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# FUZZY IDEALS IN NEAR-RINGS

SUNG MIN HONG, YOUNG BAE JUN AND HEE SIK KIM

ABSTRACT. In this paper, we give another proof of Theorem 2.13 of [4] without using the sup property. For the homomorphic image $f(\mu)$ and preimage $f^{-1}(\nu)$ of fuzzy left (resp. right) ideals $\mu$ and $\nu$ respectively, we establish the chains of level left (resp. right) ideals of $f(\mu)$ and $f^{-1}(\nu)$, respectively. Moreover, we prove that a necessary condition for a fuzzy ideal $\mu$ of a near-ring $R$ to be prime is that $\mu$ is two-valued.

## 1. Introduction

S. Abou-Zaid [1] introduced the notion of a fuzzy subnear-ring, and studied fuzzy left (resp. right) ideals of a near-ring, and gave some properties of fuzzy prime ideals of a near-ring. In [4], S. D. Kim and H. S. Kim proved that the homomorphic image of a fuzzy left (resp. right) ideal which has the "sup property" is a fuzzy left (resp. right) ideal. In this paper, we give another proof of Theorem 2.13 of [4] without using the sup property. For the homomorphic image $f(\mu)$ and preimage $f^{-1}(\nu)$ of fuzzy left (resp. right) ideals $\mu$ and $\nu$ respectively, we establish the chains of level left (resp. right) ideals of $f(\mu)$ and $f^{-1}(\nu)$, respectively. Moreover, we prove that a necessary condition for a fuzzy ideal $\mu$ of a near-ring $R$ to be prime is that $\mu$ is two-valued.

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## 2. Preliminaries

By a *near-ring* [8] we mean a non-empty set $R$ with two binary operations "+" and "·" satisfying the following axioms:

(1) $(R, +)$ is a group,
(2) $(R, \cdot)$ is a semigroup,
(3) $x \cdot (y + z) = x \cdot y + x \cdot z$ for all $x, y, z \in R$.

Precisely speaking, it is a left near-ring because it satisfies the left distributive law. We will use the word "near-ring" in stead of "left near-ring". We denote $xy$ instead of $x \cdot y$. Note that $x0 = 0$ and $x(-y) = -xy$ but in general $0x \neq 0$ for some $x \in R$. Let $R$ and $S$ be near-rings. A map $f : R \to S$ is called a (*near-ring*) *homomorphism* if $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$ for any $x, y \in R$. An *ideal* $I$ of a near-ring $R$ is a subset of $R$ such that

(4) $(I, +)$ is a normal subgroup of $(R, +)$,
(5) $RI \subseteq I$,
(6) $(r + i)s - rs \in I$ for any $i \in I$ and any $r, s \in R$.

Note that $I$ is a *left ideal* of $R$ if $I$ satisfies (4) and (5), and $I$ is a *right ideal* of $R$ if $I$ satisfies (4) and (6).

We note that the intersection of a family of left (resp. right) ideals is a left (resp. right) ideal, and that the onto homomorphic image of a left (resp. right) ideal is also a left (resp. right) ideal.

We now review some fuzzy logic concepts (see [2], [9] and [10] for details). A *fuzzy set* $\mu$ in a set $R$ is a function $\mu : R \to [0, 1]$. Let $\mathrm{Im}(\mu)$ denote the image set of $\mu$. Let $\mu$ be a fuzzy set in a set $R$. For $\alpha \in [0, 1]$, the set

$$R_\mu^\alpha := \{x \in R | \mu(x) \geq \alpha\}$$

is called a *level subset* of $\mu$.

Let $f$ be a mapping from a set $R$ to a set $S$ and let $\mu$ and $\nu$ be fuzzy sets in $R$ and $S$, respectively. Then $f(\mu)$, the *image* of $\mu$ under $f$, is a fuzzy set in $S$:

$$f(\mu)(y) := \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise,} \end{cases}$$

for all $y \in S$. $f^{-1}(\nu)$, the *preimage* of $\nu$ under $f$, is a fuzzy set in $R$:

$$f^{-1}(\nu)(x) := \nu(f(x))$$

for all $x \in R$.

We say that a fuzzy set $\mu$ in $R$ has the *sup property* if, for any subset $T$ of $R$, there exists $t_0 \in T$ such that

$$\mu(t_0) = \sup_{t \in T} \mu(t).$$

Let $f$ be a mapping from a set $R$ to a set $S$ and let $\mu$ be a fuzzy set in $R$. Then $\mu$ is said to be *$f$-invariant* if $f(x) = f(y)$ implies $\mu(x) = \mu(y)$ for all $x, y \in R$. Clearly, if $\mu$ is $f$-invariant then $f^{-1}(f(\mu)) = \mu$.

## 3. Fuzzy Ideals

Let $R$ be a near-ring and $\mu$ be a fuzzy set in $R$. We say that $\mu$ is a *fuzzy subnear-ring* of $R$ if, for all $x, y \in R$,

(7) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$,

(8) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$.

$\mu$ is called a *fuzzy ideal* of $R$ if $\mu$ is a fuzzy subnear-ring of $R$ and

(9) $\mu(y + x - y) \geq \mu(x)$,

(10) $\mu(xy) \geq \mu(y)$,

(11) $\mu((x + z)y - xy) \geq \mu(z)$,

for any $x, y, z \in R$.

Note that $\mu$ is a fuzzy left ideal of $R$ if it satisfies (7), (8), (9) and (10), and $\mu$ is a fuzzy right ideal of $R$ if it satisfies (7), (8), (9) and (11) (see [1]).

LEMMA 1 ([1, Theorem 4.2]). *Let $\mu$ be a fuzzy set in a near-ring $R$. Then the level subset $R_\mu^\alpha$ is a subnear-ring (resp. an ideal) of $R$ for all $\alpha \in [0, 1]$, $\alpha \leq \mu(0)$ if and only if $\mu$ is a fuzzy subnear-ring (resp. a fuzzy ideal).*

The following proposition will be used in the sequel.

PROPOSITION 1. *Let $f$ be a mapping from a set $R$ to a set $S$, and let $\mu$ be a fuzzy set in $R$. Then for every $\alpha \in (0,1]$,*

$$S^{\alpha}_{f(\mu)} = \bigcap_{0<\beta<\alpha} f(R^{\alpha-\beta}_{\mu}).$$

*Proof.* Let $\alpha \in (0,1]$. For $y = f(x) \in S$, assume that $y \in S^{\alpha}_{f(\mu)}$. Then

$$\alpha \leq f(\mu)(y) = f(\mu)(f(x)) = \sup_{z \in f^{-1}(f(x))} \mu(z).$$

Hence for every real number $\beta$ with $0 < \beta < \alpha$, there exists $x_0 \in f^{-1}(y)$ such that $\mu(x_0) > \alpha - \beta$, and so $y = f(x_0) \in f(R^{\alpha-\beta}_{\mu})$. Therefore $y \in \bigcap_{0<\beta<\alpha} f(R^{\alpha-\beta}_{\mu})$.

Conversely, let $y \in \bigcap_{0<\beta<\alpha} f(R^{\alpha-\beta}_{\mu})$. Then $y \in f(R^{\alpha-\beta}_{\mu})$ for every $\beta$ with $0 < \beta < \alpha$, which implies that there exists $x_0 \in R^{\alpha-\beta}_{\mu}$ such that $y = f(x_0)$. It follows that $\mu(x_0) \geq \alpha - \beta$ and $x_0 \in f^{-1}(y)$, so that

$$f(\mu)(y) = \sup_{z \in f^{-1}(y)} \mu(z) \geq \sup_{0<\beta<\alpha} \{\alpha - \beta\} = \alpha.$$

Hence $y \in S^{\alpha}_{f(\mu)}$, and the proof is complete. □

S. D. Kim and H. S. Kim [4] proved the following theorems.

THEOREM 1. ([4, Theorem 2.12]). *A near-ring homomorphic preimage of a fuzzy left (resp. right) ideal is a fuzzy left (resp. right) ideal.*

THEOREM 2 ([4, Theorem 2.13]). *A near-ring homomorphic image of a fuzzy left (resp. right) ideal having the sup property is a fuzzy left (resp. right) ideal.*

Now we give another proof of Theorem 2 without using the sup property.

THEOREM 3. *Let $f : R \to S$ be an onto near-ring homomorphism and let $\mu$ be a fuzzy left (resp. right) ideal of $R$. Then $f(\mu)$ is a fuzzy left (resp. right) ideal of $S$.*

*Proof.* In view of Lemma 1 it is sufficient to show that $S^{\alpha}_{f(\mu)}$, $\alpha \in [0, \mu(0)]$, is a left (resp. right) ideal of $S$. Note that $S^{0}_{f(\mu)} = S$, and if $\alpha \in (0, 1]$ then $S^{\alpha}_{f(\mu)} = \bigcap_{0<\beta<\alpha} f(R^{\alpha-\beta}_{\mu})$ by Proposition 1. Since $R^{\alpha-\beta}_{\mu}$ is a left (resp. right) ideal of $R$ and since $f$ is onto, $f(R^{\alpha-\beta}_{\mu})$ is a left (resp. right) ideal of $S$. Therefore $S^{\alpha}_{f(\mu)}$ is an intersection of a family of left (resp. right) ideals is also a left (resp. right) ideal of $S$, ending the proof. □

THEOREM 4. *Let $f$ and $\mu$ be as in* Theorem 3. *Then there is a one-to-one correspondence between the set of all $f$-invariant left (resp. right) fuzzy ideals of $R$ and the set of all left (resp. right) fuzzy ideals of $S$.*

*Proof.* Straightforward in view of Theorem 1, Theorem 3 and the following results:

(i) $f^{-1}(f(\mu)) = \mu$, where $\mu$ is any $f$-invariant left (resp. right) fuzzy ideal of $R$;

(ii) $f(f^{-1}(\nu)) = \nu$, where $\nu$ is any left (resp. right) fuzzy ideal of $S$. □

THEOREM 5. *Let $f : R \to S$ be an onto homomorphism of near-rings and let $\mu$ and $\nu$ be left (resp. right) fuzzy ideals of $R$ and $S$, respectively such that*

$$\mathrm{Im}(\mu) = \{\alpha_0, \alpha_1, ..., \alpha_n\} \text{ with } \alpha_0 > \alpha_1 > ... > \alpha_n, \text{ and}$$

$$\mathrm{Im}(\nu) = \{\beta_0, \beta_1, ..., \beta_m\} \text{ with } \beta_0 > \beta_1 > ... > \beta_m.$$

*Then*

(i) $\mathrm{Im}(f(\mu)) \subset \mathrm{Im}(\mu)$ *and the chain of level left (resp. right) ideals of $f(\mu)$ is*

$$f(R^{\alpha_0}_{\mu}) \subset f(R^{\alpha_1}_{\mu}) \subset ... \subset f(R^{\alpha_n}_{\mu}) = S.$$

(ii) $\mathrm{Im}(f^{-1}(\nu)) = \mathrm{Im}(\nu)$ *and the chain of level left (resp. right) ideals of $f^{-1}(\nu)$ is*

$$f^{-1}(S^{\beta_0}_{\nu}) \subset f^{-1}(S^{\beta_1}_{\nu}) \subset ... \subset f^{-1}(S^{\beta_m}_{\nu}) = R.$$

*Proof.* (i) Since $f(\mu)(y) = \sup_{x \in f^{-1}(y)} \mu(x)$ for all $y \in S$, obviously $\mathrm{Im}(f(\mu)) \subset \mathrm{Im}(\mu)$. Note that for any $y \in S$,

$$\begin{aligned} y \in f(R_\mu^{\alpha_i}) &\Leftrightarrow \text{there exists } x \in f^{-1}(y) \text{ such that } \mu(x) \geq \alpha_i \\ &\Leftrightarrow \sup_{z \in f^{-1}(y)} \mu(z) \geq \alpha_i \\ &\Leftrightarrow f(\mu)(y) \geq \alpha_i \\ &\Leftrightarrow y \in S_{f(\mu)}^{\alpha_i}. \end{aligned}$$

Hence $f(R_\mu^{\alpha_i}) = S_{f(\mu)}^{\alpha_i}$ for $i = 0, 1, \cdots, n$, and therefore the chain of level left (resp. right) ideals of $f(\mu)$ is

$$f(R_\mu^{\alpha_0}) \subset f(R_\mu^{\alpha_1}) \subset \cdots \subset f(R_\mu^{\alpha_n}) = S.$$

(ii) Since $f^{-1}(\nu)(x) = \nu(f(x))$ for all $x \in R$ and since $f$ is onto, we have $\mathrm{Im}(f^{-1}(\nu)) = \mathrm{Im}(\nu)$. Note that for all $x \in R$,

$$\begin{aligned} x \in f^{-1}(S_\nu^{\beta_i}) &\Leftrightarrow f(x) \in S_\nu^{\beta_i} \\ &\Leftrightarrow \nu(f(x)) \geq \beta_i \\ &\Leftrightarrow f^{-1}(\nu)(x) \geq \beta_i \\ &\Leftrightarrow x \in R_{f^{-1}(\nu)}^{\beta_i}, \end{aligned}$$

so that $f^{-1}(S_\nu^{\beta_i}) = R_{f^{-1}(\nu)}^{\beta_i}$ for all $i = 0, 1, \cdots, m$. Hence the chain of level left (resp. right) ideals of $f^{-1}(\nu)$ is

$$f^{-1}(S_\nu^{\beta_0}) \subset f^{-1}(S_\nu^{\beta_1}) \subset \cdots \subset f^{-1}(S_\nu^{\beta_m}) = R.$$

This completes the proof. □

LEMMA 2. *Let $\mu$ and $\nu$ be fuzzy left (resp. right) ideals of $R$ and $f(R)$ respectively, where $f : R \to S$ is a near-ring homomorphism. Then $f(\mu)(0) = \mu(0)$ and $f^{-1}(\nu)(0) = \nu(0)$.*

*Proof.* Straightforward. □

Let $\rho$ and $\delta$ be two fuzzy sets in a near-ring $R$. The *product* $\rho \circ \delta$ is defined by

$$\rho \circ \delta(x) := \begin{cases} \sup\limits_{x=yz} \{\min\{\rho(y), \delta(z)\}\}, \\ 0 \quad \text{if } x \text{ is not expressible as } x = yz. \end{cases}$$

A fuzzy ideal $\mu$ of a near-ring $R$ is said to be *prime* [1] if $\mu$ is not a constant function and for any fuzzy ideals $\rho$ and $\delta$ of $R$, $\rho \circ \delta \subset \mu$ implies $\rho \subset \mu$ or $\delta \subset \mu$.

For a fuzzy left (resp. right) ideal $\delta$ of a near-ring $R$, let

$$\delta_0 := \{x \in R | \delta(x) = \delta(0)\}.$$

LEMMA 3 ([1, Theorem 3.7]). *Let $\delta$ be a fuzzy prime ideal of a near-ring $R$. Then $\delta_0$ is a prime ideal of $R$.*

PROPOSITION 2. *Let $f : R \to S$ be a near-ring homomorphism and let $\delta$ be a fuzzy left (resp. right) ideal of $R$. Then $f(\delta_0) \subseteq f(\delta)_0$, with equality if $\delta$ has the sup property.*

*Proof.* Let $x \in \delta_0$. Then

$$f(\delta)(f(0)) \geq f(\delta)(f(x)) \geq \delta(x) = \delta(0) = f(\delta)(f(0)),$$

and so $f(\delta)(f(x)) = f(\delta)(f(0)) = f(\delta)(0)$. Hence $f(x) \in f(\delta)_0$ or $f(\delta_0) \subseteq f(\delta)_0$. Assume that $\delta$ has the sup property and let $x \in R$ be such that $f(x) \in f(\delta)_0$. Then

$$\delta(0) = f(\delta)(f(x)) = \sup\{\delta(y) | f(y) = f(x)\} = \delta(y)$$

for some $y \in R$ such that $f(y) = f(x)$ since $\delta$ has the sup property. Thus $y \in \delta_0$, and so $f(x) = f(y) \in \delta_0$. This completes the proof. □

THEOREM 6. *Let $\mu$ be a fuzzy prime ideal of a near-ring $R$. Then $|\mathrm{Im}(\mu)| = 2$, i.e., $\mu$ is two-valued. In particular, $\mu(0) = 1$.*

*Proof.* Note that $|\mathrm{Im}(\mu)| \geq 2$ since $\mu$ is not constant. Assume that $|\mathrm{Im}(\mu)| \geq 3$. Let $\mu(0) = \alpha$ and $\lambda =$glb$\{\mu(x)|x \in R\}$. Then there exist $\gamma, \beta \in \mathrm{Im}(\mu)$ such that $\lambda \leq \gamma < \beta < \alpha$. Let $\rho$ and $\delta$ be fuzzy sets in $R$ such that $\rho(x) := \frac{1}{2}(\gamma + \beta)$ for all $x \in R$ and

$$\delta(x) := \begin{cases} \lambda & \text{if } x \notin R_\mu^\beta, \\ \alpha & \text{otherwise.} \end{cases}$$

Clearly, $\rho$ is a fuzzy ideal of $R$. We now prove that $\delta$ is a fuzzy ideal of $R$. Let $x, y \in R$. If $x, y \in R_\mu^\beta$, then $x - y \in R_\mu^\beta$ and $\delta(x - y) = \alpha = \min\{\delta(x), \delta(y)\}$. If $x \in R_\mu^\beta$ and $y \notin R_\mu^\beta$ (or $x \notin R_\mu^\beta$ and $y \in R_\mu^\beta$) then $x - y \notin R_\mu^\beta$ and

$$\delta(x - y) = \lambda = \min\{\delta(x), \delta(y)\},$$

since

$$\delta(x) \text{ (or } \delta(y))\text{=}\alpha > \lambda = \delta(y) \text{ (or } \delta(x)).$$

If $x \notin R_\mu^\beta$ and $y \notin R_\mu^\beta$ then $\delta(x) = \delta(y) = \lambda$ and so

$$\delta(x - y) \geq \lambda = \min\{\delta(x), \delta(y)\}.$$

Hence $\delta(x - y) \geq \min\{\delta(x), \delta(y)\}$ for all $x, y \in R$. Similarly, we know that

$$\delta(xy) \geq \min\{\delta(x), \delta(y)\} \text{ for all } x, y \in R.$$

Hence $\delta$ is a fuzzy subnear-ring of $R$. For any $y \in R$, if $y \in R_\mu^\beta$ then $xy \in R_\mu^\beta$ for all $x \in R$, and so $\delta(xy) = \alpha = \delta(y)$. If $y \notin R_\mu^\beta$, then $\delta(xy) \geq \lambda = \delta(y)$. Hence $\delta(xy) \geq \delta(y)$ for all $x, y \in R$. Let $x, y \in R$. If $x \in R_\mu^\beta$ then $y + x - y \in R_\mu^\beta$ and $\delta(y + x - y) = \alpha = \delta(x)$. If $x \notin R_\mu^\beta$, then $\delta(y + x - y) \geq \lambda = \delta(x)$. This proves that $\delta$ is a fuzzy left ideal of $R$. Let $x, y, z \in R$. If $z \in R_\mu^\beta$, then $(x + z)y - xy \in R_\mu^\beta$ and $\delta((x+z)y-xy) = \alpha = \delta(z)$. If $z \notin R_\mu^\beta$, then $\delta(z) = \lambda \leq \delta((x+z)y-xy)$. Hence $\delta((x + z)y - xy) \geq \delta(z)$ for all $x, y, z \in R$, and therefore $\delta$ is a fuzzy ideal of $R$. Now we show that $\rho \circ \delta \subseteq \mu$. Consider the following cases:

Case (i) $x = 0$. Then

$$\rho \circ \delta(x) = \sup_{x=yz}\{\min\{\rho(y), \delta(z)\}\} \leq \frac{1}{2}(\gamma + \beta) < \alpha = \mu(0).$$

Case (ii) $0 \neq x \in R_\mu^\beta$. Then $\mu(x) \geq \beta$, and

$$\rho \circ \delta(x) = \sup_{x=yz}\{\min\{\rho(y), \delta(z)\}\} \leq \frac{1}{2}(\gamma + \beta) < \beta \leq \mu(x).$$

Case (iii) $0 \neq x \notin R_\mu^\beta$. For any $y, z \in R$ such that $x = yz$, we have $z \notin R_\mu^\beta$. Thus $\delta(z) = \lambda$ and so

$$\rho \circ \delta(x) = \sup_{x=yz}\{\min\{\rho(y), \delta(z)\}\} = \lambda \leq \mu(x).$$

Thus in each case, $\rho \circ \delta(x) \leq \mu(x)$ or $\rho \circ \delta \subseteq \mu$.

Next we show that neither $\rho \subseteq \mu$ nor $\delta \subseteq \mu$. We can find $x \in R$ such that $\mu(x) = \gamma$. Then

$$\rho(x) = \frac{1}{2}(\gamma + \beta) > \gamma = \mu(x).$$

Hence $\rho \not\subseteq \mu$. We also know that $\mu(y) = \beta$ for some $y \in R$. It follows that $y \in R_\mu^\beta$ and $\delta(y) = \alpha > \beta = \mu(y)$. Therefore $\delta \not\subseteq \mu$. This shows that $\mu$ is not a fuzzy prime ideal of $R$, which is a contradiction. Hence $|\mathrm{Im}(\mu)| = 2$. Now let $|\mathrm{Im}(\mu)| = \{\alpha, \gamma\}$ and $\gamma < \alpha$. Then $\mu(0) = \alpha$ since $\mu(0) \geq \mu(x)$ for all $x \in R$. Assume that $\alpha \neq 1$. Then there exists $\beta \in [0, 1]$ such that $\alpha < \beta \leq 1$. Let $\rho$ and $\delta$ be fuzzy sets in $R$ such that $\rho(x) := \frac{1}{2}(\alpha + \gamma)$ for all $x \in R$ and

$$\delta(x) := \begin{cases} \beta & \text{if } x \in \mu_0, \\ \gamma & \text{otherwise.} \end{cases}$$

Clearly $\rho$ is a fuzzy ideal of $R$. Since $\mu_0$ is an ideal of $R$, $\delta$ is a fuzzy ideal of $R$. It can be easily checked that $\rho \circ \delta \subseteq \mu$. Since $\mu(0) = \alpha < \beta = \delta(0)$, we have $\delta \not\subseteq \mu$. Note that there exists $x \in R$ such that $\mu(x) = \gamma < \frac{1}{2}(\alpha + \gamma) = \rho(x)$, so that $\rho \not\subseteq \mu$. This is a contradiction to the hypothesis. Hence $\mu(0) = 1$, ending the proof. □

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## References

[1] S. Abou-Zaid, *On fuzzy subnear-rings and ideals*, Fuzzy Sets and Sys. **44** (1991), 139-146.
[2] P. S. Das, *Fuzzy groups and level subgroups*, J. Math. Anal. and Appl. **84** (1981), 264-269.
[3] V. N. Dixit, R. Kumar and N. Ajal, *On fuzzy rings*, Fuzzy Sets and Sys. **49** (1992), 205-213.
[4] S. D. Kim and H. S. Kim, *On fuzzy ideals of near-rings*, Bull. Korean Math. Soc. **33** (1996), 593-601.
[5] R. Kumar, *Fuzzy irreducible ideals in rings*, Fuzzy Sets and Sys. **42** (1991), 369-379.
[6] ———, *Certain fuzzy ideals of rings redefined*, Fuzzy Sets and Sys. (1992), 251-260.
[7] D. S. Malik, *Fuzzy ideals of artinian rings*, Fuzzy Sets and Sys. **37** (1990), 111-115.
[8] J. D. P. Meldrum, *Near-rings and their links with groups*, Pitman, Boston (1985).
[9] A. Rosenfeld, *Fuzzy groups*, J. Math. Anal. Appl. **35** (1971), 512-517.
[10] L. A. Zadeh, *Fuzzy sets*, Inform. and Control **8** (1965), 338-353.

Sung Min Hong and Young Bae Jun, Department of Mathematics, Gyeong-sang National University, Chinju 660-701, Korea
*E-mail*: ybjun@nongae.gsnu.ac.kr

Hee Sik Kim, Department of Mathematics, Hanyang University, Seoul 133-791, Korea
*E-mail*: heekim@email.hanyang.ac.kr