Journal of the Korean
OR/MS Society
Vol. 23, No 2, June 1998

A Lagrangian Relaxation Approach to Capacity Planning for a Manufacturing System with Flexible and Dedicated Machines

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Abstract

We consider a multiperiod capacity planning problem for determining a mix of flexible and dedicated capacities under budget restriction. These capacities are controlled by purchasing flexible machines and/or new dedicated machines and disposing old dedicated machines. Acquisition and replacement schedules are determined and operations are assigned to the flexible or dedicated machines for the objective of minimizing the sum of discounted costs of acquisition and operation of flexible machines, new dedicated machines, and old dedicated machines. In this research, the problem is formulated as a mixed integer linear program and solved by a Lagrangian relaxation approach. A subgradient optimization method is employed to obtain lower bounds and a multiplier adjustment method is devised to improve the bounds. We develop a linear programming based Lagrangian heuristic algorithm to find a good feasible solution of the original problem. Results of tests on randomly generated test problems show that the algorithm gives relatively good solutions in a reasonable amount of computation time.

1. Introduction

To survive today's highly competitive environments, many companies are planning to upgrade their production equipment, and some are already moving towards flexible automation. In general, to upgrade production equipment, flexible manufacturing modules (FMMs) and/or new dedicated machines may be acquired, and old dedicated machines may be disposed at the same time. A typical FMM consists of a multi-functional machining center with automatic tool and pallet changers and a tool magazine, and a material handling device. An FMM can perform a wide variety of operations, while a dedicated machine can perform only one type of operations such as drilling operations or milling operations. On the other hand, acquisition and operation costs required for a dedicated machine are lower than those for an FMM. Therefore, both flexiblities of FMMs and lower costs of dedicated machines should be considered simultaneously in upgrading production equipment.

There are some previous research results related to acquisition of flexible machines and disposal/replacement of existing dedicated machines. Fine and Freund [3] and Roth *et al.* [16] determine optimal capacities of flexible and dedicated systems using stochastic and dynamic model, respectively. With a deterministic model, Rajagopalan [14] determines an optimal mix of flexible and dedicated capacities for a case of multiple products with a nondecreasing demand pattern. By relaxing the assumption of nondecreasing demand, Rajagopalan and Soteriou [15] develop a more realistic model that deals with capacity acquisition, disposal and replacement decisions. As in other studies on the deterministic capacity acquisition problem, Li and Tirupati [10, 11] solve capacity expansion problems considering tradeoffs between economies of scale and scope for cases with general and dynamic demand patterns.

Capacity planning and operations assignment have been considered simultaneously in other research. Bard and Feo [1] develop a single-period nonlinear cost model that can be used to determine the number of machines to be purchased as well as the fraction of time each machine should be configured for a particular type of operations. As extensions of the model, Suresh [17] and Lim and Kim [13] develop multiperiod replacement models under phased implementation strategies. Although budget restriction and demand uncertainty are not considered explicitly in most of previous research, Karabakal *et al.* [8] and Lim and Kim [13] consider the former in their deterministic model, while Chakravarty [2], Gupta *et al.* [7] and Li and Tirupati [12] consider the latter. On the other hand, Li and Qiu [9] study the impact of operational factors on the decisions of capacity acquisition and technology choice.

In this paper, we consider a multiperiod capacity planning problem for determining a mix of flexible and dedicated capacities under budget restriction. These capacities are controlled by purchasing flexible machines and/or new dedicated machines and disposing old dedicated machines. Acquisition and replacement schedules are determined and operations are assigned to the flexible or dedicated machines for the objective of minimizing the sum of discounted costs of acquisition and operation of flexible machines, new dedicated machines, and old dedicated machines. This problem is a generalized version of the problem studied in Lim and

Kim [13], which is based on the assumptions that new dedicated machines cannot be purchased during the planning horizon and acquired FMMs are fully utilized up to their capacities if possible. In the generalized problem of the current study, however, it is assumed that new dedicated machines are available and that FMMs may be underutilized if using dedicated machines costs less. For the generalized problem, we present a mixed integer linear programming model and develop a solution procedure based on the Lagrangian relaxation approach.

2. Mathematical Model

First, we state the assumptions made in this research.

1) Technological improvements can be predicted, that is, technologies (FMMs and dedicated machines) currently available and anticipated within the planning horizon are known in advance.
2) Demands and process plans for the products are known.
3) Acquired FMMs and dedicated machines are not retired during the planning horizon, and initially, there is no FMM.
4) A dedicated machine disposed in any period of the planning horizon cannot be redeployed during the remaining periods.
5) A dedicated machine can process only one type of operations. In this research, operations are classified into types according to the similarity of operations (e.g. drilling operations and turning operations).
6) There is no salvage value for dedicated machines. (Salvage values are often low enough to be ignored, especially for obsolete dedicated machines.)

The following is notation used in the formulation for the capacity planning problem.

- i index for operation types or dedicated machines, $i = 1, 2, ..., I$
- j index for FMMs, $j = 1, 2, ..., J$
- t index for time periods, $t = 1, 2, ..., T$
- D_{it} work amount (requirements) of operation type i (to be performed) in period t (This is determined from process plans and demands for the products.)
- A^{F}_{jt} acquisition costs of a type-j FMM in period t
- A^{ND}_{it} acquisition costs of a new dedicated machine of type i in period t
- C^{F}_{ijt} operation costs for processing one unit work of operation type i on a type-j FMM in period t
- C^{ND}_{it} operation costs for processing one unit work of operation type i on a new dedicated machine of type i in period t

C_{it}^{D} operation costs for processing one unit work of operation type i on an old dedicated machine of type i in period t

W_{ijt}^{F} amount of work that a type-j FMM can do to process operation type i in period t when the FMM is configured to process only that type

W_{it}^{ND} amount of work that a new dedicated machine of type i can do to process operation type i in period t if the capacity of the machine is fully utilized

W_{it}^{D} amount of work that an old dedicated machine of type i can do to process operation type i in period t if the capacity of the machine is fully utilized

B present worth of the budgets that can be spent over the planning horizon

r_t discount factor in period t (i.e. $(1+p)^{-t}$, where p is the discount rate)

y_{jt} number of type-j FMMs acquired in period t

v_{it} number of new dedicated machines of type i acquired in period t

z_{it} equals 1 if one or more old dedicated machines of type i are used in period t, and 0 otherwise (i.e. z_{it} = 0 if all of the old dedicated machines are disposed in period t or earlier)

h_{ijt} fraction of type-j flexible capacity which is allocated to operation type i in period t

e_{it} fraction of capacity of newly acquired dedicated machines of type i which is used to satisfy demand of operation type i in period t

s_{it} fraction of capacity of existing dedicated machines of type i which is used to satisfy demand of operation type i in period t

Now, we present a mixed integer linear programming formulation of the problem.

(P) Minimize

$$\sum_j \sum_t r_t A_{jt}^F y_{it} + \sum_i \sum_j \sum_t r_t C_{ijt}^F W_{ijt}^F h_{ijt} + \sum_i \sum_t r_t C_{it}^D W_{it}^D s_{it} + \sum_i \sum_t r_t A_{it}^{ND} v_{it} + \sum_i \sum_t r_t C_{it}^{ND} W_{it}^{ND} e_{it} \quad (1)$$

$$\sum_j W_{ijt}^F h_{ijt} + W_{it}^D s_{it} + W_{it}^{ND} e_{it} \geq D_{it} \quad \forall i,\ t \quad (2)$$

$$\sum_j \sum_t r_t A_{jt}^F y_{jt} + \sum_i \sum_t r_t A_{it}^{ND} v_{it} \leq B \quad (3)$$

$$\sum_i h_{ijt} \leq \sum_{u=1}^{t} y_{ju} \quad \forall j,\ t \quad (4)$$

$e_{it} \le \sum_{u=1}^{t} v_{iu}$ $\quad \forall i, \ t \quad$ (5)

$s_{it} \le z_{it}$ $\quad \forall i, \ t \quad$ (6)

$z_{it} \le z_{i,t-1}$ $\quad \forall i, \ t \quad$ (7)

$y_{it} \ge 0$ and integer $\quad \forall j, \ t \quad$ (8)

$v_{it} \ge 0$ and integer $\quad \forall i, \ t \quad$ (9)

$z_{it} \in \{0,1\}$ $\quad \forall i, \ t \quad$ (10)

$h_{ijt} \ge 0$ $\quad \forall i,j,t \quad$ (11)

$e_{it} \ge 0$ $\quad \forall i, \ t \quad$ (12)

$0 \le s_{it} \le 1$ $\quad \forall i, \ t \quad$ (13)

The objective function to be minimized denotes the sum of acquisition and operation costs of FMMs and new dedicated machines, and costs for operating old dedicated machines. Constraints (2) ensure that demands for each operation type during the planning horizon should be satisfied by acquired FMMs, old dedicated machines, or new dedicated machines. Constraint (3) represents the budget limit, while constraints (4) represent allocations of flexible capacities to operation types. Constraints (5) and (6) determine fractions of new and existing dedicated capacities which are used to satisfy work requirements for the operations, respectively. Constraints (7) specifies restrictions imposed by one of the assumptions made in this study, i.e., an old dedicated machine may not be redeployed during the planning horizon once it is disposed.

3. Lagrangian Relaxation Approach

In this research, the Lagrangian relaxation approach is employed to solve (P). Foundations of the Lagrangian relaxation theory and some early successful applications are summarized in Fisher [4, 5] and Geoffrion [6]. In this study, the original problem (P) is relaxed by dualizing constraint (2) with Lagrangian multipliers $\lambda_{it} \le 0$. The resulting relaxed problem is:

(LR_λ) Minimize

$$\sum_j \sum_t r_t A^F_{jt} y_{jt} + \sum_i \sum_j \sum_t (\lambda_{it} + r_t C^F_{ijt}) W^F_{ijt} h_{ijt}$$
$$+ \sum_i \sum_t (\lambda_{it} + r_t C^D_{it}) W^D_{it} s_{it}$$
$$+ \sum_i \sum_t r_t A^{ND}_{it} v_{it} + \sum_i \sum_t (\lambda_{it} + r_t C^{ND}_{it}) W^{ND}_{it} e_{it}$$
$$- \sum_i \sum_t \lambda_{it} D_{it}$$

subject to (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), (13) and

$\lambda_{it} \le 0$ for all i and t. (14)

This relaxed problem can be decomposed into two independent subproblems, (SP1) and (SP2), as follows. While (SP1) is associated with acquisition of FMMs and new dedicated machines and allocations of their capacities to operation types, (SP2) is associated with use of capacities of old dedicated machines and disposal of the machines.

(SP1) Minimize

$$\sum_j \sum_t r_t A^F_{jt} y_{it} + \sum_i \sum_j \sum_t (\lambda_{it} + r_t C^F_{ijt}) W^F_{ijt} h_{ijt}$$
$$+ \sum_i \sum_t r_t A^{ND}_{it} v_{it} + \sum_i \sum_t (\lambda_{it} + r_t C^{ND}_{it}) W^{ND}_{it} e_{it}$$

subject to (3), (4), (5), (8), (9), (11), (12) and (14)

(SP2) Minimize

$$\sum_i \sum_t (\lambda_{it} + r_t C^D_{it}) W^D_{it} s_{it}$$

subject to (6), (7), (10), (13) and (14)

We can compute a lower bound on the objective function value of the optimal solution (to be called the *optimal value* for brevity throughout the paper) of the original problem by solving these subproblems. For given λ_{it}, L(λ), which is a solution of (LRλ) and also a lower bound for (P), can be obtained using solutions of (SP1) and (SP2), denoted by L1(λ) and L2(λ), respectively, as follows. Here, λ denotes the vector of which the elements are λ_{it}.

$$\mathrm{L}(\lambda) = \mathrm{L1}(\lambda) + \mathrm{L2}(\lambda) - \sum_i \sum_t \lambda_{it} D_{it}.$$

3.1. Solution procedures for the relaxed problems

Solving (SP1) for given λ

First, we give a property which characterizes an optimal solution of (SP1). For given λ_{it}, let $\hat{i}_{jt} = \arg\min_i (\lambda_{it} W^F_{ijt} + r_t C^F_{ijt} W^F_{ijt})$ for all j and t. Throughout the remainder of this section, we drop the subscripts of $\hat{i}$ for notational simplicity.

Proposition 1. *Assume y^*_{jt} and v^*_{it} are optimal to (SP1) for given λ_{it}. Then, in an optimal solution for (SP1), h_{ijt} and e_{it} satisfy the following:*

$$(a)\quad h^*_{ijt} = \begin{cases} \sum_{u=1}^{t} y^*_{ju} & \text{if } i = \hat{i} \text{ and } (\lambda_{it} W^F_{ijt} + r_t C^F_{ijt} W^F_{ijt}) < 0 \\ 0 & \text{otherwise} \end{cases} \qquad (15)$$

$$(b)\quad e^*_{it} = \begin{cases} \sum_{u=1}^{t} v^*_{iu} & \text{if } (\lambda_{it} W^{ND}_{it} + r_t C^{ND}_{it} W^{ND}_{it}) < 0 \\ 0 & \text{otherwise} \end{cases} \qquad (16)$$

Proof. Note that h^* and e^* can be determined independently. We first prove the optimality of h^*. If h^* satisfies constraints (4) and the operation costs in the objective function of (SP1), $\sum_i \sum_j \sum_t (\lambda_{it} + r_t C^F_{ijt}) W^F_{ijt} h_{ijt}$, have a minimum value at h^*, h^* is optimal. Since y^* is an optimal solution of (SP1), h^* given by (15) satisfies constraints (4). Obviously, $h_{ijt} = 0$ is an optimal solution when $\lambda_{it} W^F_{ijt} + r_t C^F_{ijt} W^F_{ijt} \geq 0$. When $\lambda_{it} W^F_{ijt} + r_t C^F_{ijt} W^F_{ijt} < 0$, $\sum_i \sum_j \sum_t (\lambda_{it} + r_t C^F_{ijt}) W^F_{ijt} h_{ijt}$ is minimum if h_{ijt} is set as equation (15) since $\lambda_{\hat{i}t} W^F_{\hat{i}jt} + r_t C^F_{\hat{i}jt} W^F_{\hat{i}jt} \leq \lambda_{it} W^F_{ijt} + r_t C^F_{ijt} W^F_{ijt}$ for all i. This completes the proof for part (a). The optimality of e^* can be proved similarly. ■

In proposition 1, the term $\lambda_{it}W_{ijt}^F$(or $\lambda_{it}W_{it}^{ND}$) can be considered as benefits that can be obtained by using FMM j(or a new dedicated machine of type i) to process operation type i in period t, if λ_{it} is interpreted as value of operation type i in period t. In addition, $r_tC_{ijt}^FW_{ijt}^F$ (or $r_tC_{it}^{ND}W_{it}^{ND}$) is the discounted operation cost incurred when a type-j FMM (or a new dedicated machine of type i) processes operation type i in period t. Therefore, (a) means that it is best to use an FMM to process an operation type that is best for the FMM if net profits (benefits minus costs) are positive. Similarly, (b) means that it is optimal to use all available capacities of new dedicated machines if net profits of using them are positive.

Constraints (4) and (5) can be eliminated using proposition 1. Since it is optimal to assign the whole capacity of each FMM to its best-suited operation type if the net profits are positive, the following substitution can be made in (4) for all j and t and in the objective function of (SP1).

$$h_{ijt}=\begin{cases}\sum_{u=1}^{t}y_{ju} & \text{if } i=\hat{i}\text{ and}(\lambda_{it}+r_tC_{ijt}^F)W_{ijt}^F<0\\ 0 & \text{otherwise}\end{cases}$$

Similarly, the following substitution can be made for e_{it}.

$$e_{it}=\begin{cases}\sum_{u=1}^{t}v_{iu} & \text{if}(\lambda_{it}+r_tC_{it}^{ND})W_{it}^{ND}<0\\ 0 & \text{otherwise}\end{cases}$$

With these substitutions, (4) and (5) can be eliminated and the objective function can be modified to

$$\sum_j\sum_t[r_tA_{jt}^Fy_{jt}+(\sum_{u=1}^{t}y_{ju})\min\{(\lambda_{\hat{i}t}+r_tC_{\hat{i}jt}^F)W_{\hat{i}jt}^F,0\}]+$$
$$\sum_i\sum_t[r_tA_{it}^{ND}v_{it}+(\sum_{u=1}^{t}v_{iu})\min\{(\lambda_{it}+r_tC_{it}^{ND})W_{it}^{ND},0\}].\tag{17}$$

To simplify this objective function, we introduce the following notation.

$$M_{jt}\equiv r_tA_{jt}^F+\sum_{p=t}^{T}\min\{\lambda_{\hat{i}p}W_{\hat{i}jp}^F+r_pC_{\hat{i}jp}^FW_{\hat{i}jp}^F,0\},$$

$$L_{it}\equiv r_tA_{it}^{ND}+\sum_{p=t}^{T}\min\{\lambda_{ip}W_{ip}^{ND}+r_pC_{ip}^{ND}W_{ip}^{ND},0\}.$$

With this notation, the first and second term of the modified objective function can be simplified as follows.

$$\sum_j\sum_t[r_tA_{jt}^Fy_{jt}+(\sum_{u=1}^{t}y_{ju})\min\{(\lambda_{\hat{i}t}+r_tC_{\hat{i}jt}^F)W_{\hat{i}jt}^F,0\}]$$
$$=\sum_j[\{r_1A_{j1}^Fy_{j1}+\min(\lambda_{\hat{i}1}W_{\hat{i}j1}^F+r_1C_{\hat{i}j1}^FW_{\hat{i}j1}^F,0)y_{j1}\}+$$
$$\{r_2A_{j2}^Fy_{j2}+\min(\lambda_{\hat{i}2}W_{\hat{i}j2}^F+r_2C_{\hat{i}j2}^FW_{\hat{i}j2}^F,0)$$
$$(y_{j1}+y_{j2})\}+....+$$
$$\{r_TA_{jT}^Fy_{jT}+\min(\lambda_{\hat{i}T}W_{\hat{i}jT}^F+r_TC_{\hat{i}jT}^FW_{\hat{i}jT}^F)$$
$$(y_{j1}+y_{j2}+...+y_{jT})\}]$$
$$=\sum_j[\{r_1A_{j1}^F+\sum_{p=1}^{T}\min(\lambda_{\hat{i}p}W_{\hat{i}jp}^F+r_pC_{\hat{i}jp}^FW_{\hat{i}jp}^F,0)\}y_{j1}+$$
$$\{r_2A_{j2}^F+\sum_{p=2}^{T}\min(\lambda_{\hat{i}p}W_{\hat{i}jp}^F+r_pC_{\hat{i}jp}^FW_{\hat{i}jp}^F,0)\}y_{j2}+...+$$
$$\{r_TA_{jT}^F+\min(\lambda_{\hat{i}T}W_{\hat{i}jT}^F+r_TC_{\hat{i}jT}^FW_{\hat{i}jT}^F,0)y_{jT}\}]$$
$$=\sum_j\{M_{j1}y_{j1}+M_{j2}y_{j2}+...+M_{jT}y_{jT}\}$$
$$=\sum_j\sum_tM_{jt}y_{it}.$$

The second term of (17) can be simplified similarly using L_{it}. As a result, we have the following objective function of a simpler form.

$$\sum_j \sum_t M_{jt} y_{jt} + \sum_i \sum_t L_{it} v_{it}$$

By the above elimination and simplification, (SP1) can be converted into the following knapsack problem.

(SP1K)

$$Z(B) = \text{Max} \sum_j \sum_t \{-M_{jt} y_{jt}\} + \sum_i \sum_t \{-L_{it} v_{it}\}$$

subject to

$$\sum_j \sum_t r_t A^F_{jt} y_{it} + \sum_i \sum_t r_t A^{ND}_{it} v_{it} \le B$$

$$y_{jt} \ge 0 \text{ and integer} \qquad \forall j, t$$

$$v_{it} \ge 0 \text{ and integer} \qquad \forall i, t$$

We solve this knapsack problem using a dynamic programming recursion. Since the computation time for the DP procedure used in this study depends on the number of variables (y_{jt} *and* v_{it}) and the budget, it is necessary to reduce the number of variables and to scale input parameters. We can reduce the number of variables with the following property, for which the proof is omitted here because it is trivial.

Proposition 2. *For given* λ_{it}, *the following are satisfied in an optimal solution of (SP1).*

(*a*) *If* $M_{jt} > 0$, *then* $y^*_{jt} = 0$.

(*b*) *If* $L_{it} > 0$, *then* $v^*_{it} = 0$.

Note that M_{jt} is composed of the acquisition cost of a type-j FMM in period t and the sum of positive net profits (or negative costs) from period t to the end of the planning horizon, i.e. $\sum_{p=t}^{T} \min\{\lambda_{ip} W^F_{ijp} + r_p C^F_{ijp} W^F_{ijp}, 0\}$, which can be obtained from processing an operation type that is best for the FMM. Similarly, L_{it} consists of the acquisition cost of a new dedicated machine of type i in period t and the sum of positive net profits (or negative costs) from period t to the end of the planning horizon, i.e. $\sum_{p=t}^{T} \min\{\lambda_{ip} W^{ND}_{ip} + r_p C^{ND}_{ip} W^{ND}_{ip}, 0\}$. Therefore, proposition 2 implies that the sum of positive net profits should be greater than acquisition costs in order to justify purchase of an FMM or a new dedicated machine. Using the above proposition, we can eliminate variables that should be equal to 0 in an optimal solution.

Since an optimal solution of (SP1K) is also optimal for (SP1), the optimal value for (SP1) is

$$L1(\lambda) = Z(B).$$

<u>*Solving (SP2) for given* λ</u>

The following proposition characterizes an optimal solutionm, s^*_{it} and z^*_{it} of(SP2). Let $\hat{t}_i = \arg\max\{t \mid s^*_{it} = 1\}$.

Proposition 3. *In an optimal solution of (SP2), we have*

$$(a)\quad s^*_{it} = \begin{cases} 1 & \text{if } (\lambda_{it} + r_t C^D_{it}) W^D_{it} < 0 \\ 0 & \text{otherwise} \end{cases}$$

$$(b)\ z_{it}^{*}=\begin{cases}1 & t=1,2,\ldots,\hat{t}_i\\ 0 & t=\hat{t}_i+1,\ldots,T\end{cases}$$

Proof. (a) and (b) are obvious since s_{it}^{*} and z_{it}^{*} determined by (a) and (b) are feasible and the objective function, $\sum_i\sum_t(\lambda_{it}+r_tC_{it}^{D})W_{it}^{D}s_{it}$, has a minimum value at s_{it}^{*}. ∎

Therefore, the optimal value for (SP2), denoted by L2(λ), is

$$\mathrm{L2}(\lambda)=\sum_i\sum_t(\lambda_{it}+r_tC_{it}^{D})W_{it}^{D}s_{it}^{*}.$$

With the above solutions for the two subproblems for given λ, we obtain a solution of (LR$_\lambda$), which gives a lower bound on the optimal value of (P), as follows.

$$\mathrm{L}(\lambda)=\mathrm{L1}(\lambda)+\mathrm{L2}(\lambda)-\sum_i\sum_t\lambda_{it}D_{it}.$$

3.2. Finding the Best Multipliers

To find a better lower bound, we need to find better values for the Lagrangian multipliers, λ_{it}. In the following, we present a solution method for the following Lagrangian dual problem (PL) to find the best multipliers.

(PL) Maximize L(λ)
subject to $\lambda\le 0$

Note that solutions of (PL) serve as lower bounds for (P).

<u>*Subgradient Optimization Method*</u>

To solve (PL), we use the subgradient optimization algorithm described in [4]. At iteration r, subgradient vector θ^{r} is determined by

$$\theta_{it}^{r}=\sum_j W_{ijt}^{F}h_{ijt}^{*}+W_{it}^{D}s_{it}^{*}+W_{it}^{ND}e_{it}^{*}-D_{it},$$

where h_{ijt}^{*}, s_{it}^{*} and e_{it}^{*} are the optimal solutions for (LR$_\lambda$) obtained at iteration r. Given the multipliers at iteration r, λ_{it}^{r}, multipliers for the next iteration are generated by $\lambda_{it}^{r+1}=\min\{\lambda_{it}^{r}+\omega_r\theta_{it}^{r},0\}$ for all i and t, where ω_r is a positive scalar step size. A commonly used step size at iteration r is $\omega_r=\mu_r(Z^{*}-\mathrm{L}(\lambda^{r}))/\|\theta^{r}\|^2$ where $\mu_r\le 2$ is a positive scalar, Z^{*} is an upper bound on the optimal value of (PL) and ‖ • ‖ denotes the norm of vector •.

Initially, the value for μ_r is set to 2 ($\mu_0=2$) and it is halved when the solution of (LR$_\lambda$) has not increased for a given number of iterations. It is reported that this rule performs well empirically, even though it is not guaranteed to satisfy the sufficient condition for convergence to the optimal solution [4].

<u>*Multiplier Adjustment Method*</u>

We develop a multiplier adjustment method to improve the lower bound computed at each iteration of the subgradient optimization method. The basic idea of the multiplier adjustment method is to increase the constant term in (LRλ), $-\sum_i\sum_t \lambda_{it} D_{it}$, by decreasing some multipliers further down to negative values which are set to zero in the subgradient optimization method. We describe the multiplier adjustment method in a form of a procedure in the following. This procedure is applied to results from each iteration of the subgradient optimization method.

Procedure 1 (*Multiplier adjustment*)

Step 1. Let $R = \{(i, t) \mid \lambda_{it}=0$ and $D_{it} - W^D_{it} > 0\}$. If R is empty, stop this procedure. Otherwise, go to step 2.

Step 2. Compute

$$\delta^j_{it} = \frac{(r_t C^F_{ij\hat{t}} W^F_{ij\hat{t}} + \lambda_{\hat{i}t} W^F_{ij\hat{t}}) - r_t C^F_{ijt} W^F_{ijt}}{W^F_{ijt}}$$

for all $(i,t) \in R$ and j.

Step 3. Compute

$$\nabla_{it} = \max_j \delta^j_{it} \quad \text{for all} \quad (i,t) \in R$$

for all $(i, t) \in R$.

Step 4. Adjust multipliers

$$\lambda_{it} \leftarrow \lambda_{it} + \max\{\nabla_{it}, -r_t C^{ND}_{it}\}$$

for all $(i, t) \in R$.

Note that coefficients of h_{ijt}, $j=1, 2, \ldots, J$, are changed if λ_{it} (currently zero since $(i, t) \in R$) is decreased to a negative value. When $\lambda_{it} = \delta^j_{it}$, the coefficient of h_{ijt} becomes $r_t C^F_{ij\hat{t}} W^F_{ij\hat{t}} + \lambda_{\hat{i}t} W^F_{ij\hat{t}}$. In addition, is δ^j_{it} negative since $(\lambda_{\hat{i}t} W^F_{ij\hat{t}} + r_t C^F_{ij\hat{t}} W^F_{ij\hat{t}}) < r_t C^F_{ijt} W^F_{ijt}$. Therefore, coefficients of h_{ijt}s become greater than or equal to those of $h_{ij\hat{t}}$s when $\lambda_{it} = \max_j \delta^j_{it}$. Consequently, M_{jt} does not change since all $\hat{t}$s remain the same, if λ_{it}s for all $(i, t) \in R$ are changed from zero to ∇_{it}. Similarly, L_{it} does not change even though λ_{it} is decreased to $-r_t C^{ND}_{it}$ since the coefficient of e_{it} in the objective function, $\lambda_{it} W_{it}^{ND} + r_t C^{ND}_{it} W_{it}^{ND}$, becomes zero. Therefore, the optimal value of (SP1) does not change since M_{jt} and L_{it} are not changed by the above multiplier adjustment.

On the other hand, the optimal value of (SP2) may be decreased by change in the multipliers. Since N_{it} may be changed by modifying the multipliers, $\hat{t}_i$ may be changed as well. When $\hat{t}_i$ is changed by a unit, the optimal value of (SP2) can be decreased by up to $\nabla_{it} W^D_{it}$, which is the amount of decrease in N_{it}. However, this decrease can be compensated for by the increase in the objective function of (PL), which is $-\nabla_{it} D_{it}$. Since $D_{it} - W^D_{it} > 0$ for $(i, t) \in R$, the net increase in the lower bound, $\sum_{(i,t)\in R} \nabla_{it}(D_{it} - W^D_{it})$, is always greater than or equal to zero. Therefore, a lower bound obtained with the multiplier adjustment method is always greater than or equal to the bound that can be obtained without it.

Initial Multipliers

Commonly, initial multipliers are set to zero in the subgradient optimization method. However, we suggest another method, in which initial multipliers are set as $\lambda_{it}^0 = \min[0, \max\{(\max_j - r_t C_{ijt}^F), \ -r_t C_{it}^{ND} \ -r_t C_{it}^D\}]$ for all i and t. The basic idea of this initialization is to increase the constant term $(-\sum_i \sum_t \lambda_{it} D_{it})$ as much as possible by decreasing multipliers while fixing both of the optimal values of (SP1) and (SP2) at zero.

Note that $\max_j(-r_t C_{ijt}^F)$ is a value for λ_{it} which makes coefficients of h_{ijt}'s in the objective function of (SP1) geater than or equal to zero. Similarly, $-r_t C_{it}^{ND}$ and $-r_t C_{it}^D$ are values for λ_{it} that make coefficients of e_{it} and s_{it} equal to zero in the objective functions of (SP1) and (SP2), respectively. Consequently, all coefficients in the objective functions of (SP1) and (SP2) become greater than or equal to zero when initial multipliers are set with this method.

4. Lagrangian Heuristic

In this paper, we develop a Lagrangian heuristic in which a linear programming based procedure is used for finding good feasible solutions. A basic idea of the procedure is to purchase FMMs and new dedicated machines for which the corresponding variables, y_{jt}'s and v_{it}'s became positive frequently (or those that were frequently selected) during the subgradient optimization method. Based on the frequency of being selected, priorities of FMMs and new dedicated machines are determined in each period. That is, priorities of FMMs and new dedicated machines, denoted by P_{jt} and Q_{it} are set zero initially, and then increased by one if y_{jt}^* and v_{it}^* became positive after each iteration of the subgradient optimization method.

In the subgradient optimization method, an upper and a lower bound values are commonly set to a large number and zero, respectively. However, we compute an initial upper and a lower bound values to obtain good initial step size (ω_0) that may help to improve convergence of the subgradient optimization method. The initial lower bound is set to the maximum value of two lower bounds, i.e., the optimal value of a linear program obtained by relaxing integrality constraints of y_{it}, v_{it} and z_{it} in (P), and the lower bound obtained from the initial Lagrangian multipliers. On the other hand, the initial upper bound is obtained using the procedure (Procedure A) given in the appendix with priorities determined as

$$P_{jt} = \sum_i W_{ijt}^F / |I| r_t A_{jt}^F \quad \text{for all } j,t,$$

$$Q_{it} = W_{it}^{ND} / r_t A_{it}^{ND} \quad \text{for all } i,t.$$

The following procedure describes a Lagrangian heuristic developed for the capacity planning problem considered in this study. The heuristic can generate several alternative feasible solutions to obtain a good feasible solution by setting two parameters (I_1 and I_2).

Procedure 2 (*Lagrangian heuristic*)

Step 0. Set $N = 0$, $P_{jt}=0$ and $Q_{it}=0$ for all i, j, t and initialize Lagrangian multipliers using the method suggested in this paper and compute initial lower and upper bounds.

Step 1. Solve the subproblems, (SP1) and (SP2) using a DP algorithm and proposition 3. Let $P_{jt} \leftarrow P_{jt}+1$ and $Q_{it} \leftarrow Q_{it}+1$ and for all (j, t) and (i, t) pairs such that $y_{it}^* > 0$ and $v_{it}^* > 0$. If the solutions are feasible to (P), terminate. The current solution is optimal. Otherwise, go to Step 2.

Step 2. If N is a multiple of I_1, go to Step 3. Otherwise, modify Lagrangian multipliers using the subgradient optimization method. Apply Procedure 1 and let $N \leftarrow N+1$. Go to Step 1.

Step 3. Find a feasible solution using Procedure A. Update the incumbent solution if a better solution is found. Reset $P_{jt}=0$ and $Q_{it}=0$ for all i, j, and t. If N is greater than a predetermined maximum iteration count (I_2), terminate. Otherwise, modify Lagrangian multipliers using the subgradient optimization method. Apply Procedure 1 and let $N \leftarrow N+1$. Go to Step 1.

In the suggested algorithm, two parameters, I_1 and I_2, are set to 200 and 5000, respectively. Therefore, 26 feasible solutions (including an initial feasible solution) are generated to find a feasible solution. Although the chance to find a better feasible solution may be increased as the number of alternative feasible solutions to be generated is increased, those values for I_1 and I_2 were selected considering computational burden and solution quality of the Lagrangian heuristic. Note that the same feasible solutions may be generated from two consecutive values of N, when N is large (since Lagrangian multipliers may converge).

5. Computational Results

To evaluate the performance of the

heuristic algorithm, computatuional tests were done on 135 test problems, five problems for each of all combinations of three levels for the number of operation types (12, 16, 20), three levels for the number of FMM types (8, 12, 16) and three levels for the planning horizon (5, 10, 15). Other data were generated from probability distributions with parameters that are summarized in Table 1, in which DU(*a*, *b*) and U(*a*, *b*) denote the discrete uniform distribution with range [*a*, *b*], and the uniform distribution with range (*a*, *b*), respectively. For computational convenience, the discount factor is not considered (r_t is set to 1 for all *t*) in the test problems, but obviously, the difficulty of the problem remains the same with $r_t \neq 1$. The heuristic algorithm was coded on C++ and a subroutine of CPLEX 4.0 was used to solve linear programs in Procedure A. Tests were done on a personal computer with a Pentium II processor operating at 266MHz clock speed.

Test results are given in Table 2, which show the average, minimum and maximum duality gaps of the heuristic solutions and CPU times for each problem set. The (percentage) gap represents the percentage deviation of the heuristic solution value from the best lower bound obtained from the Lagrangian relaxation. The overall average gap was 3.98% (the gap was 7.15% in the

〈Table 1〉 Parameters used for problem generation

Data	Probability distributions
Acquisition costs of an FMM (A^F_{jt})	DU(10, 20)
Operation cost of an FMM (C^F_{ijt})	U(4×10^{-5}, 6×10^{-5})
Capacity of an FMM (W^F_{ijt})	U(2.5×10^5, 3.5×10^5)
Acquisition costs of a new dedicated machine (A^{ND}_{it})	DU(5, 15)
Operation cost of a new dedicated machine (C^{ND}_{it})	U(3×10^{-5}, 5×10^{-5})
Capacity of a new dedicated machine (W^{ND}_{it})	U(2×10^5, 3×10^5)
Operation cost of an old dedicated machine (C^D_{it})	U(5×10^{-5}, 7×10^{-5})
Capacity of an old dedicated machine (W^D_{it})	U(1.4×10^5, 2.6×10^5)
Demands (D_{it})	U(1.5×10^5, 2.5×10^5)
Budget (*B*)	100 (constant)

〈Table 2〉 Results of the computational experiments

			Duality Gap (%) ‡			CPU Time (seconds)		
$I^{\dagger}$	J	T	Mean	Minimum	Maximum	Mean	Minimum	Maximum
12	8	5	2.87	1.95	3.51	143.2	131.0	153.0
12	8	10	2.36	1.83	2.90	488.4	459.0	524.0
12	8	15	2.32	1.45	3.02	933.0	701.0	1058.0
12	12	5	2.47	1.95	3.20	200.6	176.0	218.0
12	12	10	1.94	1.25	2.58	579.4	540.0	600.0
12	12	15	2.28	1.38	3.43	1082.2	990.0	1152.0
12	16	5	3.34	2.38	4.87	218.8	171.0	255.0
12	16	10	3.31	2.72	4.11	576.4	476.0	644.0
12	16	15	2.59	1.74	3.60	1350.4	1263.0	1447.0
16	8	5	4.34	3.20	5.49	248.0	227.0	264.0
16	8	10	5.02	3.43	6.31	666.8	631.0	688.0
16	8	15	4.84	4.11	5.57	1371.4	1304.0	1448.0
16	12	5	3.75	3.28	4.85	295.0	275.0	305.0
16	12	10	4.72	3.14	6.04	790.4	731.0	831.0
16	12	15	4.32	3.63	5.14	1719.6	1610.0	1845.0
16	16	5	3.84	3.12	4.55	330.4	319.0	346.0
16	16	10	5.04	3.99	6.29	868.8	721.0	971.0
16	16	15	4.51	3.13	6.59	2059.6	1976.0	2119.0
20	8	5	3.84	2.20	5.05	316.2	289.0	336.0
20	8	10	5.33	4.28	7.15	814.4	749.0	874.0
20	8	15	5.09	3.90	6.08	1758.6	1527.0	2014.0
20	12	5	4.11	3.65	5.25	307.2	256.0	337.0
20	12	10	5.21	4.27	5.99	1017.2	962.0	1076.0
20	12	15	5.85	4.94	7.10	2198.6	1827.0	2428.0
20	16	5	4.13	3.72	4.58	382.2	311.0	413.0
20	16	10	4.79	3.81	6.03	1164.4	1090.0	1199.0
20	16	15	5.12	4.26	6.00	2688.4	2483.0	2911.0

† I, J and T denote the number of operation types, the number of FMM types and the length of the planning horizon, respectively.

‡ $\text{Duality gap (\%)} = \dfrac{(\textit{Heuristic Solution Value} - \textit{Best Lower Bound})}{\textit{Best Lower Bound}} \times 100$

worst case). The average gap increases as the number of operation types increases. This may be because it is more difficult to find optimal acquisition and replacement plans when the number of operation types is larger. Note that solution qualities of the heuristic are affected by acquisition and replacement plans in the heuristic, since operations assignments are determined optimally by solving linear programs once acquisition and replacement plans are given. Computation times are strongly affected by the number of linear programs solved in Procedure A. As a result, computation times increase as the problem size of the linear programs increases.

To see the quality of the solutions in terms of the deviation from optimal solutions, we generate 30 test problems, using the same method as the one used in generating 135 test problems above. (Optimal solutions could not be obtained for the 135 problems because of the computational complexity.) Results are given in Table 3. The table shows the average values of the percentage gaps, percentage errors, and the number of problems for which the algorithm found optimal solutions. Here, *error* represents the percentage deviation of the heuristic solution value from the optimal value. The optimal solutions were obtained using CPLEX 4.0. As shown in the table, the heuristic algorithm found near-optimal solutions for the small sized test problems. The overall average error was 0.32% with the worst case error of 1.90%. The heuristic algorithm found optimal solutions of 13 test problems among 30 test problems.

Although it is not shown in the tables, the subgradient optimization procedure converged slightly faster with the method suggested in this study for determining initial multipliers. When the initialization method was used, the initial lower bounds obtained from the

〈Table 3〉 Performance of the heuristic algorithm

I	J	T	Duality Gap (%)	Error (%)†	# of optimal solutions found	CPU Time (seconds) Heuristic	CPU Time (seconds) Optimal
8	6	5	3.61	0.87	1	128.0	1.0
8	8	8	2.09	0.60	2	355.0	4.0
8	10	10	1.39	0.01	2	441.4	7.8
10	6	5	3.01	0.08	2	189.4	5.8
10	8	8	1.92	0.30	3	374.8	6.8
10	10	10	1.43	0.04	3	678.2	43.2
Average			2.24	0.32			

† $\text{Error (\%)} = \dfrac{(\textit{Heuristic Solution Value} - \textit{Optimal Solution Value})}{\textit{Optimal Solution Value}} \times 100$

subgradient optimization procedure were approximately 84% of the final lower bounds on average, while they were zero without the method. When the initialization method was not used, 315 iterations, on average, were necessary to obtain the same bounds as those obtained from the initialization method. Although the final lower bounds obtained with and without the initialization method were not much different, the initialization method may be especially useful in reducing computation time for branch and bound algorithms that find lower bounds using the Lagrangian relaxation approach.

6. Conclusion

In this paper, we considered a problem of planning manufacturing capacity by retiring old dedicated machines, and acquiring FMMs and/or new dedicated machines under budget restriction over a finite planning horizon. The problem was formulated as a mixed integer linear program and solved using a Lagrangian relaxation approach. In the approach, the relaxed problem was divided into two independent subproblems. Using several properties that help to solve the subproblems easily, we developed a linear programming based Lagrangian heuristic which uses information of solutions of relaxed problems to find good feasible solutions. A subgradient optimization method with a multiplier adjustment method is employed to obtain better lower bounds. Results of tests on randomly generated problems showed that the heuristic gave good solutions in a reasonable amount of computation time.

The model considered in this research can be considered as a more practical one than those suggested previously in other research since the model considers the budget limitation and the machine flexibility by taking into account multifunctional features of FMMs. However, it is necessary to consider other flexibilities such as routing flexibility resulting from FMMs for the model to become a more practical tool for capacity planning. Also, it is needed to study the impacts of demand uncertainty or nonlinear cost structures on capacity planning decisions.

Appendix. *A procedure for obtaining a feasible solution of* (*P*)

First, we define three linear programs, (LP0), (LP1) and (LP2), that are used in the procedure.

(LP0) Min

$$\sum_j\sum_t r_t A_{jt}^F y_{it} + \sum_i\sum_j\sum_t r_t C_{ijt}^F W_{ijt}^F h_{ijt} +$$
$$\sum_i\sum_t r_t C_{it}^D W_{it}^D s_{it} + \sum_i\sum_t r_t A_{it}^{ND} v_{it} + \sum_i\sum_t r_t C_{it}^{ND} W_{it}^{ND} e_{it}$$

subject to (2), (3), (4), (5), (6), (7), (11), (12), (13) and

$$y_{jt} \geq 0 \qquad \forall j,t \qquad (8')$$

$$v_{it} \geq 0 \qquad \forall i,t \qquad (9')$$

$$0 \le z_{it} \le 1 \qquad \forall i,t \qquad (10')$$

(LP1) Min

$$\sum_j \sum_t r_t A_{jt}^F y_{jt} + \sum_i \sum_j \sum_t r_t C_{ijt}^F W_{ijt}^F h_{ijt} +$$
$$\sum_i \sum_t r_t C_{it}^D W_{it}^D s_{it} + \sum_i \sum_t r_t A_{it}^{ND} v_{it} + \sum_i \sum_t r_t C_{it}^{ND} W_{it}^{ND} e_{it}$$

subject to (2), (3), (4), (5), (6), (7), (8'), (9'), (10'), (11), (12), (13) and $\sum_{(j,t)\in R_1} y_{jt} + \sum_{(i,t)\in R_2} v_{it} = 0$ for $R_1 = \{(j,t) | P_{jt} = 0\}$ and $R_2 = \{(i,t) | Q_{it} = 0\}$

(LP2) Min

$$\sum_i \sum_j \sum_t r_t C_{ijt}^F W_{ijt}^F h_{ijt} + \sum_i \sum_t r_t C_{it}^D W_{it}^D s_{it}$$
$$+ \sum_i \sum_t r_t C_{it}^{ND} W_{it}^{ND} e_{it}$$

subject to (2), (11), (12), (13) and

$$\sum_i h_{ijt} \le \sum_{u=1}^{t} \tilde{y}_{ju} \qquad \forall j,t \qquad (4')$$

$$e_{it} \le \sum_{u=1}^{t} \tilde{v}_{iu} \qquad \forall i,t \qquad (5')$$

$$s_{it} \le \tilde{z}_{it} \qquad \forall i,t \qquad (6')$$

where $\tilde{y}_{jt}$, $\tilde{v}_{it}$ and $\tilde{z}_{it}$ are given values.

Procedure A (*Obtaining a feasible solution*)

Step 1. Solve (LP1), and (LP0) if (LP1) is infeasible. Let $\hat{y}_{jt}$, $\hat{v}_{it}$ and $\hat{z}_{it}$ be an optimal solution of (LP1) or (LP0) if (LP1) is infeasible. Let $\tilde{y}_{jt}$, $\tilde{v}_{it}$ and $\tilde{z}_{it}$ be the smallest integers greater than or equal to $\hat{y}_{jt}$, $\hat{v}_{it}$ and $\hat{z}_{it}$ respectively. If $\tilde{y}_{jt}$, $\tilde{v}_{it}$ and $\tilde{z}_{it}$ satisfy (3) and (7), go to Step 2. Otherwise go to Step 3.

Step 2. Solve (LP2). If (LP2) is feasible, go to Step 5. Otherwise, for m = 0 to T do:

Set $\tilde{z}_{ir}=1$ for all i and for r = 0, 1, ..., m. Solve (LP2) with new $\tilde{z}_{it}$ values. If (LP2) is infeasible, increase m by one and solve (LP2) again with new m and $\tilde{z}_{it}$ values. Go to Step 5 if (LP2) is feasible.

If (LP2) is infeasible when m = T, terminate. (A feasible solution cannot be found with this procedure.)

Step 3. Repeat decreasing the value of one of $\tilde{y}_{jt}$s or $\tilde{v}_{it}$s by one in a nondecreasing order of their priorities, P_{jt} and Q_{it}, until (3) is satisfied. Go to Step 4.

Step 4. For each i, let $\tilde{z}_{it}=1$ for t = 1, ..., t', and $\tilde{z}_{it}=0$ for t = t'+1, ..., T, where $t' = \operatorname{argmax}_t\{ t \mid \tilde{z}_{it} > 0\}$. Go to Step 2.

Step 5. Repeat decreasing the value of one of $\tilde{y}_{jt}$s or $\tilde{v}_{it}$s by one in a nondecreasing order of their priorities until they cannot be decreased any longer. Solve (LP2) with new $\widetilde{y_{jt}}$ and $\widetilde{v_{it}}$, and update $\widetilde{y_{jt}}$, $\widetilde{v_{it}}$ and the incumbent solution whenever a better solution is found.

Step 6. From $n = T$ down to 0 do:

Let $\widetilde{z_{is}}=0$ for all i and for $s = n, n+1, \ldots, T$. Solve (LP2) with new $\widetilde{z_{it}}$ values. If the resulting (LP2) is feasible, update the solution.

Terminate.

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