrator. □

COROLLARY 2.5. *Let N_1^n be a closed, simply connected manifold and N_2^m be a finite product of closed surfaces with non-zero Euler characteristic. Then $N_1 \times N_2$ is a codimension 2 fibrator.*

Proof. Rewrite $N_1 \times N_2$ as $N_1' \times N_2'$, where N_1' is simply connected and N_2' is a product of closed surfaces with negative Euler characteristic. Since $\pi_1(N_2')$ is hyperhopfian[6], N_2' is a closed aspherical manifold with hyperhopfian fundamental group. □

THEOREM 2.6. *Let N_1^n be a closed, simply connected manifold, N_2^m be a closed, aspherical manifold with hopfian fundamental group, and $\chi(N_1 \times N_2) \neq 0$. Then $N_1 \times N_2$ is a codimension 2 fibrator.*

Proof. Since the fundamental group of $N_1 \times N_2$ is isomorphic to the hopfian group $\pi_1(N_2)$, it suffices to show that $N_1 \times N_2$ is a hopfian manifold by [6, Theorem 5.10]. According to Corollary 2.3 and the same method of the proof in Theorem 2.4, it is easy to show that $N_1 \times N_2$ is a hopfian manifold. □

REMARK. Note that every codimension 2 fibrator N must be a codimension 2 PL fibrator.

3. Codimension k PL fibrators

Throughout this section, for a PL map $p : M \to B$, v will denote a vertex of a polyhedron B, $L = \text{link}(v, B), S = \text{star}(v, B) = v * L$, $L' = p^{-1}L$ and $S' = p^{-1}S$. These are understood to arise in the first barycentric subdivision of triangulations on which p is simplicial.

LEMMA 3.1 [7]. *If X is a CW-complex such that $\pi_i(X) = 0$ for $1 < i \leq k$ and if the map $f : X \to X$ induces an isomorphism $\pi_1(X) \to$*

$\pi_1(X)$, then f also induces isomorphisms

$$f_* : H_i(X) \to H_i(X) \text{ and } f^* : H^i(X) \to H^i(X)\ (i \leq k).$$

LEMMA 3.2 [8]. *Let N be a closed n-manifold which is a codimension $k-1$ PL fibrator, $k > 2$, and whose fundamental group is normally cohopfian and contains no Abelian normal subgroup, and let $p : M \to B$ be an N-like PL map defined on a PL $(n+k)$-manifold. Then p has Property $R \cong$.*

LEMMA 3.3 [8]. *Suppose N^n is a closed hopfian manifold and $p : M^{n+k} \to B$ is an N-like PL map such that $H^n[p]$ is locally constant. Then p is an approximate fibration if and only if p has Property $R \cong$.*

PROPOSITION 3.4. *Suppose N^n is a hopfian manifold with hopfian fundamental group and m, k are integers, $1 < m \leq k$, such that $\pi_i(N) = 0$ for $1 < i \leq m$ and $H_i(N) = 0$ for $m < i \leq k$, and suppose $p : M^{n+k} \to B$ is an N-like PL map. Then p is an approximate fibration if and only if p has Property $R \cong$.*

Proof. Due to Lemma 3.3, it suffices to show that $H^n[p]$ is locally constant. For each $v \in B$, there is a collapse $R : S' \to p^{-1}v$. Since p has Property $R \cong$, $R|p^{-1}(c) : p^{-1}(c) \to p^{-1}(v)$, $c \in L$, induces an isomorphism at the fundamental group level and it also does so for i-th cohomology groups, $0 \leq i \leq k$, by Lemma 3.1. Hence, the i-th cohomology sheaf $H^i[p]$ is locally constant in the same range. According to [9, Theorem 3.6], $H^n[p]$ is locally constant. □

THEOREM 3.5. *Let N^n be a closed hopfian manifold which is a codimension 2 PL fibrator. Suppose $\pi_i N = 0$ for $2 \leq i \leq m-1 (m \geq 3)$, $\pi_1 N$ is normally cohopfian and has no proper normal subgroup isomorphic to $\pi_1 N/A$, where A itself is an Abelian normal subgroup of $\pi_1 N$. Then N is a codimension m PL fibrator.*

Proof. It is easily checked that N is a codimension $(m-1)$ PL fibrator by Lemma 3.2 and Proposition 3.4. Let $p : M^{n+m} \to B^m$ be any PL N-like map. Lemma 3.2 implies that p has Property $R \cong$. Since N is a codimension $(m-1)$ PL fibrator, $p|L' : L' \to L$ is an approximate fibration. From the complete movability criterion [2], it suffices to show that $R : p^{-1}c \to p^{-1}v$ is a homotopy equivalence for any $c \in L$. Since N is a hopfian manifold, it is enough to show that R is a degree one map.

Applying the fact that $\pi_i(N) = 0$ for $1 < i \leq m-1$ and Lemma 3.1, R induces isomorphisms

$$R_* : H_i(N) \to H_i(N) \text{ and } R^* : H^i(N) \to H^i(N) \text{ for } 1 < i \leq m-1.$$

Then B^m is an m-dimensional manifold by [8, Corollary 3.6]. This implies that L^{m-1} is a homotopy $(m-1)$-sphere.

First, the homology sequence of (S', L') shows

$$\cdots H_m(S', L') \to H_{m-1}(L') \to H_{m-1}(S') \to H_{m-1}(S', L') \cdots,$$

where the first term is $H_m(S', L') \cong H^n(N) \cong Z$ and the last term is $H_{m-1}(S', L') \cong H^{n+1}(N) \cong 0$ by the Alexander duality.

Consider the following diagram

$$(*) \quad \begin{array}{ccccccc} H_m(S', L') & \longrightarrow & H_{m-1}(L') & \longrightarrow & H_{m-1}(S') & \longrightarrow & 0 \\ \downarrow{\scriptstyle p_*} & & \downarrow{\scriptstyle p'_*} & & & & \\ H_m(S, L) & \xrightarrow{\cong} & H_{m-1}(L) & \longrightarrow & H_{m-1}(S) \cong 0 & & \end{array}$$

Now, we show that $p'_* : H_{m-1}(L') \to H_{m-1}(L)(\cong Z)$ is an epimorphism. Since $p|L' : L' \to L$ is an approximate fibration, we have the homotopy exact sequence;

$$\cdots \to \pi_{m-1}(N) \to \pi_{m-1}(L') \to \pi_{m-1}(L) \to \pi_{m-2}(N) \to \cdots$$

If $m \geq 4$, then $\pi_i(N) = 0$ for $i = m-2$ and $m-1$. Thus, $p_\# : \pi_{m-1}(L') \to \pi_{m-1}(L)$ is an isomorphism.

If $m = 3$, we have the following homotopy exact sequence

$$\cdots \to \pi_2(N) \to \pi_2(L') \to \pi_2(L) \to \pi_1(N) \to \cdots$$

By hypothesis, $\pi_2(N)$ is zero. Then $p_\# : \pi_2(L') \to \pi_2(L)$ is an isomorphism by using the fact that $\pi_1(N)$ has no proper normal subgroup isomorphic to $\pi_1(N)/A$, where A itself is an Abelian normal subgroup of $\pi_1(N)$. As a result, $p_{\#} : \pi_{m-1}(L') \to \pi_{m-1}(L)$ is an isomorphism for $m \geq 3$.

On the other hand, we have the natural following diagram

$$\begin{array}{ccc} \pi_{m-1}(L') & \xrightarrow{\cong} & \pi_{m-1}(L) \\ \downarrow & & \downarrow \cong \\ H_{m-1}(L') & \xrightarrow{p'_*} & H_{m-1}(L) \end{array}$$

where the vertical homomorphism $\pi_{m-1}(L) \to H_{m-1}(L)$ is an isomorphism because L is a homotopy $(m-1)$-sphere. This shows that $p'_* : H_{m-1}(L') \to H_{m-1}(L)$ is an epimorphism.

Next, we apply the Wang sequences for approximate fibration $p|L' : L' \to L$[8];

$$\cdots \to H_{m-1}(N) \to H_{m-1}(L') \to H_0(N) \to H_{m-2}(N) \to H_{m-2}(L').$$

Here the last term is $H_{m-2}(L') \cong H_{m-2}(S')$ due to the homology exact sequence of (S', L') and the last homomorphism is an isomorphism since S' collapses to $p^{-1}v$ and $R| : p^{-1}c \to p^{-1}v$ induces an isomorphism of $H_{m-2}(p^{-1}c)$ to $H_{m-2}(p^{-1}v)$ by Lemma 3.1. Therefore, $H_{m-1}(L') \cong \mathrm{Im}H_{m-1}(N) \oplus Z$ and we easily check that in (*), $p_* : H_m(S', L') \to H_m(S, L)$ is an isomorphism by the diagram chasing, and then we see that $p_* : H_m(S', S' - p^{-1}v) \to H_m(S, S - v)$ is an isomorphism. Similarly, we obtain an isomorphism $p_* : H_m(S', S' - p^{-1}c) \to H_m(S, S - c)$ for any $c \in S$ sufficiently close to v.

Then the following commutative diagram holds, where U is a connected open neighborhood of v in$S$ having compact closure and $c \in U$.

$$\begin{array}{ccc} H^{n+m}(p^{-1}v) \cong H_m(S', S' - p^{-1}v) & \xrightarrow{\cong} & H_m(S, S - v) \cong H^0(v) \\ \uparrow & & \uparrow \cong \\ H_m(S', S' - cl(p^{-1}U)) & \xrightarrow{\cong} & H_m(S, S - cl(U)) \cong H^0(clU) \\ \downarrow & & \downarrow \cong \\ H^{n+m}(p^{-1}c) \cong H_m(S', S' - p^{-1}c) & \xrightarrow{\cong} & H_m(S, S - c) \cong H^0(c) \end{array}$$

This implies that R^* on the cohomology is constantly 1 near v, and implies the same on the homology by the universal coefficient theorem. As a consequence, we conclude that N is a codimension m PL fibrator.□

COROLLARY 3.6. *Let N^n be a closed aspherical manifold with hyperhopfian, normally cohopfian group and $\pi_1(N)$ has no proper normal subgroup isomorphic to $\pi_1(N)/A$, where A itself is an Abelian normal subgroup of $\pi_1(N)$. Then N^n is a PL fibrator.*

Proof. Since N is a closed, aspherical manifold with hyperhopfian fundamental group, it is easy to check that N is a hopfian manifold. According to Lemma 2.1, N is a codimension 2 PL fibrator. Thus, Theorem 3.5 implies that N is a PL fibrator . □

COROLLARY 3.7. *Let N_1^n be a closed aspherical manifold whose fundamental group is hyperhopfian, normally cohopfian and has no proper normal subgroup isomorphic to $\pi_1(N)/A$, where A itself is an Abelian normal subgroup of $\pi_1(N)$, and let N_2^k be a closed manifold with $\pi_i(N_2) = 0$, for $1 \leq i \leq m-1$ and some integer m. Then $N_1 \times N_2$ is a codimension m PL fiberator.*

Proof. By Corollary 2.3 and the method of the proof in Theorem 2.4, $N_1 \times N_2$ is a hopfian manifold. Thus, Theorem 3.5 implies that $N_1 \times N_2$ is a codimension m PL fibrator. □

COROLLARY 3.8. *Let N_1^n be a closed aspherical manifold whose fundamental group is hyperhopfian, normally cohopfian and has no proper normal subgroup isomorphic to $\pi_1(N)/A$, where A itself is an Abelian normal subgroup of $\pi_1(N)$. Then $N_1^n \times S^m$ is a codimension m PL fiberator, where S^m is a sphere.*

COROLLARY 3.9. *Let N_1^n be a finite product of closed orientable surfaces with negative Euler characteristic. Then $N_1^n \times S^m$ is a codimension m PL fibrator.*

Proof. The fundamental group of N_1 is hyperhopfian[6] and a normally cohopfian group with no nontrivial Abelian normal subgroup[8]. Therefore, the conclusion follows from Theorem 3.5. □

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